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FISH AND WILDLIFE SERVICE, Arnie J. Suomela, *Commissioner*

NATURAL MORTALITY RATE OF GEORGES BANK HADDOCK

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Fishery Bulletin 126

From Fishery Bulletin of the Fish and Wildlife Service

VOLUME 58

UNITED STATES GOVERNMENT PRINTING OFFICE • WASHINGTON • 1958

For sale by the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C.
Price 15 cents

ABSTRACT

Data on the catch per day at each age and the annual fishing effort for Georges Bank haddock are available for the period 1932 to 1951. These data provide five estimates of the natural mortality rate. The estimates indicate that sampling for the older ages in the catch has been inadequate for the purpose of computing natural mortalities. Between ages 3 and 4, a negative natural mortality coefficient is obtained, indicating the possibility of incomplete recruitment at age 3.

The data between ages 4 and 6 provide two estimates of natural mortality, both near zero. It is concluded that an upper limit of the natural mortality coefficient of 0.2, corresponding to an annual expectation of death of 15 percent, is sufficiently conservative for predicting the effect of increases in mesh size on yield.

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By Clyde C. Taylor, *Fishery Research Biologist*, Bureau of Commercial Fisheries

When a regulation to increase the mesh size of nets used by trawlers engaged in the Georges Bank haddock fishery was first considered, the importance of an accurate estimate of the rate of natural mortality of haddock was fully realized (Graham 1952). The average annual-total mortality rate over the period 1931 to 1948 was known to be 45 percent, but the proper partition of this rate into fishing and natural mortality was unknown.

To assess the changes resulting from increases in mesh size which, in effect, increase the age at which the fish are first subject to capture by the gear, yield curves were calculated using the following annual expectations of death from natural causes: 0, 7.5, 15.0, 22.5, and 30.0 percent. Experiments with nets had shown that the small-mesh net had a 50 percent selection point for haddock at 25 centimeters, corresponding to an age for haddock of about 1½ years. Yield computations showed that substantial increases in yield could be expected with larger-meshed nets at the lower natural mortality rates and that even if the natural mortality were as high as 30 percent, no decrease in yield would result from an increase in mesh size designed to raise the age of haddock at first capture to 2½ years (Graham 1952).

Before recommending an increase in mesh size to the International Commission for the Northwest Atlantic Fisheries, the Scientific Advisors to Panel 5 of that Commission reviewed the available data bearing on the haddock fishery. It was the considered opinion of that group that the natural mortality rate probably did not exceed 15 percent annually. In the course of these deliberations, it appeared that an objective estimate could not be obtained from the available data. Efforts to apply a method for estimating natural mortality from information on the catch per day and the annual fishing intensity had produced meaningless results.

I have reviewed these early attempts to estimate natural mortality of the Georges Bank haddock. The reason for their failure is apparent. Using the same data and a method of analysis differing in treatment but not in principle, estimates of the magnitude of natural mortality were obtainable and are presented here. This additional information is necessary to refine the criteria establishing the age and size at which haddock should be harvested to obtain the maximum yield.

Calculation of M, Natural Mortality Coefficient

Beverton and Holt (1956, Appendix D) show the derivation of an equation relating the log ratio of the annual mean abundance in pairs of successive years to the fishing intensity:

$$\log_e\left(\frac{{}_vN_x}{{}_{v+1}N_{x+1}}\right)+\log_e\left\{\frac{(cf_x+M)\,(1-e^{-(cf_{x+1}+M)})}{(cf_{x+1}+M)\,(1-e^{-(cf_x+M)})}\right\}=cf_x+M \quad (1)$$

where:

- ${}_vN_x$ = mean abundance of age group v in year x,
- ${}_{v+1}N_{x+1}$ = mean abundance a year later,
- f_x = fishing intensity in year x,
- f_{x+1} = fishing intensity in year x+1,
- c = constant relating fishing intensity to the instantaneous fishing mortality rate, and
- M = natural mortality coefficient.[1]

The magnitude of the second term on the left side of equation (1) departs from zero with the magnitude of the change in fishing intensity between each year in a pair. Thus, given a series of pairs of years with fishing intensities and corresponding indices of abundance of the age groups, a linear relation between fishing intensity and the total mortality coefficient should be obtained. The slope of the best fitting regression line provides an estimate of c, and the y-intercept an estimate of M.

We note from (1) that a direct solution for c and M cannot be found because of the second

[1] M is an instantaneous rate. The annual natural mortality rate is equal to $1-e^{-M}$. If M=0.1, for example, the annual natural mortality is $1-e^{-0.1}$, or 0.0952, or 9.52 percent.

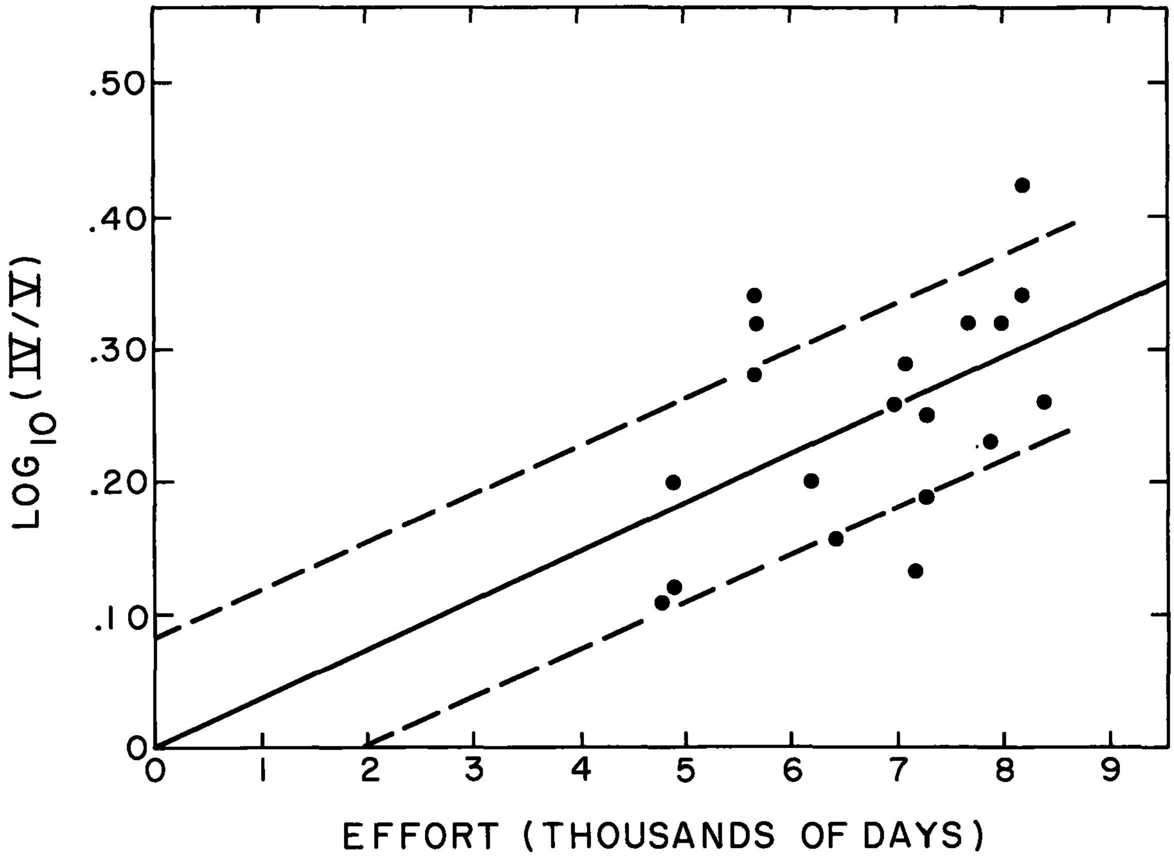

FIGURE 1.—Regression of the log-ratio of abundance, ages 4 and 5, on effort.

term on the left side of the equation, which also contains c and M. We may, however, solve for these values by an iterative process, first considering the second term to be zero and computing the line of best fit for the logs of the abundance ratios for each pair of years plotted against the effort in the first year of each pair (fig. 1). The estimates of c and M thus obtained are used to evaluate the second term of equation (1) to obtain more exact estimates. The procedure is repeated until successive estimates of c and M do not differ significantly. An illustrative example of the computational procedure is shown in Appendix A, p. 6.

Table 1 shows the catch per day in numbers of Georges Bank haddock, ages 3 to 8, for the years 1932 to 1951, together with the fishing intensity expressed as thousands of days fished within the Georges Bank area. The effort is a "calculated" effort based on the effort, catch, and catch per day of a standard study group of large otter trawlers fishing on Georges Bank. The total effort within the Georges Bank area is estimated by dividing the total catch from the area by the catch per day of the standard study group.

From the data of table 1, we may attempt five estimates of the natural mortality rate (for age groups III and IV, IV and V, etc.). The values of c and M obtained by applying equation (1) to table 1 are summarized in table 2 together with the standard error of the regression line, (S_c), the standard error of M, (S_m), the standard error of estimate, ($S_{y.x}$) and the confidence limits. A plot of the data for ages 4 and 5 is shown in figure 1. Note that the log ratio of abundance is plotted to base 10 and not base e. The value of the slope and intercept are multiplied by 2.3026 to satisfy the requirements of equation (1).

DISCUSSION

Examination of the 1951 worksheets used to compute natural mortality shows that the attempt was abandoned when a negative value of c, the slope of the regression line, was obtained. Fishing mortality is related to fishing intensity as follows:

$$F = cf \qquad (2)$$

where F is the instantaneous fishing mortality rate. A negative value of c means that increases in fishing result in decreases in fishing mortality. This meaningless result might come about from various errors or bias in the data, such as inadequate sampling resulting in unreliable age composition data, bias in the effort data, variations in the availability of fish from year to year, or great variability in M. One might also suspect that variations in the fishing intensity, f, during the period covered by the data had not been sufficiently large to cause measurable variations in the second term of equation (1).

On examining the computations, I find that the procedure adopted was to calculate the log-ratio of the catch per day of ages 3 to 9+ in one year to the catch per day of ages 4 to 9+ in the following year.[2] This procedure is theoretically correct if the following assumptions can be safely made: (1) the fish are fully recruited to the fishery at age 3, (2) the natural mortality rate is constant over the range of ages, and (3) the sampling and age composition data are equally reliable throughout the age range.

Table 2 sheds some light on the validity of the foregoing assumptions. Between ages 3 and 4 the natural mortality estimate has a high negative value. This result raises some doubt that we are safe in concluding that recruitment is complete at age 3. Possibly the losses through natural mortality between ages 3 and 4 may be more than compensated by additional recruitment at age 4.

It has been generally assumed in the work with haddock that 3-year olds were fully available and therefore mortality rates have been computed onwards from this age. Indeed, we find that if we compute the average mortality rate from age 4, the total mortality coefficient does not differ significantly from that calculated from age 3. Earlier estimates of total mortality are not, then, invalidated. Nevertheless, the data of table 1 suggest that the mortality rate from age 3 to 4 is somewhat less than from age 4 to 5, as we might expect if recruitment is not entirely complete at age 3.

Before considering assumption (2), we need to examine assumption (3), that the data are equally reliable throughout the age range. In evaluating the reliability of these results, we find that on the average over the period in question, 87.9 percent by numbers of a year class are taken during their first 5 years in the fishery; that 6.1 percent consist of 6-year old fish; 3.0 percent are 7-year olds; and only 1.3 percent are 8-year olds. The numbers of these ages taken in the samples from the catch are of corresponding relative magnitude, so that age composition of the larger fish is determined from considerably smaller samples than are available for determining the proportions of ages among the smaller fish. Thus, while the number of ratios used in computing the regression of equation (1) may be the same, the reliability of the ratios themselves differ.

A further consideration affecting estimates derived from the older ages is evidence that, during certain periods of the year, the older fish tend to be found in deeper water not so heavily fished (Colton 1955).

Between ages 6 and 7, we obtain on the first iteration, from the data of table 1, a negative value of c (−.1264) and a high positive value of M (1.575). Again, between ages 7 and 8, we obtain a high value of M (0.491) but this time the slope of the regression line is positive ($c=0.0657$). Since these estimates of natural mortality are as high or considerably higher than observed total mortalities in some year classes during their lifetime in the fishery, we conclude that the data for haddock older than 6 years are not sufficiently accurate for use in estimating the magnitude of natural mortality.

Since we must question the reliability of data from age 6 onwards which are based on relatively smaller numbers of fish than for the younger ages, we are not able to decide whether the natural mortality rate is constant from ages 3 to 9 (assumption 2).

We see, then, that the three assumptions necessary for making a gross comparison of ages 3 to 9+ and 4 to 9+ are either invalid or in doubt. It is not, therefore, surprising that the attempt to use this procedure met with failure.

Considering now the estimates of natural mortality derived from the data for ages 4 and 5,

[2] Age determinations are not made for haddock older than 9 years. All fish 9 years or older are grouped as 9+ and comprise less than 1 percent by numbers of the landings.

and 5 and 6, we obtain two estimates of M which do not differ significantly (table 2). These estimates are close to zero but the confidence limits indicate the estimates are not precise.

In estimating M, which is the value of the Y-intercept of the regression line, one is extrapolating considerably beyond the range of the data. In these circumstances it is statistically sound to use an estimate of the error which depends both on the standard error, and the standard error of the slope of the regression line (Snedecor 1946, p. 120). Interpreted biologically and in the light of what is known about the Georges Bank haddock fishery, these confidence limits merely indicate the precision of the data and not the probability of the natural mortality actually exceeding these limits.

The average total mortality rate of Georges Bank haddock over the period 1931 to 1948 was 0.6 (45 percent annually) (Graham 1952). The upper 95 percent confidence limits for the estimates between ages 4 and 6 are close to this value (table 2). More than 90 million pounds of haddock, on the average, were annually removed from the fishery between 1931 and 1950. Information on the relative abundance of ages made possible accurate predictions of the catch (e. g., Schuck, 1952). The probability of the natural mortality being equal to the total mortality is far more remote than the confidence limits indicate.

Assessment of the effect of mesh regulation on the haddock fishery is an operational problem requiring a likely working value of M. Examining table 2, we note that the values of c, the slope of the regression of mortality on effort, are quite consistent from ages 3 to 6, the ages for which the abundance ratios are most accurately determined. This consistency suggests that the standard error of the regression line, $S_{y.x}$, indicates the likely limits of M. On this basis, we note from table 2 that there is one chance in three that the natural mortality coefficient exceeds 0.196 for ages 4 and 5, and only five chances out of a hundred that it exceeds 0.380. For ages 5 and 6, we obtain similar estimates at somewhat lower levels of confidence.

In view of the rather low estimates of M obtained from the abundance and effort data, it is concluded that a working value of about 0.2 is sufficiently high for a conservative prediction of the effect of mesh regulation but that models must be explored with values of M ranging up to 0.4.

On the basis of a 45 percent total-annual mortality, a natural mortality coefficient of 0.2 is equivalent to an annual expectation of death from natural causes of 15 percent, while a coefficient of 0.4 is equivalent to 30 percent.

SUMMARY

A review of an early and unsuccessful attempt to establish a natural mortality estimate for Georges Bank haddock shows that while the method applied was correct in principle, it failed to produce a meaningful result because three basic assumptions were either not fulfilled or were in doubt. These assumptions are: (1) haddock are fully recruited at age 3, (2) the natural mortality rate is constant over the range of ages, and (3) the sampling and age-composition data are equally reliable throughout the age range.

Using the same data and method, but treating the age groups individually rather than grouped, five estimates of the natural mortality coefficient are obtained. These estimates indicate that sampling for the older ages in the catch has been inadequate for the present purpose. Between ages 3 and 4, a negative natural mortality coefficient is obtained, indicating the possibility of incomplete recruitment at age 3. The data between ages 4 and 6 provide two estimates of natural mortality, both near zero.

The standard error of the regression line indicates that it is unlikely the natural mortality coefficient exceeds a value of about 0.3. Although a value of 0.2, which is equivalent to an annual mortality of 15 percent from natural causes, is sufficiently conservative for analytical purposes at the present time, other yield models with values of M ranging up to 0.4 must be explored.

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TABLE 1.—*Statistics of the Georges Bank Haddock Fishery* [1]

Year	Catch per day at Age—						
	III	IV	V	VI	VII	VIII	Effort
1932	2,849						9.11
1933	722	1,144					8.41
1934	1,073	711	683				4.84
1935	1,284	583	520	438			6.45
1936	1,796	931	408	244	231		6.22
1937	1,309	704	542	256	121	65	8.19
1938	966	482	267	207	120	44	7.87
1939	2,378	654	282	126	114	43	8.02
1940	1,658	1,021	312	186	94	28	7.22
1941	1,280	1,046	752	231	122	38	7.33
1942	2,566	1,038	624	363	158	34	5.73
1943	3,608	1,591	536	504	155	64	4.88
1944	1,414	2,610	949	416	96	90	5.66
1945	419	1,246	1,219	486	193	62	4.89
1946	1,997	398	854	562	218	48	7.28
1947	1,190	862	250	314	179	88	8.22
1948	2,406	704	400	174	120	68	7.71
1949	1,669	1,307	330	149	88	56	7.14
1950	708	831	680	240	114	53	5.72
1951	3,886	344	416	374	133	52	6.49

[1] Catch per day is expressed in numbers and effort in thousands of days.

TABLE 2.—*Estimated values of c and M, together with errors and confidence limits*

Ages	c	S_C	M	S_M	95 percent confidence limits of M	$S_{y.x}$	Upper confidence limits of M based on $S_{y.x}$	
							67 percent	95 percent
III to IV	0.1022	±0.021	−0.170	±0.341	−0.886 to 0.546	±0.272	0.102	0.374
IV to V	.0845	±.016	.012	±.240	−.495 to .514	±.184	.196	.380
V to VI	.0823	±.022	.004	±.337	−.675 to .519	±.239	.243	.482
VI to VII	*−.1264		*1.575					
VII to VIII	.0657	±.029	.491	±.457	−.489 to 1.472	±.292	.783	1.075

* Based on first iteration only.

APPENDIX A

COMPUTATIONAL PROCEDURE FOR SOLVING EQUATION (1) FOR c AND M

The procedure for solving equation (1), page 1 may be outlined as follows (Beverton 1954):

1. Let $f_x = f_{x+1}$ in the second term on the left hand side of equation (1). This reduces the term to zero.

2. Plot the logs of the abundance ratios for each pair of years against the effort in the first year of each pair. Compute the regression coefficients, which are the first estimates of c and M. Call them c_1 and M_1.

3. Replot the data, computing a value of the log correction term, using c_1 and M_1 for each pair of years, and adding it to the log ratios previously plotted. Again compute regression coefficients, calling these more exact values c_2 and M_2.

4. Do a third iteration using c_2 and M_2 to estimate c_3 and M_3. Repeat the procedure until successive iterations do not differ significantly. These are the best estimates of c and M.

Work sheet for first iteration

Year class	A_x $\mathrm{III}^{N}x$	B_x $\mathrm{IV}^{N}x+1$	C_x A_x/B_x	D_x $\mathrm{Log}_{10}C_x$	E_x effort
1929	2849	1144	2. 49	0. 396	9. 11
1930	722	711	1. 02	. 009	8. 41
1931	1073	583	1. 84	. 265	4. 84
1932	1284	931	1. 38	. 140	6. 45
1933	1796	704	2. 55	. 406	6. 22
1934	1309	482	2. 72	. 434	8. 19
1935	966	654	1. 48	. 170	7. 87
1936	2378	1021	2. 33	. 367	8. 02
1937	1658	1046	1. 58	. 199	7. 22
1938	1280	1038	1. 23	. 090	7. 33
1939	2566	1591	1. 61	. 207	5. 73
1940	3608	2610	1. 38	. 140	4. 88
1941	1414	1246	1. 13	. 053	5. 66
1942	419	398	1. 05	. 021	4. 89
1943	1997	862	2. 32	. 365	7. 28
1944	1190	704	1. 69	. 228	8. 22
1945	2406	1307	1. 84	. 265	7. 71
1946	1669	831	2. 01	. 303	7. 14
1947	708	344	2. 06	. 314	5. 72
1948	(3886)				(6. 49)

$$\text{Column } D_x = \log_{10}\left(\frac{v^{N}x}{v+1^{N}x+1}\right) = \log_{10}\left(\frac{\mathrm{III}^{N}x}{\mathrm{IV}^{N}x+1}\right) = \log_{10} 2.49 = 0.396$$

$N = 19$ $\qquad \Sigma E^2_x = 932.9313$

$\Sigma D_x = 4.372$ $\qquad \Sigma D_x E_x = 31.13234$

$\Sigma E_x = 130.89$

	Base 10	*Base e*
c_1	0. 0325	0. 0748
M_1	. 0065	. 015

Work sheet for second iteration

Year class	D_x	E_x	F_x E_x (.0748)	G_x $F_x+.0150$	H_x $1-e^{-G_x}$	I_x $\frac{G_x H_{x+1}}{G_{x+1} H_x}$	J_x $\log_{10} I_x$	K_x J_x+D_x
1929	0. 396	9. 11	0. 681	0. 696	0. 5014	1. 02	0. 009	0. 405
1930	. 009	8. 41	. 629	. 644	. 4748	1. 13	. 053	. 062
1931	. 265	4. 84	. 362	. 377	. 3141	. 95	—. 023	. 242
1932	. 140	6. 45	. 482	. 497	. 3917	1. 00	000	. 140
1933	. 406	6. 22	. 465	. 480	. 3812	. 94	—. 027	. 379
1934	. 434	8. 19	. 613	. 628	. 4664	1. 01	. 004	. 438
1935	. 170	7. 87	. 589	. 604	. 4534	. 99	—. 004	. 166
1936	. 367	8. 02	. 600	. 615	. 4593	1. 03	. 013	. 380
1937	. 199	7. 22	. 540	. 555	. 4260	1. 00	000	. 199
1938	. 090	7. 33	. 548	. 563	. 4305	1. 06	. 025	. 115
1939	. 207	5. 73	. 429	. 444	. 3586	1. 03	. 013	. 220
1940	. 140	4. 88	. 365	. 380	. 3161	. 98	—. 009	. 131
1941	. 053	5. 66	. 423	. 438	. 3547	1. 03	. 013	. 066
1942	. 021	4. 89	. 366	. 381	. 3168	. 92	—. 036	—. 015
1943	. 365	7. 28	. 544	. 559	. 4283	. 97	—. 013	. 352
1944	. 228	8. 22	. 615	. 630	. 4674	1. 01	. 004	. 232
1945	. 265	7. 71	. 577	. 592	. 4468	1. 02	. 009	. 274
1946	. 303	7. 14	. 534	. 549	. 4225	1. 05	. 021	. 324
1947	. 314	5. 72	. 428	. 443	. 3579	. 98	—. 009	. 305
1948	----------	(6. 49)	. 485	. 500	. 3944	----------	----------	----------

Column $F_x=c_1 f_x$
Column $G_x=c_1 f_x+M_1$
Column $J_x=\log_{10}\left\{\frac{c_1 f_x+M}{c_1 f_{x+1}}+M\left[\frac{1-e^{-(c_1 f_{x+1}+M)}}{1-e^{-(c_1 f_x+M)}}\right]\right\}$

$N=19$ $\quad \sum E_x^2=932.9313$
$\sum E_x=130.89$ $\quad \sum E_x K_x=31.70379$
$\sum K_x=4.415$

	Base 10	Base e
c_2	.0413	.0951
M_2	—.0525	—.121

The fourth iteration yields the following values for c_4 and M_4:

	Base 10	Base e
c_4	0. 0444	0. 1022
M_4	—0. 0740	—0. 170

U. S. GOVERNMENT PRINTING OFFICE: 1958 O—434981