DEVELOPMENT OF A MATHEMATICAL RELATIONSHIP BETWEEN ELECTRIC-FIELD PARAMETERS AND THE ELECTRICAL CHARACTERISTICS OF FISH

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ABSTRACT

A few of the electrical characteristics of fish were defined and studied as the criteria for producing a specific, observed reaction. Voltages and currents applied to a small test cell containing a live fish were measured. With the use of an electrical analogue of the cell contents, these data were reduced to the equivalent parameters applied to a mathematical model representing the fish. The resulting values of resistivity and power density computed for this model were then used to describe the corresponding fish characteristics. Application of pulsed voltages to the cell demonstrated a minimum power density below which no reaction was observed. Equations were then derived to relate this value to corresponding electrical conditions that would exist at an electrical guiding installation.

The use of electric fields is presently being investigated as a possible method of diverting fish for safe passage around hydroelectric dams, thus aiding in the perservation of the salmon fisheries.[1]

A major problem of this investigation is the evaluation of an electric field as a motivating stimulus. Prior to the research reported upon here, few, if any, methods have been developed to observe or determine accurately those portions of a field pattern which motivate the behavior of a fish. Therefore, the optimum electrical parameters of a particular, effective guiding installation may not apply at some other site. In other words, so little is known about how an electrical field stimulates a fish that it is difficult to design a successful electrical-guiding device. Extensive and costly experimentation has been necessary at each new installation to obtain the necessary parameters for an effective electrical-guiding device.

An investigation of this problem indicates that an adequate solution would include a mathematical method of relating electric-field parameters to the electrical characteristics of the fish. These characteristics control the response of the fish to electricity and thus define for the fish specific regions in an electrical field. However, because of the complexity of the biological structure involved, a precise description of these characteristics is impossible. The mathematical process therefore, must utilize an electrical analogue to facilitate such a description.

The research presented here has three objectives: (1) To develop a mathematical model to serve as the electrical analogue of a fish, (2) to find an evaluation procedure which will determine the electrical properties and processes within the model, and (3) to develop a method to relate these model characteristics to the parameters of the electric fields to be used in proposed fish-guiding devices.

MATERIALS AND METHODS

The experiments were conducted at the Bureau of Commercial Fisheries Biological Laboratory,

NOTE.—Approved for publication April 17, 1962.

[1] Mason, James E., and Rea Duncan. Manuscript in preparation. The development and appraisal of methods of diverting fingerling salmon with electricity at Lake Tapps. Bureau of Commercial Fisheries Biological Laboratory, Seattle, Wash.

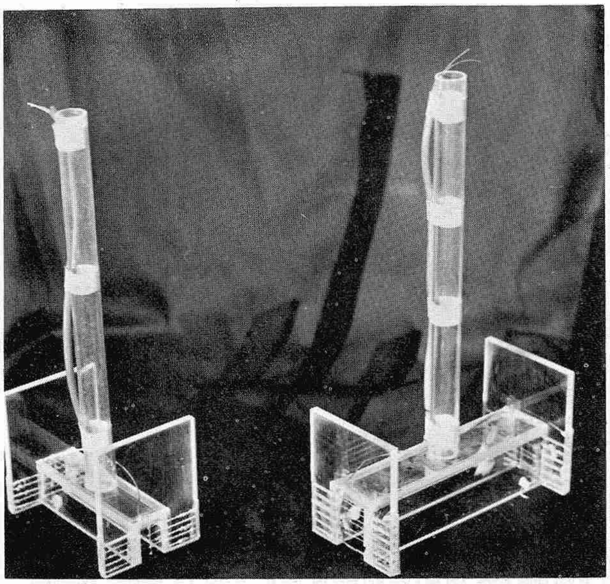

FIGURE 1.—Plastic cells used to contain fish while testing.

Seattle, Wash., from November 27, 1958, to January 9, 1959. The fish used were 0 age-group sockeye salmon (*Oncorhynchus nerka*). These fish were obtained from the Winthrop National Fish Hatchery in September, 1958, and were kept in the laboratory holding facilities until their transfer to the experimental area.

The general method of laboratory investigation involved measuring voltages and currents applied to a small, plastic test cell (fig. 1) containing a live fish. Measurements were carried out with the fish facing either the positive or the negative electrode. These parameters were measured coincident with a specific observable reaction of the fish. Measurements of applied voltage were made from electrodes positioned within, and at opposite ends of, the cell. These electrodes were designed so the electrical resistance between them would be completely controlled by the contents of the cell. It was also necessary to reduce the volume of the surrounding body of water to a minimum in order to obtain measurements which were analogous to functions of the electrical characteristics of a fish. Reactions produced in the fish were created by applying the necessary voltage across the cell.

The cell voltages used in the calculations were the lowest applied values to which the fish appeared to respond. This response was selected because it could be easily observed and because the stimulus threshold was well defined. When the applied voltage reached the threshold value, a violent swimming motion was observed. This reaction could be considered equivalent to the muscle response caused by the rheobase potential (Mitchell, 1948), and would correspond to the threshold reaction reported in later publications (Fisher, 1950; Cattley, 1955). Since this observed threshold phenomenon is a property of the biological structure of the fish, any corresponding electric-field gradient should produce the same effect. Therefore, this response defined a characteristic of the fish that could be related to field-pattern parameters.

HOLDING FACILITIES

Holding facilities were set up adjacent to the experimental area in order to provide ease of operation and to minimize any change in water conditions between the holding troughs and the cell. Fish were held here at least 4 hours prior to testing.

Equipment for this installation included three wooden troughs, 22 cm. wide by 24 cm. deep by 196 cm. long, and a 1042-liter wooden tank. Dechlorinated city water was supplied to the tank through a float valve. From the tank, water was pumped to a head trough which in turn supplied the three holding troughs. Approximately 70 percent of the flow from the holding troughs was diverted back into the main tank, and 30 percent was drawn off as waste. A thermostatically controlled refrigeration unit kept the water temperature at approximately 50° F.

APPARATUS

The major portion of the experimental apparatus was made up of the plastic tank and test cells and the electronic measuring and testing equipment. The plexiglass tank had inside dimensions of 25 cm. by 25 cm. by 38 cm. and had one fixed partition and one adjustable partition. A test cell (fig. 2) was placed between these partitions, and the adjustable partition was positioned and clamped in such a manner as to hold the cell tightly between the two partitions. Electrodes made of duralumin were placed at both ends of the tank to permit the application of the test currents.

Two cells of different size were used in order to accommodate the range of fish sizes tested. Cell number one had inside dimensions of 2.54 cm. wide, 3.81 cm. deep, and 15.88 cm. long; whereas

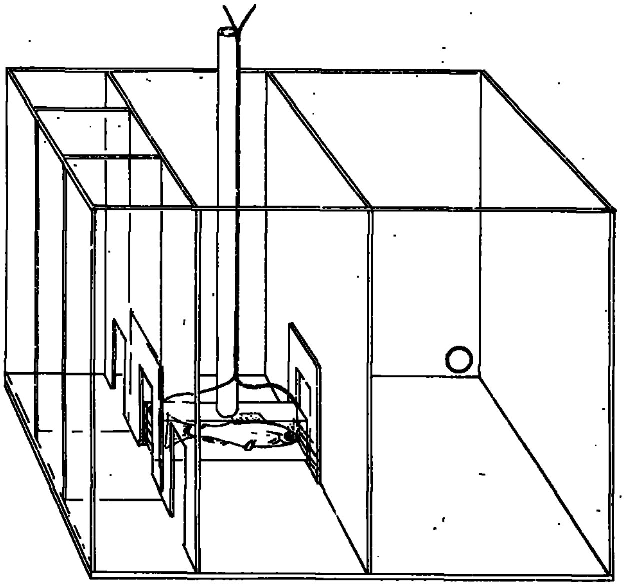

FIGURE 2.—A test cell in position within the experimental tank.

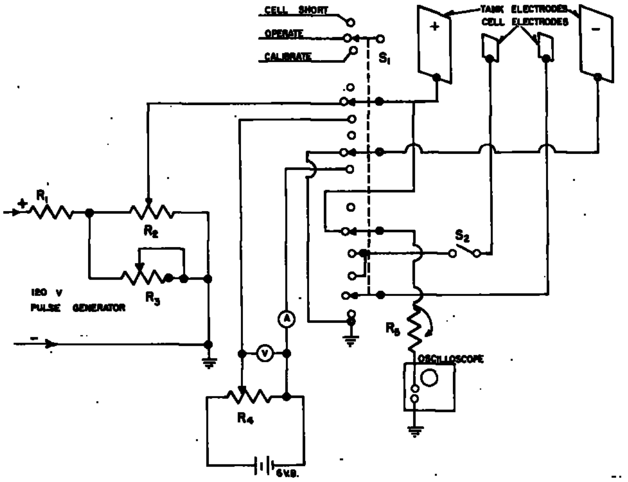

FIGURE 3.—Circuit diagram of the complete testing unit.

cell number two was 1.59 cm. wide, 2.54 cm. deep, and 11.43 cm. long. Each cell was equipped with nylon restraining threads stretched horizontally across both ends. The quanity of restraining material was kept at a minimum by using only six threads per end for cell number one and three threads per end for cell number two. Monotoring electrodes were placed in each cell opening to measure the voltage drop across the cell. The leads for these electrodes were routed through the cell wall and up the handle to the external circuit.

Three types of monitoring electrodes were used. The first arrangement consisted of a ¼-inch wide, stainless steel strip, cemented to the top wall, just inside each cell opening. The second type was a ¼-inch mesh screen which covered the two ends of the cell, and the third type consisted of a single wire stretched horizontally across the cell openings. Experimentation with all three types revealed that the fish were not sensitive to the proximity of a conducting plane. This information was important for consideration in the design of future cells.

Water was obtained from the holding facilities and was gravity fed to the tank from a 14-gallon reservoir. The rate of flow through the test cell was kept at approximately 2 gallons per hour. Water resistivity was controlled by the introduction of NaCl.

The electronic circuitry enabled a square wave pulse of the desired amplitude and duration to be applied to the cell (fig. 3). Electronic switching equipment, controlled by the investigator (described in detail by Volz (1962)) connected the output of a d.c. (direct current) generator to the test circuit at preset intervals and for preset durations. The voltage applied to the circuit was essentially a square wave pulse with an amplitude of 120 volts and a duration of 50 milliseconds.

The circuit functions as follows: When switch S_1 is in the operate position, the functioning circuit is the operate circuit (fig. 4a). In this case, resistors R_1, R_2, and R_3 combine to form an adjust-

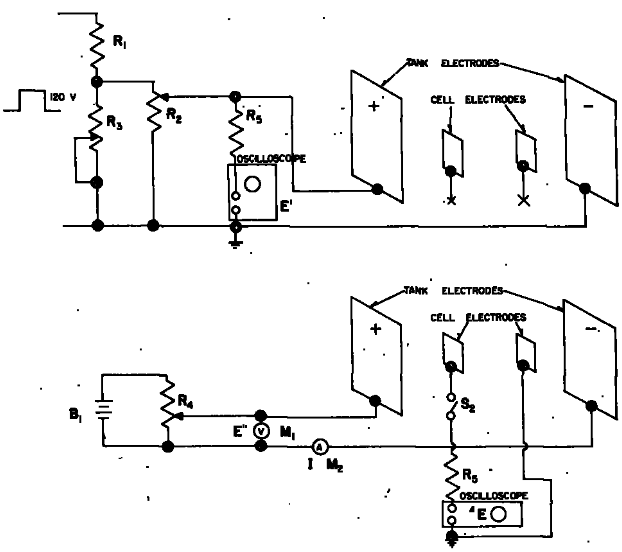

FIGURE 4.—Circuit diagram of the testing unit: (above) operate configuration; (below) calibrate configuration.

able voltage divider that reduces the 120-volt input pulse to the required voltage applied to the tank. This voltage is applied to the tank electrodes and is monitored on the oscilloscope by means of the monitoring electrodes. Resistor R_5 is used to change scale readings on the oscilloscope by one-half, when necessary, in order to achieve the maximum possible accuracy.

When switch S_1 is placed in the calibrate position, the functioning circuit is the calibrate circuit (fig. 4b). In this case battery B and resistor R, supply an adjustable d.c. voltage to the tank electrodes. The voltage drop across the cell is now measured on the oscilloscope. Switch S_2 prevents current from flowing in the monitoring circuit until the operator is ready to record the reading. This operation is necessary in order to prevent the electrodes from becoming polarized and giving erroneous readings. After the readings have been taken, depolarization is accomplished by closing switch S_2 and placing switch S_1 in the "cell short" position. With the switch in this position, the cell electrodes are shorted, thus returning the circuit to equilibrium.

MEASUREMENT PROCEDURE

After preliminary experimentation, a specific measurement procedure was adopted. The fish to be tested was taken from the holding trough and was placed in the plexiglass tank. The test cell was then lowered over the fish and positioned so that the openings at the ends of the cell coincided with the openings in the partitions. The resistivity of the water entering the test cell was then measured with a standard resistivity bridge.

After the fish was allowed a two-minute rest, switch S_1 was placed in the operate position, and the electronic pulsing device was triggered. Pulses with a duration of 50 milliseconds were then automatically applied at intervals of 20 seconds. Between each pulse application, the potentiometer (R_2) was manually rotated to increase the pulse amplitude. When the reaction level was reached, the fish jumped violently, and the operator, observing this reaction, recorded the pulse voltage from the oscilloscope.

The battery voltage (B_1) was used to establish the relationship between the pulse voltage across the tank and the actual voltage across the cell. The desired level of current was set, and the voltage was recorded from the voltmeter (M_1). With switch S_2 closed, switch S_1 was moved to the cell short position and the cell electrodes were shorted for 30 seconds. With switch S_2 open, switch S_1 was turned to the calibrate position. Switch S_2 was then closed, and the voltage was immediately recorded from the oscilloscope. Three levels of current were used for each fish tested, and measurements were repeated at least twice at each level to insure accuracy.

After being tested, each fish was lifted from the cell in a net, and after the excess water, clinging to the fish, was eliminated, the fish was placed in a graduated cylinder partially filled with water. The volume of the displaced water was then recorded as the volume of the fish.

The complete testing procedure took approximately 15 minutes per fish. After the volume measurement was completed, each fish was returned to the appropriate trough, where it was held for several days. No mortalities occurred among the fish tested.

ANALYSIS PROCEDURE

The symbols listed below are used in the discussions that follow:

ρ_f=fish resistivity.
ρ_w=water resistivity.
V_f=fish volume.
V_c=cell volume.
L=fish length.
D=distance between cell electrodes.
R=resistance of the cell containing the fish.
R_a=resistance of the cell without the fish.
E=d.c. voltage drop across the cell.
E''=d.c. voltage applied to the tank.
E'=reaction voltage applied to the tank.
V=guiding-field gradient.
P_{fv}=power density in the fish.
P_{wv}=power density in the water.
I=d.c. current through the cell.
m=empirical function of ρ_w and ρ_f.

Appendix A contains a glossary which may be useful to readers who are not familiar with electrical laws and terms. The following discussion is purposely brief; however, the subject is covered in more detail in appendix B. The mathematical model of the fish was developed by constructing an electrical analogue of the contents of the test cell. The analogue was developed by dividing the cell into a number of rectangular parallelepipeds, one of which represented the fish and the remaining ones represented the water within the cell. The size of the parallelepiped representing the fish was dependent upon the size of the fish, whereas the dimensions of the remaining parallelepipeds were functions of both cell and fish geometry.

The correct partitioning of the cell's contents into parallelepipeds was governed by the following requirements: (1) the calculated fish resistivity must remain constant as the water resistivity is varied over a specified range, (2) there must be a practical limitation on the complexity of the resulting equations, and (3) the accuracy requirements of the resulting equations must not exceed the capabilities of existing measuring techniques. These are the restrictions which were used in devising the mathematical model of the fish.

In the partitioning scheme, each parallelepiped of the cell model was treated as a resistive-circuit component. The end or cross sectional boundaries served as connecting terminals, whereas the remaining surfaces were considered nonconducting. The resistance of each of the parallelepipeds was related to the resistivity of its enclosed medium by using Ohm's Law. The result was a set of equations called component equations which related cell dimensions, fish dimensions, water resistivity, and fish resistivity to the resistance of a corresponding circuit component. Thus, an analogue circuit of the cell's contents was devised by replacing each parallelepiped of the cell model with an equivalent two-terminal resistive component. The total resistance of this analogue circuit was designated to be the cell's resistance. An equation was obtained by using the known properties of resistors to relate this resistance to the resistance of each component. This equation was called the circuit equation. By substitution of the component equations for the resistance values in the circuit equation, a relationship was obtained between the cell resistance and the resistivity of the fish model. Thus, the resulting equation provided a method of evaluating a specific property of the fish.

The transformation from the actual cell to that of the model of the cell's contents can be more clearly visualized by mentally subdividing the actual cell volume into infinitesimal volume elements. These elements would be of equal volume and can be called differential volume elements (similar to the units used in the differential theory of calculus).

The transformation consists of transferring each element of the cell volume into a parallelepiped in the model of the cell's contents. The correct distribution of volume elements should be carried out in such a way that the total power in each parallelepiped will be equal to the power dissipated in a corresponding circuit component for all values of water resistivity. In order to complete the transformation, the lines of current flow can be rotated in each differential element without changing the power dissipated by the element. This allows these current lines to be arranged to correspond with those current lines which would be expected to exist in each parallelepiped. After considerable preliminary experimentation, two configurations were chosen which best fulfilled the previously established criteria.

The first partitioning plan divided the cell's contents into four parallelepipeds; three represented the total water in the cell. The fourth parallelepiped, which represented the fish, was designated as the fish model. The corresponding analogue circuit was composed of four resistors (R_1, the fish; and R_2, R_3, and R_4, the water) connected in a series-parallel network. This arrangement was satisfactory in all ranges of water resistivities tested except when the resistivity of the fish was greater than the resistivity of the surrounding water. It was in this region that this arrangement had a point of discontinuity.

In order that this difficulty might be avoided, the volume elements were resolved in a different manner for this area of operation. In this partitioning system, the cell's contents were divided laterally into two rectangular parallelepipeds, one representing the fish and the other representing the total water in the cell. The corresponding analogue circuit was composed of two resistors (R_1, the fish; and R_2, the water) connected in series.

EVALUATION PROCEDURE

The resistivity of the fish model was found by using the measured resistivity of the water in the cell and the resistance of the cell. The values of R were found from the slope of a plot of cell voltage (E) vs. cell current (I) for each fish tested. Figure 5 shows an example of one of these graphs. A similar graph (fig. 5) was made from data taken with the cell empty in order to find the value of R_a. One of these graphs was made for each series of fish tested. Because the value of R_a is directly proportional to the resistivity of the

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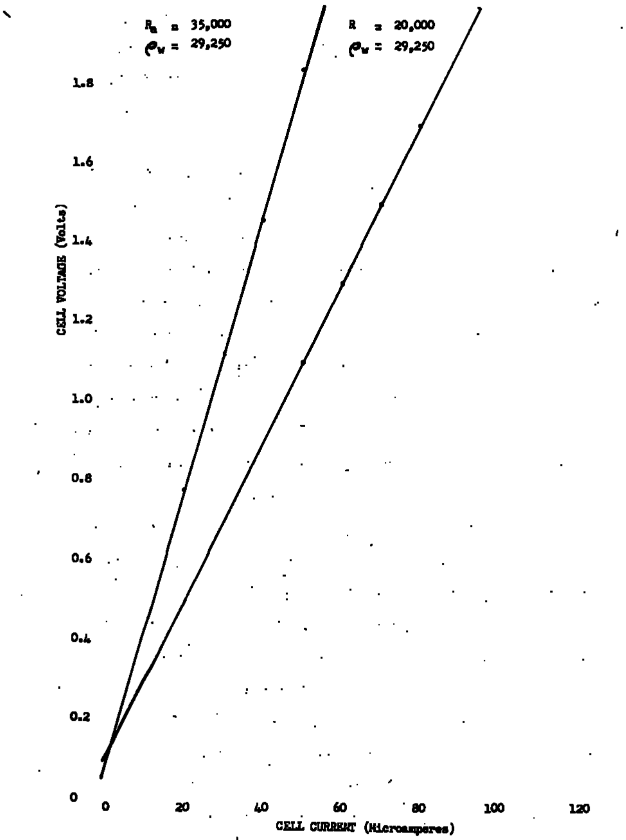


Figure 5.—A plot of cell voltage (E) vs. cell current (I); first for an empty cell and then for the cell with the test fish enclosed. The resistance of the empty cell (R_a) and the resistance of the cell with the fish enclosed (R) were found from the slope of their respective graphs.

water in the cell, it was easy to calculate from this single value of R_a the R_a's for the rest of the series. With the use of these values and other measured parameters, such as fish volume, fish length, and distance between electrodes, the values of fish resistivity (ρ_f) were found. The figures for fish length were obtained from the graph illustrated in figure 6.[2]

The power density in the fish model was determined by applying Kirchhoff's Voltage Law to the analogue circuit of the cell's contents. With this approach, the total power in the fish model was related to a voltage applied across the cell's contents. The voltage that was applied to the cell was considered as being applied to the analogue circuit terminals, as represented by the ends of the cell. The voltage value used in the computation was the applied cell voltage which produced the specific swimming like motion chosen as our desired reaction. By application of Kirchhoff's Voltage Law and our component equations, the actual voltage applied across the fish model was determined. The power density in the fish model was determined by using Joule's Law, the fish model's dimensions, the voltage applied to the model, and the component resistance of the model. The equation describing this relationship provided a method of evaluating the rate at which an electrical process takes place within a fish.

The range of water resistivities, over which the configurations and their corresponding equations were useful, was established by varying the water resistivity (ρ_w), calculating the resistivity of the fish (ρ_f), and observing within what range of water resistivities the resistivity of the fish remained relatively constant.

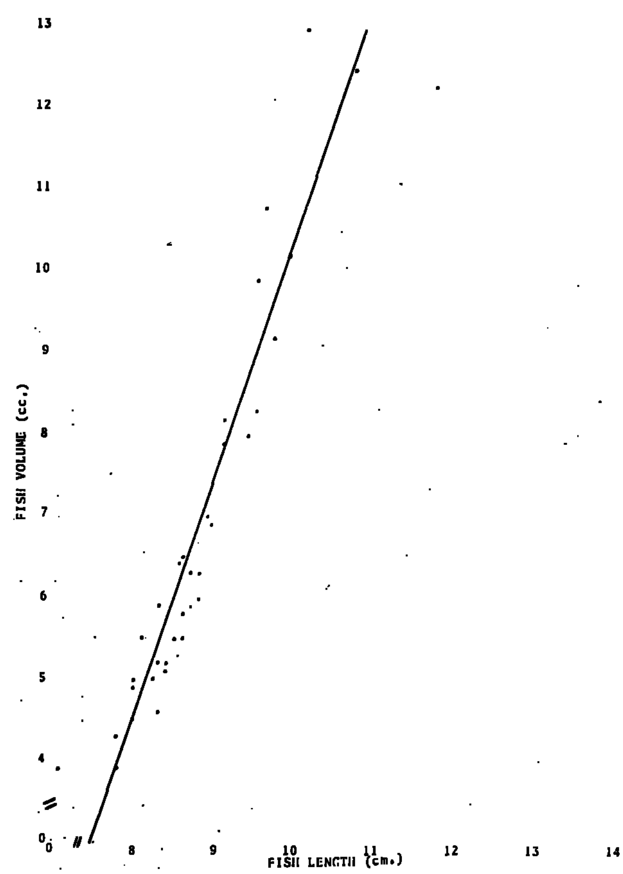


Figure 6.—Length-volume relationship used to provide the additional parameter fish length (L) in the four-body equation.

[2] When the four-body equation was derived, it became necessary to add a previously unneeded parameter, fish length (L). The lengths and volumes of a random sample of experimental fish were measured and a plot of fish length (L) vs. fish volume (V_f) was made to obtain this measurement. From this graph, approximate values of fish length were obtained by using the recorded values of fish volume (V_f) from the previous experiments.

The theory and complete mathematical derivations of the formulas used in computing fish resistivity and power density are found in appendix B.

In order that the third objective of our research might be fulfilled, the power density in the fish (P_{fv}) was related to the corresponding power density (P_{wv}) and voltage gradient (V) in an open body of water. The complete derivation and explanation of the relationship between P_{fv} and P_{wv} can be found in appendix C.

RESULTS AND DISCUSSION

Table 1 shows the models of the test cell that were derived, the corresponding analogue circuits, the range of water resistivities for each and the equations used, to solve for ρ_f and P_{fv}.

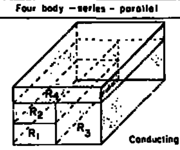
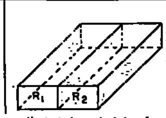
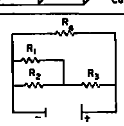
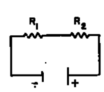
The formula derived for relating cell voltage gradients and power densities to open water gradients and power is as follows:

$$\rho_w Pwv = (m^2 \rho_f P_{fv}) = V^2 \text{ where } m = f(\rho_w \rho_f)$$

The complete derivation and explanation of this formula can be found in appendix C.

The symmetry of the equations and configurations is similar to those experienced with standard series and parallel electrical circuits. Basic laws of electricity can be applied to these configurations just as if they were composed of common commercial resistors. For example, in a resistance network with two resistors in parallel, if one resistance approaches zero and the other remains constant, the total resistance approaches zero.

TABLE 1.—*Formulas derived for computing fish resistivity and power density; shown with the corresponding test-cell models and analogue circuits. The limits of usefulness, based on water resistivity, are also shown.*

DERIVED TEST	Four body —series – parallel	Two body —series
CELL MODELS	 Conducting planes are illustrated as shaded surfaces	
ANALOGUE CIRCUIT		
EQUATION FOR ρ_f	$\rho_f = \frac{\rho_w[R R_a V_f L - 2V_f R_a(R_a-R)(D-L)]}{R R_a V_f L - V_f R_a(R_a-R)(D-L) + \rho_w D^2 L(R_a-R)}$	$\rho_f = \rho_w + \frac{\rho_w^2 D^2}{R_a^2 V_f}(R-R_a)$
EQUATION FOR P_{fv}	$P_{fv} = \left(\frac{EE'}{E''}\right)\left(\frac{\rho_w D}{\rho_f \rho_w D^2 L + 2R_a V_f(D-L)(\rho_w-\rho_f)}\right)^2 L^2 \rho$	$P_{fv} = \left(\frac{E'E}{E''}\right)^2\left(\frac{R_a}{R}\right)^2\left(\frac{\rho_f}{\rho^2 D^2}\right)$
LIMITS OF USEFULNESS	$1 \leq \frac{\rho_w}{\rho_f} \leq 20$	$1 \geq \frac{\rho_w}{\rho_f} \geq 0.75$

If, on the other hand, two resistors are connected in series and the value of the first resistor approaches zero and that of the second resistor remains constant, the total resistance of the circuit approaches the value of the second resistor. These same principles are applied to the resistance elements of the model of the cell's contents.

In order that the situation in which the resistivity of the water (ρ_w) is greater than the resistivity of the fish (ρ_f) might be accurately represented, the configuration and corresponding equation starting with two resistors in parallel are used. This equation reaches an extreme limit when the resistivity of the water (ρ_w) is infinite. When the resistivity of the water (ρ_w) is infinite, the resistance of the cell is equal to the resistance of the fish. By expanding this two-body, parallel configuration into a four-body, series-parallel network, this limit disappears. In the four-body, series-parallel network, when the resistivity of the water (ρ_w) is infinite, the resistance of the cell (R) is infinite; and when the resistivity of the water (ρ_w) is zero the resistance of the cell (R) also is zero.

In order that an accurate reflection of the situation in which the resistivity of the water (ρ_w) is less than the resistivity of the fish (ρ_f) may be achieved, the configuration and corresponding equation with two resistors in series are used. This equation reaches an extreme limit where the resistivity of the water (ρ_w) is zero. In this case, the resistance of the cell (R) is equal to the resistance of the fish.

The equation developed to relate power density in the cell to power density in an open body of water is not completely satisfactory. When the volume of the cell is expanded to infinity in our configuration, the boundaries no longer enclose finite regions and therefore exhibit an impossible field condition. For elimination of the effects of this property it was necessary to put an empirical function (m) of ρ_w and ρ_f into the equation. This empirical function will have to be determined by further experimentation.

Appendix D illustrates some qualitative values of fish resistivity and power density that were calculated to observe the practicability and validity of the derived analytical processes. With the use of the formulas that were derived, according to their restrictions, the average value of fish resistivity was calculated for the fish tested. When measured from end to end, with the fish acclimated at 49° F., this resistivity was 1,380 ohm centimeters. The large deviations in the values for the fish tested are primarily due to two causes: (1) a lack of accurate data on fish length and, (2) leakage current between the cell flange and the tank partitions. If the cell was not placed exactly in the same position each time, the dimension changes of the minute spaces between the cell flange and the tank partitions caused an error between the resistance of the cell with the fish in (R) and the resistance of the cell empty (R_a). Because of these errors, further work should be done before the absolute numerical values can be considered correct.

During the experiment, several interesting phenomena were observed. One of these was the apparent dependence of the reaction threshold upon pulse-rise time. Although the available equipment prohibited the variation of pulse-rise time, the fish was observed to withstand, without irritation, a much greater level of voltage when it was applied as a continuous voltage gradient. Mitchell (1948) found this to be true also in work with individual nerves.

It was also noticed that the reaction level appears to be a funtion of fish size when the fish is facing the positive pole; yet, when the fish faces the negative pole, this level is constant. When the fish was facing the negative pole, the reaction level was generally lower than it was when the fish was facing the positive pole (table 2). This has also been observed by other investigators (Eggen and Sheckels, 1954; McMillan, Holmes, and Everest, 1937[3]).

CONCLUSIONS

1. A mathematical model can be devised to serve as the electrical analogue of a fish.
2. The techniques developed in this work can provide a means of evaluating the electrical properties and processes within the model and thus provide a practical method of evaluating some electrical characteristics of a fish.
3. Power density can be used to describe an electrical criterion required to produce a specific reaction in a fish.

[3] McMillan, F. O., H. B. Holmes, and F. Alton Everest. 1937. The response of fish to impulse voltages. A report on investigations conducted at the Tablerock site near Medford, Oreg., between August 25, 1937 and September 18, 1937. 15 p. Typewritten. A copy of this report is available at the Bureau of Commercial Fisheries Biological Laboratory, Seattle, Wash.

TABLE 2.—*The relationship of the average reaction power density to fish volume and polarity orientation*

Pole fish is facing		Fish volume (cc.)		P_{fe},[1] (microwatt per cc.)	
Positive	Negative	Range	Average	Range	Average
	x	3.9 to 9.0	6	0.51 to 2.51	0.9
	x	11.0 to 22.0	15	0.32 to 1.64	0.8
x		3.9 to 9.0	6	0.61 to 3.31	1.4
x		11.0 to 22.0	15	0.84 to 8.44	3.8

[1] P_{fe} must be applied within 2 milliseconds, since a pulse with a rise time of not greater than 2 milliseconds was used.

4. The mathematical techniques developed here indicate that power density, as a reaction criterion, can be related to electric field parameters.
5. Additional experimentation will be required to develop the relationship between power density and the corresponding voltage gradient in an unbounded body of water.

RECOMMENDATIONS

The experimentation reported here should be furthered, and the following improvements should be employed:

(a) The test cell used in future experiments should resemble, in principle at least, the cell in figure 7. This design reduces the possibility of leakage currents, provides easier access to the cell, and enables closer observations of the fish.

(b) The electronic equipment should be well shielded and grounded.

(c) Alternating current should be used wherever possible to avoid polarization effects.

(d) The ends of the cell should be perforated conducting planes, erected perpendicular to the major cell axis.

(e) The incremental increase in pulse amplitude should be small in magnitude and constant in rate.

(f) A consistent handling procedure should be used for each fish.

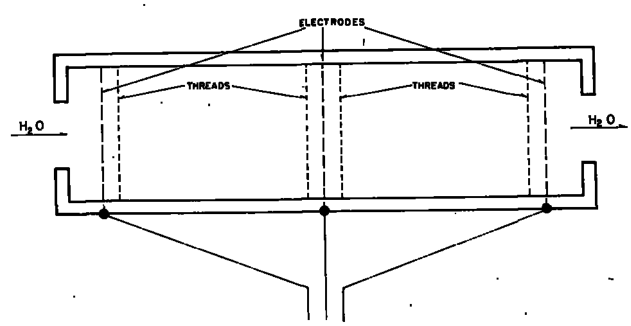


FIGURE 7.—Proposed test cell, incorporating the recommended improvements to eliminate leakage currents.

(g) The length of each fish tested should be accurately measured and recorded.

SUMMARY

Electrical fish guiding is being developed for use in protecting natural fish runs where their migration is threatened by manmade obstacles. A major problem in this development is the evaluation of an electric field as a motivating stimulus. In order that a precise method of evaluation might be established, a method of analysis was developed which employed a mathematical model of a fish.

The fish was confined in a small plastic test cell, through which water was circulated. The electrical characteristics of the cell's contents (water and fish) were determined by measuring voltages and currents applied to the cell. A mathematical model of the cell's contents was then devised by mentally dividing the test-cell volume into several parallelepipeds. One of the parallelepipeds was designated to represent the fish, whereas the others represented the water within the cell. The characteristics of this model were defined as those properties necessary to produce an electrical analogue of the cell's contents. Since the properties of the water portion of the analogue were known, the characteristics of the fish portion were determined by measuring the electrical parameters applied to the cell. The entire process utilized: (1) the voltage applied between the cell electrodes; (2) the electrical resistance of the cell, with the fish and without the fish; (3) the water resistivity; (4) the length of the fish; (5) the volume of the fish; (6) the cell length or separation of the cell electrodes; and (7) the formulas derived to relate the resistivity and power density of the model to the measured parameters.

One model characteristic, power density, was determined to be a criterion for producing an observed reaction. The applied voltages used in these power calculations were the minimum values to which the fish appeared to react. Therefore, the resulting power density value was a property controlled by the fish.

Study of the variations in the threshold of reaction revealed that the power density necessary for this reaction increased as the size of the fish increased, provided the fish was facing the positive pole. If the polarity was reversed, the threshold remained constant over the range of fish sizes tested. The two levels of power density converged as the fish size decreased. With the fish facing the positive pole, the average value of power density ranged from 1.4 to 3.8 $\mu w/cc.$ over a corresponding fish-volume variation of 6 to 15 cc. When the fish faced the negative pole, values of 0.8 to 0.9 $\mu w/cc.$ were obtained over the same volume range.

By expanding the cell dimensions, in our calculations, we found it possible to obtain a simple relationship between power density and the equivalent voltage gradient in an unbounded body of water. This gradient is the variable that would be applied in an electrical fish-guiding field pattern. The calculated gradient would permit the identification of portions of any field pattern as being capable or incapable of producing the reaction observed in the laboratory. Because of the manner in which the test cell's contents were subdivided, the equations derived for the open-water gradient contain a proportionality function, with constants that must be evaluated empirically. Therefore, this particular phase of the work is incomplete. Examination of the results of this work, however, indicates that a mathematical model can be devised to serve as the electrical analogue of a fish, and the techniques that were developed can provide a practical method of evaluating some electrical characteristics of fish. Additional experimentations, incorporating the techniques developed herein and incorporating the recommended improvements, will certainly be of value to the electrical-guiding program.

LITERATURE CITED

CATTLEY, J. G.
1955. Your guide to electrical fishing (1). World Fishing, vol. 4, No. 3, pp. 125–127.

EGGEN, ARMON M., and G. DALE SHECKELS.
1954. Susceptibility of rainbow trout to electric shocks. Montana State College Bulletin, Engineering Experiment Station, No. 19, 14 pp.

FISHER, KENNETH C.
1950. Physiological considerations involved in electrical methods of fishing. The Canadian Fish Culturist, No. 9, 26 pp.

MITCHELL, PHILIP H.
1948. General physiology. 4th ed. McGraw-Hill Book Company, Inc., New York, 927 pp.

VOLZ, CHARLES D.
1962. Ignition-pulsed electric fence guides migrating salmon. Electronics, vol. 35, No. 16, pp. 50–53.

APPENDIX A

GLOSSARY

1. Joule's Law: That portion of the power input to any device which is equal to the product of the resistance of the conductors forming the winding of the device and the square of the current through this winding is always converted into heat. That is, when a current I flows through a resistance r, heat is always "dissipated" in this resistance, and the rate of dissipation is $P_h=rI^2$ [1]

2. Kirchhoff's Network Laws: (a) The algebraic sum of the currents coming to any junction in a network of conductors is always zero. (b) The algebraic sum of the potential drops around any closed loop in a network of conductors is always zero.[1]

3. Ohm's Law: If a steady difference of potential V (in volts) is impressed across a conductor which (a) is held at constant temperature and in which (b) there is no internal emf, $V=rI$ where I is the steady current in amperes which will flow through the conductor and r is the factor of proportionality called the resistance of the conductor. The drop in potential V is therefore equivalent to the drop in potential rI, this latter being called the resistance drop.[1]

4. Resistance: The opposition offered by a substance or body to the passage through it of an electric current.[2]

5. Resistivity: The proportionality factor between current density and electric intensity. A property of a medium having the same value as the resistance measured between opposite faces of a unit cube of the medium, expressed in ohm-units.[3]

[1] This definition is taken from the following source: Eshbach, Ovid W. 1955. Handbook of Engineering Fundamentals. 2d ed., John Wiley and Sons, Inc., New York. 1262 pp.

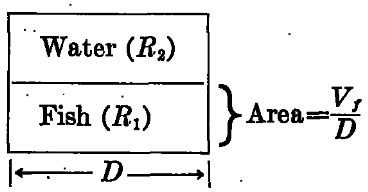
[2] This definition is taken from the following source: 1946. Webster's Collegiate Dictionary. 2d ed. G. C. Merriam Co., Springfield, Mass. 1,174 pp.

[3] Author's definition.

APPENDIX B

Theory and mathematical derivation of the formulas used in computing ρ_f and P_{fv}.

The two-body equations for (ρ_f) and (P_{fv}) were developed in the following manner:



Then, for the total cell resistance R:

$$R=\frac{R_1R_2}{R_1+R_2} \quad (1)$$

Where:

$$R_1=\frac{\rho_f D^2}{V_f} \quad R_2=\frac{\rho_w D^2}{V_c-V_f}$$

$$V_c=\frac{\rho_w D^2}{R_a} \quad R_2=\frac{\rho_w D^2 R_a}{\rho_w D^2-R_aV_f}$$

Substituting for R_1 and R_2 in (1) and solving for ρ_f we get:

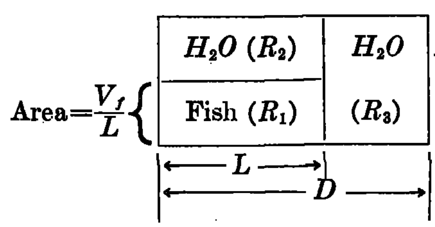
$$\rho_f=\frac{\rho_w R R_a V_f}{RR_aV_f+\rho_w D^2(R_a-R)} \quad (2)$$

and for power density:

$$P_{fv}=\left(\frac{E'E}{E''}\right)^2\frac{1}{R_1V_f}=\left(\frac{E'E}{E''}\right)^2\frac{1}{\rho_f D^2} \quad (3)$$

The expansion to a three-body and a four-body arrangement was done in the following manner:

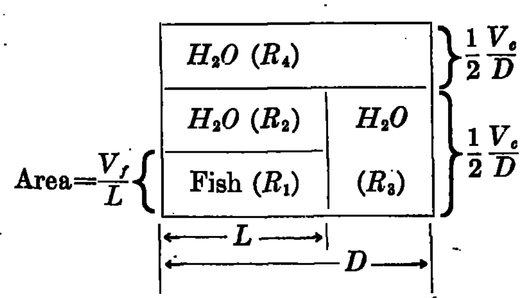
Three-Body



$$R=R_3+\frac{R_2R_1}{R_1+R_2}$$

(4)

Four-body



$$R=\frac{(R_4)\left(R_3+\frac{R_1R_2}{R_1+R_2}\right)}{R+\left(R_3+\frac{R_1R_2}{R_1+R_2}\right)} \qquad (5)$$

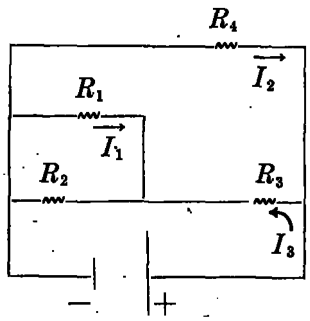
$$R_4=\frac{\rho_f L^2}{V_f},\; R_2=\frac{\rho_w L}{\frac{V_c}{2D}-\frac{V_f}{L}}=\frac{2\rho_w L^2 R_a}{(\rho_w LD-2V_f R_a)}$$

$$R_3=\frac{\rho_w(D-L)}{\frac{V_c}{2D}}=\frac{2R_a(D-L)}{D},\; R_4=\frac{\rho_w D}{\frac{V_c}{2D}}=2R_a$$

Substituting into (5) and solving for ρ_f gives:

$$\rho_f=\frac{\rho_w[RR_aV_fL-2V_fR_a(R_a-R)(D-L)]}{RR_aV_fL-2V_fR_a(R_a-R)(D-L)+\rho_wD^2L(Ra-R)} \qquad (6)$$

Then, for power density we solve for the current in R_1 in the following network.



I_1, I_2, I_3=loop currents

$E_c=\frac{E'E}{E''}$

$$\begin{vmatrix} I_1 & I_2 & I_3 \\ (R_1+R_2) & R_2 & R_2 \\ R_2 & (R_4+R_3+R_2) & (R_2+R_3) \\ R_2 & (R_2+R_3) & (R_2+R_3) \end{vmatrix} \begin{matrix} \\ =0 \\ =0 \\ =E_c \end{matrix}$$

The power density in the fish is:

$$P_{fv}=I_1^2R_1\frac{1}{V_f} \qquad (7)$$

Solving for the current I_1:

$$I_1=\frac{E_c\begin{vmatrix} R_2 & R_2 \\ (R_4+R_3+R_2) & (R_2+R_3) \end{vmatrix}}{\begin{vmatrix} (R_1+R_2) & R_2 & R_2 \\ R_2 & (R_4+R_3+R_2) & (R_2+R_3) \\ R_2 & (R_2+R_3) & (R_2+R_3) \end{vmatrix}}=E_c\frac{R_{31}}{\Delta R}$$

Then:

$$P_{fv}=E_c^2\left(\frac{R_{31}}{\Delta R}\right)^2\frac{R_1}{V_f} \qquad (8)$$

Expanding the determinant factors and substituting for the resistance values in the resulting equation we obtain the following:

$$P_{fv}=\left(\frac{EE'}{E''}\right)^2\left(\frac{\rho_wD}{\rho_w\rho_fD^2L+2R_aV_f(D-L)(\rho_w-\rho_f)}\right)^2L^2\rho_f \qquad (9)$$

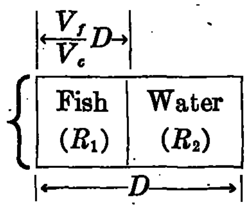
The simplicity of the resulting power density equation was achieved by the manner in which the dimensions of the individual volume elements were chosen. The geometric similarity between these elements reduced the complexity, indicated by the expansion of (8), to the simplicity of equation (9). Such a simplification is desirable for the practicability of processing data. However, it can be expected to limit the range of usefullness of the equation.

In the course of our studies, there was reason to reduce the water resistivity to a value below that obtained for the fish. We did this to check the results of previous measurements. When we attempted to evaluate our data, we found that the measured parameters forced us to work in and near a region of discontinuity that was a characteristic of our equation for (ρ_f). This condition occurred when, for (6):

$$RR_aV_fL-2V_fR_a(R_a-R)(D-L)+\rho_wD^2L(R_a-R)=0$$

To avoid this difficulty, we resolved our volume elements in a different manner for this region of operation. Hence, for the two-body case:

Area$=\frac{V_c}{D}$



$$R=R_1+R_2$$

$$R_1=\frac{\rho_f\frac{V_f}{V_c}D}{\frac{V_c}{D}}=\frac{\rho_fV_fR_a^2}{\rho_w^2D^2}$$

$$R_2=\frac{\rho_w\left(D-\frac{V_f}{V_c}D\right)}{\frac{V_c}{D}}=R\left(1-\frac{V_fR_a}{\rho_wD^2}\right)$$

Substituting and solving for ρ_f we get:

$$\rho_f=(\rho_w^2)\left(\frac{D^2}{R_a^2V_f}\right)(R-R_a)+\rho_w \tag{10}$$

And:

$$P_{fv}-\left(\frac{E'E}{E''}\right)^2\left(\frac{R_a}{R}\right)^2\frac{\rho_f}{\rho_w^2D^2} \tag{11}$$

Notice that for equation (10), there exists no point of discontinuity, as was present in equation (6).

Having derived these equations, we determined that they satisfied our requirements by using the following methods:

1. If the fish model is electrically homogeneous and has a characteristic resistivity ρ_f, then when $\rho_w=\rho_f$, $R_a=R$. Observing equations (2), (6), and (10), we see that in each case, where $R_a=R$, this condition is satisfied.

2. If the resistance R were measured continuously as ρ_w was varied from $\rho_w=0$ to $\rho_w=\infty$, we would expect the functions R and $\frac{\delta R}{\delta\rho_w}$ to be continuous over the range of variation. This is because we consider our cell to contain linear, bilateral resistive elements. To check the fulfillment of this condition, we examined the ratio $\frac{1}{R}\frac{\delta R}{\delta\rho_w}$ at the point where $\rho_w=\rho_f$. If the condition is satisfied, then:

$$\left[\frac{1}{R_{2s}}\frac{\delta R_{2s}}{\delta\rho_w}=\frac{1}{R_{4P}}\frac{\delta R_{4P}}{\delta\rho_w}\right]@\rho_w=\rho_f$$

Where:

R_{2s} = the two-body series function

R_{4P} = the four-body parallel function

Using equation (6) and solving for R as a function of ρ_w, we get:

$$R=\frac{A\rho_wB\rho_f+C(\rho_w-\rho_f)}{B\rho_f+(C+F)(\rho_w-\rho_f)} \tag{12}$$

Where:

$$A=\frac{D^2}{V_c},\ B=V_cD^2L,\ C=2V_fD^2(D-L),\ F=V_fLD^2$$

And:

$$\frac{\delta R}{\delta\rho_w}=\frac{A(C\rho_w+B\rho_f)+AC(\rho_w-\rho_f)}{B\rho_f+(F+C)(\rho_w-\rho_f)}-\frac{[AB\rho_f\rho_w+AC(\rho_w^2-\rho_f\rho_w)](F+C)}{[B\rho_f+(F+C)(\rho_w-\rho_f)]^2} \tag{13}$$

When: $\rho_w=\rho_f$ equations (12) and (13) reduce to:

$$R=\rho_fA=\rho_f\frac{D^2}{V_c}$$

And:

$$\frac{\delta R}{\delta\rho_w}=A\left(1=\frac{F}{B}\right)=\frac{D^2}{V_c}\left(1-\frac{V_f}{V_c}\right)$$

Therefore:

$$\left(\frac{1}{R_{4P}}\frac{\delta R_{4P}}{\delta\rho_w}\right)=\frac{1}{\rho_f}\left(1-\frac{V_f}{V_c}\right) \tag{14}$$

Next, we solve equation (10) for R as a function of ρ_w and obtain the following:

$$R=\frac{D^2V_f}{V_c^2}\left[\rho_w\frac{V_c}{V_f}-(\rho_w-\rho_f)\right] \tag{15}$$

And:

$$\frac{\delta R}{\delta\rho_w}=\frac{D^2}{V_c}\left(1-\frac{V_f}{V_c}\right)$$

When:

$\rho_w=\rho_f$ equation (15) reduces to:

$$R=\frac{\rho_fD^2}{V_c}$$

Then:

$$\left(\frac{1}{R_{2s}}\frac{\delta R_{2s}}{\delta\rho_w}\right)=\frac{1}{\rho_f}\left(1-\frac{V_f}{V_c}\right) \tag{16}$$

Comparing (16) with (14) we see that our requirement is satisfied.

It should be noted that in equations (2) through (11) the parameter V_c has been eliminated. This was done because the cell boundaries at the ends of the cells were not clearly defined owing to the different grid structures used. It was hoped that, by allowing this degree of freedom, the errors caused by this effect would be minimized.

APPENDIX C

Derivation and explanation of the relationship between P_{fv} and P_{wv}:

To relate our results to the electrical conditions present when the fish is not confined in a cell, we used the following procedure:

In the four-body configuration, the power density in the water of R_4 is:

$$P_{wv}=\left(\frac{EE'}{E''}\right)^2 \frac{1}{\rho_w D^2} \qquad (1)$$

From this:

$$\left(\frac{EE'}{E''}\right)^2=P_{wv}\rho_w D^2 \qquad (2)$$

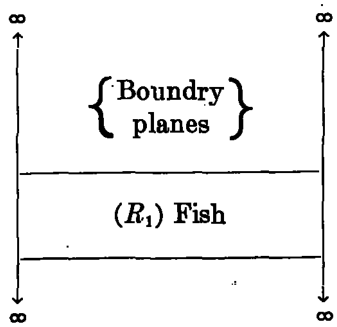
Substituting this value into equation (9) of Appendix B and expanding V_c to infinity (the expansion was carried out holding $\frac{D^2}{V_c}$ constant), we obtain the following result:

$$P_{wv}=P_{fv}\frac{\rho_f}{\rho_w}=\frac{V^2}{\rho_w}=\lim_{V_c\to\infty} P_{wv}$$

Or:

$$P_{wv}=\rho_f P_{fv}=V^2 \qquad (3)$$

By expanding V_c to infinity our volume configuration is as pictured below:



This illustration demonstrates the greatest weakness in our resolution model. That is, when we expand our configuration, our boundaries no longer enclose finite regions and therefore exhibit an impossible field condition. For correction of equation (3) and elimination of the effects of this property, it appears that it will be necessary to multiply (3) by some empirical function (f) of ρ_w and ρ_f.

Thus:

$$\rho_w P_{wv}=m^2\rho_f P_{fv}=V^2$$

Where:

$$m=f(\rho_w\rho_f)$$

This empirical function will have to be determined by further experimentation.

APPENDIX D

Tables A–1: How derived formulas were applied, and A–2: A sample of some of the numerical values of ρ_f and P_{fv} that were obtained.

TABLE 1.—*Original measurement data*

Fish number	Polarity orientation	E' (volts)	E'' (volts)	E (volts)	I (μ amps)	P_w (ohm cm.)	Water temp. (° F.)	V_f (cc.)	D (cm.)	Date
			3.18	1.70	80					
6a	Negative	2.3	2.77	1.48	70	29,250	47.0	15.3	15.8	1/5/59
			2.37	1.30	60					

TABLE 2.—*Calculation sample* (ρ_f)

Fish number	R_e (ohms)[1]	R (ohms)[1]	R_e-R (ohms)	L (cm.)[2]	$D-L$ (cm.)	A ($\times10^{11}$)[3]	B ($\times10^{11}$)[4]	$A-B$ ($\times10^{11}$)	C ($\times10^{11}$)[5]	ρ_f (ohm cm.)[6]
6a	35000	20000	15000	11.6	4.20	1.24	0.675	0.565	12.72	1,244

[1] R_e and R (from graph)
[2] $L=1/3\ V_f+6.5$ (from graph)
[3] $A=R\,R_eV_fL$
[4] $B=2V_fR_e\,(R_e-R)\,(D-L)$
[5] $C=\rho_wLD^2\,(R_e-R)$
[6] $\rho_f=\frac{\rho_w(A-B)}{A-B+C}=\frac{(29{,}250)\,(.565)}{.565+12.72}=1{,}244$ ohm cm.

TABLE 3.—*Calculation sample* (P_{fv})

Fish number	E^2 (volts)[1]	D (×10[5])	G (×10[10])[2]	H (×10[5])[3]	$D-L$ (cm.)	$\rho_w-\rho_f$ (ohm cm.)	F (×10[-6])[4]	F^2 (×10[-12])	$L^2\rho_f$ (×10[5])	P_{fv} (μ w)[5]
6a	1.54	4.62	10.55	10.71	4.20	28006	2.00	4.00	1.67	1.03

[1] $E_e^2=\left(\frac{EE'}{E''}\right)^2$

[2] $G=\rho_f\rho_w D^2L$

[3] $H=2R_aV_f$

[4] $F=\frac{\rho_w D}{G+H(D-L)(\rho_w-\rho_f)}$

[5] $P_{fv}=(E_e^2)(F^2)(L^2\rho_f)=(1.54)(4.00)(1.67)=1.03$

TABLE 4.—*A sample of the numerical values obtained for fish resistivity (ρ_f) and reaction power density (P_{fv}) using our derived equations*

Fish number	Polarity orientation (fish facing)	Water temp. (° F.)	Fish volume (cc.)	ρ_f (ohm cms.)	P_{fv} (μw/cc.)
1	−	49.0	5.9	1979	1.26
2	−	49.0	6.0	1446	1.19
3	+	49.0	5.3	1677	1.20
4	+	49.0	8.0	1092	0.94
5	+	49.5	6.1	1832	0.88
6	+	49.5	7.1	1119	1.00
7	−	49.0	9.0	1030	0.83
8	−	49.0	8.0	1000	0.50
9	−	49.0	6.5	1800	0.74
10	+	49.5	6.0	1470	0.96
11	+	49.5	6.2	790	1.39
12	−	49.5	5.4	1330	0.52