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USE OF GRIFFIN'S YIELD MODEL FOR THE GULF OF MEXICO SHRIMP FISHERY[1]

For analyzing the harvest of the Gulf of Mexico shrimp fishery, Griffin et al. (1976) have developed an equation that relates shrimp yield to freshwater discharge of the Mississippi River and fishing effort of Gulf shrimp vessels. The yield equation (referred to as Griffin's equation) is a modified Spillman production function (Heady and Dillon 1972). The Spillman function had its origin in agriculture where it was derived to predict the results of fertilizer experiments on tobacco yield in North Carolina. An important feature of the function is that it allows for environmental considerations in predicting yield. The modified form of the equation proposed by Griffin et al. (1976) is:

$$Y = \beta_0 D^{\beta_2} (1 - \beta_1^E) \quad (1)$$

where Y = yield of shrimp (million pounds),[2]
D = average daily discharge of the Mississippi River during the months that shrimp are in their nursery grounds (cubic feet per second),
E = vessel effort (thousand units),
$\beta_0, \beta_1, \beta_2$ = parameters to be estimated from data of the fishery.

The coefficients of Equation (1) were estimated from individual vessel records collected by the National Marine Fisheries Service and from measurements of water flow rates on the Mississippi River for the years 1962-74. According to Griffin and Beattie (1978), the fit was quite good, namely: "All estimated coefficients were significant at the 1% level; R^2 was 78.5; and the Durbin-Watson statistic was 2.25. The simple correlation coefficient between catch and effort was 0.64 and between catch and discharge was −0.63."

Griffin's equation has found numerous uses in the Gulf shrimp management literature. Griffin and Beattie (1978) used the equation to estimate the impact of effort reallocation as a result of Mexican extended jurisdiction; the Gulf Coast Research Laboratory at Ocean Springs, Miss., (Christmas and Etzold 1977) used the equation for similar purposes; and the Center for Wetland Resources, Louisiana State University[3] used the equation to estimate maximum sustainable yield for management considerations.

Despite the extensive usage, users have not critically reviewed Griffin's equation. Such a review is necessary because of the large-scale potential impact of proposed shrimp management plans. In view of this need, therefore, I subjected Griffin's equation to such a review.

The review consisted of two tests relevant to the usage of Griffin's equation in management decisions. In the first test, I estimated the error in expected yield introduced by the typical user who ignored the fact that the independent variables—effort and river discharge—have variances. For convenience, this was termed the "expected value test." In the second test, I depicted the error in yield estimate that would result from misspecification of model parameter estimates. For convenience, this test was termed the "sensitivity test."

The results were mixed. The expected value test produced a large absolute error in expected yield of shrimp. However, when compared with expected yield, the error was proportionally small. The sensitivity test produced some startling results. Yield turned out to be very significantly sensitive to a fixed model parameter whose constancy was conceptually questionable in the first place. This extreme sensitivity of yield raises questions regarding the reliability of Griffin's equation as a shrimp management tool.

Each test is discussed below in detail.

[1]Contribution No. 80-54M, Southeast Fisheries Center, National Marine Fisheries Service, NOAA, Miami, Fla.
[2]The original equation was estimated by Griffin et al. (1976) in nonmetric units and its nonlinear nature excludes conversion to metric units.
[3]Louisiana State University. 1979. Draft fishery management plan for shrimp fishery. Prepared by Center for Wetland Resources, L.S.U., Baton Rouge, 226 p.

Expected Value of Yield

Most users of Griffin's equation (Griffin et al. 1976; Louisiana State University (footnote 3) estimated yield by using the mean (or expected) value of the independent variables, discharge and effort. Yet it can easily be shown that, for a general two-variable function, if x, y are random variables and g an arbitrary twice differentiable function of x, y such that:

$$z = g(x,y) \tag{2}$$

then $$E[z] = E[g(x,y)] \neq g(E[x], E[y]) \tag{3}$$

or if $E[x] = \eta_x$ and $E[y]\ \eta_y$

then $$E[z] \neq g(\eta_x, \eta_y) \tag{4}$$

where $E(\cdot)$ denotes expectation of random variable.

Hence, yield estimates obtained using mean values as in Equation (4) are generally not accurate. It may be shown (Papoulis 1965) that Equation (4) may be correctly approximated as:

$$E[z] = g(\eta_x, \eta_y) + \frac{1}{2}\left\{\frac{\partial^2 g}{\partial x^2}\sigma_x^2 + \frac{\partial^2 g}{\partial y^2}\sigma_y^2 + \frac{2\partial^2 g}{\partial x \partial y}\operatorname{cov}(x,y)\right\} + \ldots \tag{5}$$

where the o^2's are the variances of variables x,y. The variance of the estimate is as follows:

$$\sigma_z^2 = \left\{\frac{\partial g}{\partial x}\right\}^2 \sigma_x^2 + \left\{\frac{\partial g}{\partial y}\right\}^2 \sigma_y^2 + 2\left\{\frac{\partial g}{\partial x}\frac{\partial g}{\partial y}\right\} \operatorname{cov}(x,y) + \ldots . \tag{6}$$

Thus, Equation (4) is only a first approximation, with Equation (5) providing the second term. Additional terms may be obtained by continuing Taylor's series expansion of $g(x,y)$ around $g(\eta_x, \eta_y)$. For the purpose of the test, however, the second term was sufficient.

For Griffin's equation the independent (random) variables were river discharge D and vessel effort E. So, the expected value of the dependent (random) variable yield Y could be expressed as:

$$E[Y] = Y(\eta_D, \eta_E) + \frac{1}{2}\left\{\frac{\partial^2 Y}{\partial D^2}\bigg|_{\eta_D} \sigma_D^2 + \frac{\partial^2 Y}{\partial E^2}\bigg|_{\eta_E} \sigma_D^2 + 2\frac{\partial^2 Y}{\partial D \partial E}\bigg|_{\eta_D, \eta_E} \operatorname{cov}(D,E)\right\}. \tag{7}$$

Similarly, the variance of the estimate was given by:

$$\sigma_y^2 = \left\{\frac{\partial Y}{\partial D}\right\}^2\bigg|_{\eta_D} \sigma_D^2 + \left\{\frac{\partial Y}{\partial E}\right\}^2\bigg|_{\eta_E} \sigma_E^2 + 2\left\{\frac{\partial Y}{\partial D}\cdot\frac{\partial Y}{\partial E}\right\}\bigg|_{\eta_D, \eta_E} \operatorname{cov}(D,E). \tag{8}$$

To compute the estimated yield and its variance we required the first, second, and cross partial derivatives of the yield equation. The derivation was tedious and hence not reproduced here. By the necessary partial differentiation of Equation (1) we could write:

$$\frac{\partial Y}{\partial D} = \beta_2 \beta_0 D^{\beta_2 - 1} (1 - \beta_1^E) \tag{9}$$

$$\frac{\partial^2 Y}{\partial D^2} = \beta_2 (\beta_2 - 1) \beta_0 D^{\beta_2 - 2} (1 - \beta_1^E) \tag{10}$$

$$\frac{\partial Y}{\partial E} = -\beta_0 D^{\beta_2} \beta_1^E \log_e \beta_1 \tag{11}$$

$$\frac{\partial^2 Y}{\partial E^2} = -\beta_0 D^{\beta_2} \beta_1^E (\log_e \beta_1)^2 \tag{12}$$

$$\frac{\partial^2 Y}{\partial D \partial E} = -\beta_0 \beta_2 D^{\beta_2 - 1} \beta_1^E \log_e \beta_1 . \tag{13}$$

Griffin et al. (1976) have estimated the equation parameters to be: $\beta_0 = 6593$, $\beta_1 = 0.995701$, and $\beta_2 = -0.60134$.

Expected yield of shrimp can be determined by using the means, variances, and covariances of river discharge and effort. Following the approach of one user of Griffin's equation (Christmas and Etzold 1977), yield was estimated by using mean values of variables for the years 1970-74. A listing of the data and numerical values of means and covariances are given in Table 1.

TABLE 1.—Data, including mean and variance of Mississippi River discharge (thousand cubic feet per second) at Tarbert Landing, Miss., and Gulf of Mexico commercial shrimp effort (thousands of days) from U.S. waters by vessel, 1970-74.[1]

	Discharge						
Year	Jan.	Feb.	Mar.	Apr.	May	Mean Jan.-May	Effort
1970	448	430	529	652	852	582.2	249.1
1971	482	481	865	449	431	541.6	259.0
1972	560	427	602	536	749	574.8	282.6
1973	842	857	779	1,284	1,373	1,027	269.7
1974	971	1,083	828	792	576	850.00	243.6
Averages				ηD =	715.1	ηE =	260.8
SD				σD =	213.9	σE =	15.74
Covariance				$\mu D, E$ =	−141.92		

[1]Sources: U.S. Army Corps of Engineers, Stages and discharges of the Mississippi River and its tributaries and other watersheds in New Orleans District. U.S. Army Engineer District, New Orleans Corps of Engineers, Louisiana, and Christmas and Etzold 1977:28).

Appropriate insertion of these numerical values in Equation (7) produced the following estimates of yield:

$$E(Y) = 89.09 \text{ million lb}, \sigma_y^2 = 247.72.$$

Ignoring the variances and covariances, as most users of Griffin's equation do, we computed the corresponding estimated yield to be: $E(Y) = 85.48$ million lb.

The expected value test indicated that an absolute error of 3.6 million lb of shrimp is introduced by ignoring the variances and covariances of the independent variables. While an error of 3.6 million lb is large in absolute terms, its significance is diminished to 4% in relative terms. Furthermore, although 4% error is sizable, it is probably insufficient to alter economic management conclusions. We may conclude, therefore, that the expected value test, if applied to other applications of Griffin's equation, would not drastically alter management conclusions.

Parameter Sensitivity Test

Griffin's equation contains three parameters, each with its own significance and sensitivity. (Absolute and relative definitions of sensitivity can be found in Tomovic and Vukobratovic (1972) and Truxal (1972).) In assessing the sensitivity of yield to these parameters, I will relate proportional changes in parameter values to proportional changes in yield. (For this application of the sensitivity test, the relative measure of sensitivity is preferred because the results obtained are independent of the units of measure used for effort and discharge.) Mathematically, the sensitivity S of yield to parameter β may be expressed as:

$$S(Y|\beta) \triangleq \frac{\partial(\log_e Y)}{\partial(\log_e \beta)} = \frac{\partial Y/Y}{\partial\beta/\beta} = \frac{\partial Y}{\partial\beta}\cdot\frac{\beta}{Y}. \quad (14)$$

In considering parameter β_0, note that it is the dimensioned constant relating effort, discharge, and yield. The sensitivity of yield to β_0 is expressed:

$$S(Y|\beta_0) = 1.0. \quad (15)$$

In Equation (15), the sensitivity is small and constant. Thus, small errors in misspecification of β_0 will not significantly affect yield estimates.

Parameter β_2 governs the relationship between discharge of the Mississippi River and yield. Its sensitivity can be expressed as:

$$S(Y|\beta_2) = \beta_2 \log_e D. \quad (16)$$

The relationship is clearly linear and the sensitivity relatively small (Figure 1). Again, the implication being that misspecification of β_2 or future changes in its value would have small impact on yield.

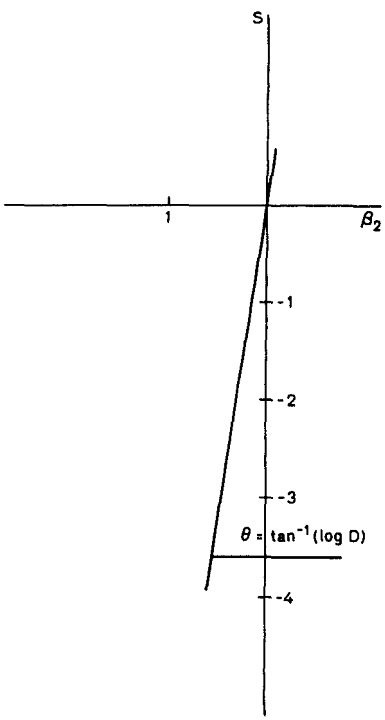

FIGURE 1.—Sensitivity (S) of yield to parameter β_2; i.e., the percentage change in yield corresponding to a 1% change in β_2.

The parameter β_1 is a little more difficult to understand. In the context of the model, β_1 is the constant that relates marginal yield corresponding to two successive units of effort. By appropriate manipulation of Equation (1) we can derive the following expression:

$$\beta_1 = \frac{\left.\frac{\partial Y}{\partial E}\right|_{\tilde{E}+1}}{\left.\frac{\partial Y}{\partial E}\right|_{\tilde{E}}}. \qquad (17)$$

Rational physical arguments may be used to show that β_1 is bounded by 0 and 1 ($0<\beta_1<1$). The law of diminishing returns provides the simplest argument, although there are others. Nevertheless, whatever the interpretation of β_1, sensitivity of yield to the parameter can be expressed as:

$$S(Y|\beta_1) = \frac{-E}{\beta_1^{-E}-1}. \qquad (18)$$

Since we have already established $0<\beta_1<1$, Equation (18) can be sketched as shown in Figure 2.

Yield is not very sensitive to β_1 for small values of $\beta_1 (\approx 0)$. The sensitivity increases hyperbolically, however, asymptotically approaching infinity as β_1 approaches the value unity.

Griffin's estimate of β_1 is 0.995701. This is about as close as one could get to the most sensitive region in Figure 2. For a value of effort E of 260.8, the sensitivity is -125.63. Thus any small misspecification of β_1 would produce very large errors in yield estimates.

It has already been shown that the parameter β_1 is related to marginal product of effort. At this point I question the assumption of β_1 as being a constant parameter. The marginal product of effort in any fishery is intimately related to stock and fleet characteristics. Realistically, one would expect marginal product and therefore its ratio to vary over time. Even if one could consider β_1 to be a constant over 1 yr, it would most certainly change over the course of the 12 yr that were used to estimate the given value (Griffin et al. 1976). Since I have already shown β_1 to be the most sensitive parameter of Griffin's equation, any misspecification or future change of β_1 would have enormous consequences on yield estimates.

Thus a user of Griffin's equation, who assumes a value of β_1 based on previous estimates of stock, effort, etc., may make incorrect predictions in the face of changing conditions. This observation severely limits the applicability of Griffin's equation for management purposes.

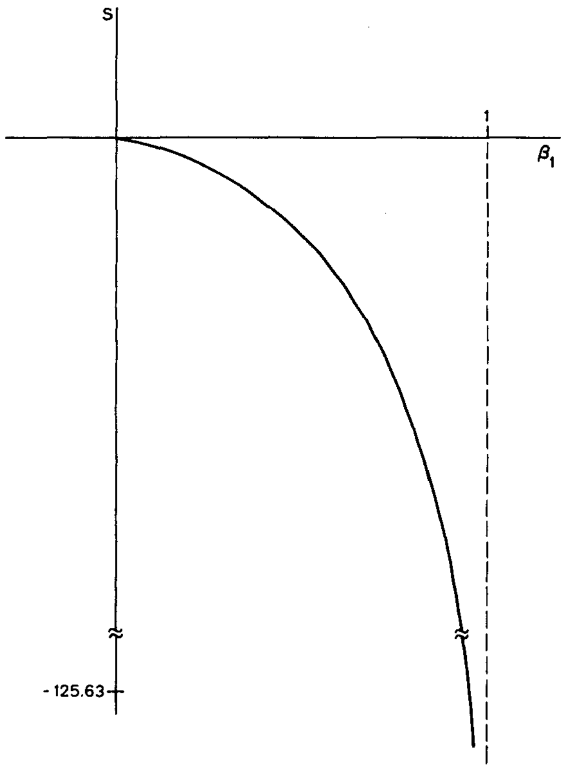

FIGURE 2.—Sensitivity (S) of yield to parameter β_1; i.e., the percentage change in yield corresponding to a 1% change in β_1.

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SEASONAL SPAWNING CYCLE OF THE PACIFIC BUTTERFISH, *PEPRILUS SIMILLIMUS* (STROMATEIDAE)

There is little information on the reproductive biology of the Pacific butterfish, *Peprilus simillimus*, which ranges from Magdalena Bay, Baja California, to the Fraser River, British Columbia, and occurs at depths of 9-91 m (Miller and Lea 1972). It is commercially fished with purse seine, lampera, and bait net (Fitch and Lavenberg 1971). In 1976, 34.18 t were taken in California (Oliphant 1979). Fitch and Lavenberg (1971) reported spawning occurs in spring and extends perhaps into July. Horn (1970) studied the systematics and biology of the genus *Peprilus*. My purpose is to describe histologically the seasonal spawning cycle of the Pacific butterfish.

Methods

Fish were collected with the use of a lampera net between depths of 2 and 20 m from the vicinity of Oceanside, southern California (lat. 33°10′ N, long. 117°25′ W), during the period September 1978 through August 1979. Only female specimens were examined. Fish were fixed and preserved in 10% Formalin.[1] Ovarian histological sections from 232 specimens were cut at 8 μm and stained with iron hematoxylin. Seasonal gonosomatic indices (ovary weight/fish weight × 100) were calculated from preserved fish. Ovaries were histologically classified into four stages (Table 1).

TABLE 1.—Monthly distribution (percent) of ovarian stages in the yearly spawning cycle of *Peprilus simillimus*, September 1978-August 1979.

Month	*N*	Regressed or regressing	Pre-vitellogenic	Vitellogenic	Pre-spawning
Sept.	25	100	0	0	0
Oct.	18	100	0	0	0
Nov.	15	100	0	0	0
Dec.	17	53	47	0	0
Jan.	21	52	43	5	0
Feb.	19	26	42	16	16
Mar.	20	10	15	0	75
Apr.	20	5	5	0	90
May	16	44	19	0	37
June	20	95	0	0	5
July	17	100	0	0	0
Aug.	24	92	8	0	0

Results

Ovaries were regressed (Stage 1) during autumn (September-November) and consisted of primary oocytes <100 μm in diameter (Table 1). Gonosomatic indices (Figure 1) were reduced at this time. The first signs of ovarian activity for the new spawning cycle were noted during December. This was determined by an abundance of previtellogenic (vacuolated) (Stage 2) oocytes (130-200 μm) which typically appear before yolk deposition begins (Table 1). Enlarging (Stage 3) vitellogenic oocytes (yolk deposition in progress) were first noted in January. The first ripe (prespawning or gravid) (Stage 4) females with ovaries containing

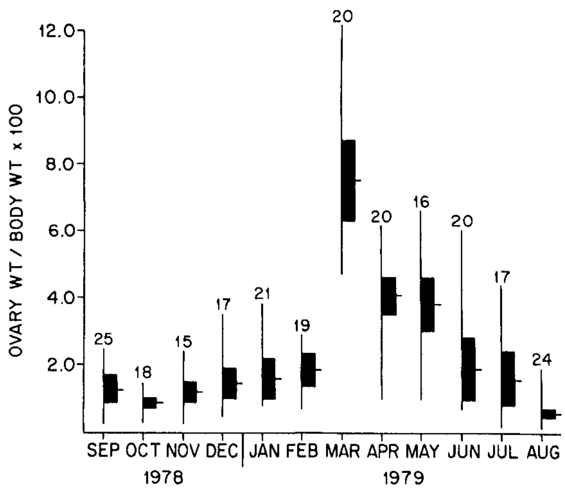

FIGURE 1.—Seasonal gonosomatic indices for *Peprilus simillimus*. Vertical line = range; horizontal line = mean; rectangle = 95% confidence interval. Sample size above each month.

[1]Reference to trade names does not imply endorsement by the National Marine Fisheries Service, NOAA.