

UNITED STATES DEPARTMENT OF THE INTERIOR, Fred A. Seaton, *Secretary*  
FISH AND WILDLIFE SERVICE, Arnie J. Suomela, *Commissioner*

# CONTRIBUTIONS OF HUDSON AND CON- NECTICUT RIVERS TO NEW YORK-NEW JERSEY SHAD CATCH OF 1956

BY KENNETH J. FISCHLER



FISHERY BULLETIN 163

From Fishery Bulletin of the Fish and Wildlife Service

VOLUME 60

PUBLISHED BY U.S. FISH AND WILDLIFE SERVICE • WASHINGTON • 1959

PRINTED BY UNITED STATES GOVERNMENT PRINTING OFFICE, WASHINGTON, D.C.

---

For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington 25, D.C.  
Price 20 cents

Library of Congress catalog card for the series, Fishery Bulletin of the Fish and Wildlife Service:

**U. S. *Fish and Wildlife Service.***

Fishery bulletin. v. 1-  
Washington, U. S. Govt. Print. Off., 1881-19

v. in illus., maps (part fold.) 23-28 cm.

Some vols. issued in the congressional series as Senate or House documents.

Bulletins composing v. 47- also numbered 1-

Title varies: v. 1-49, Bulletin.

Vois. 1-49 issued by Bureau of Fisheries (called Fish Commission, v. 1-23)

1. Fisheries—U. S. 2. Fish-culture—U. S. I. Title.

SH11.A25

639.206173

9—35239\*

Library of Congress

59r55b1j

## CONTENTS

	<b>Page</b>
Collection of data .....	161
Representativeness of samples .....	163
Analyses of meristic data .....	164
Simple discriminant function .....	164
Calculated discriminant function .....	165
Reducing error in classification .....	166
Applying the calculated discriminant function .....	167
Discussion .....	168
Summary .....	169
Literature cited .....	171
Appendix .....	172
Calculating linear discriminant function .....	172
Calculating percentage of misclassification .....	173

### ABSTRACT

Data for five meristic characters from shad sampled in 1956 in the Hudson and Connecticut Rivers were found to be representative of each shad population. These data were used to derive a calculated discriminant function which correctly classified 71.6 percent of a mixed sample of Hudson and Connecticut River shad. The percentage correctly classified was increased when the fish in the region of greatest overlap in meristic counts were not classified. Using this procedure, 79.7 percent of the fish were correctly classified and 20.3 percent were incorrectly classified.

The calculated function was applied to meristic data obtained from samples of shad taken on the New York-New Jersey coast. The proportion of shad landed on the coast classified as Hudson River or Connecticut River shad was 77 percent and 23 percent, respectively. Correcting these data for the 20.3 percent error in classification, the distribution of the 1956 coastal catch was estimated to be 90 percent Hudson River shad to 10 percent Connecticut River shad. The presence of shad in the coastal samples native to areas other than the Hudson or Connecticut Rivers was considered to be negligible. The results obtained in the meristic study compared favorably with those obtained from a tagging study which was conducted concurrently with this investigation.

# CONTRIBUTIONS OF HUDSON AND CONNECTICUT RIVERS TO NEW YORK-NEW JERSEY SHAD CATCH OF 1956

By **Kenneth J. Fischler**, *Fishery Research Biologist*  
BUREAU OF COMMERCIAL FISHERIES

In 1949 the Congress of the United States, acting on the request of the Atlantic States Marine Fisheries Commission, appropriated funds for the United States Fish and Wildlife Service to conduct an Atlantic coast study of the American shad (*Alosa sapidissima*). The purpose of this investigation was to determine the factors affecting the abundance of shad and to recommend measures whereby the fishery could be managed to obtain sustained yields. The shad is an anadromous fish which spends most of its life in the sea but ascends rivers in the spring to spawn. The young stay in the rivers until fall and then enter the ocean where they remain until sexually mature, 3 to 5 years later.

In this paper, meristic data are used to determine what percentage of the shad catch from the New York-New Jersey coast is native to the Hudson and Connecticut Rivers. Previous years' tagging experiments on the New York-New Jersey coast have shown that most of the shad caught here are native to the Hudson and Connecticut Rivers (Talbot and Sykes, 1958). Talbot (1954) and Fredin (1954), in their efforts to predict the size of the shad runs in the Hudson and Connecticut Rivers, concluded that yearly fluctuations in the catch of shad off the New York-New Jersey coast could affect the number of shad available to the fishery in these rivers. Thus, if regulations were adopted to increase the size of runs in these rivers and a large portion of the shad were landed on the coast, any benefits of the regulations to the river fisheries would be of limited value.

Hill (1959) postulated that it was possible to separate, with a high degree of accuracy, Hudson River shad and Connecticut River shad in a mixed sample belonging to both of these rivers by applying the method of discriminant function analysis

to the counts of certain meristic characters. Hill analyzed meristic data obtained from the Hudson River in 1939 and from the Connecticut River in 1945. In the present study, meristic data collected in the same year (1956) from both rivers were used to derive a discriminant function. This function was then used to determine the percentage of shad from the Hudson River and from the Connecticut River landed on the New York-New Jersey coast in 1956. In the analysis of data it was assumed that only shad native to the Hudson and Connecticut Rivers were present in the coastal samples. The results of this analysis were compared with those obtained from a tagging experiment (Nichols 1958) that was conducted concurrently with the meristic study.

Staff members of the U.S. Bureau of Commercial Fisheries Biological Laboratory, Beaufort, North Carolina, assisted in the study, and shad fishermen along the New York-New Jersey coast and on the Hudson and Connecticut Rivers generously supplied fish from which the meristic counts were obtained. The author is also indebted to Donald R. Hill for his review of the statistical methods used in the manuscript.

## COLLECTION OF DATA

In the spring of 1956, meristic data were obtained from shad landed at three locations on the New Jersey coast and at two locations each in the Hudson River and in the Connecticut River. The two sampling locations in the Hudson River and the Rocky Hill sampling location in the Connecticut River (fig. 1) were located on shad spawning grounds. Samples from the New York-New Jersey coast were obtained over an 8-week period beginning April 1. Collections were made at Beach Haven, Point Pleasant, and Port Monmouth, N.J. Since the fish obtained at Port Monmouth were actually caught in the Staten

NOTE.—Approved for publication, June 3, 1958. *Fishery Bulletin* 163.

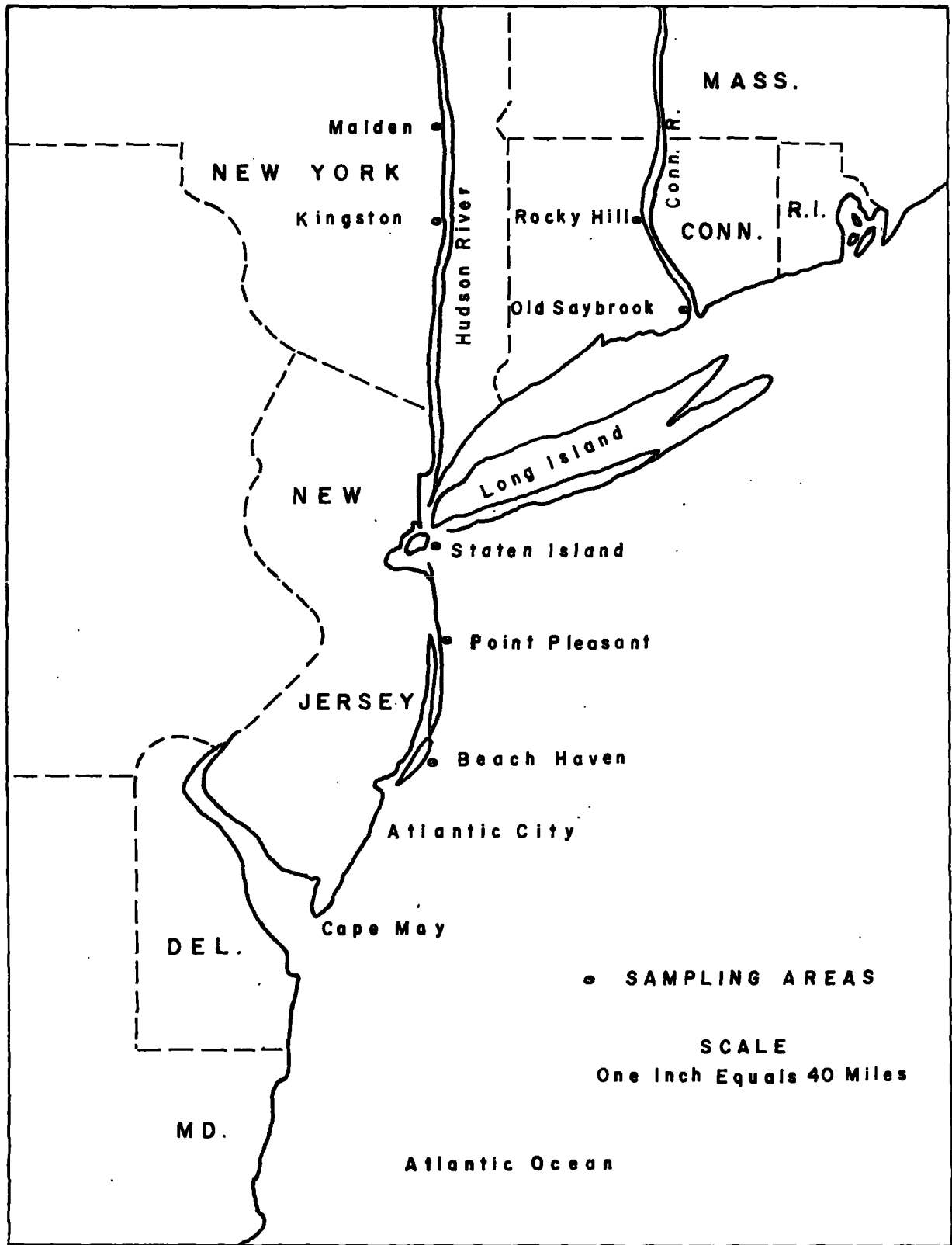


FIGURE 1.—Shad sampling areas along the New York–New Jersey coast and in the Hudson and Connecticut Rivers in 1956.

Island, N.Y., area, they will be referred to as Staten Island fish. Shad sampled at Beach Haven were taken from pound nets fished 2 miles north of Beach Haven, and the shad sampled at Point Pleasant were taken from pound nets fished 3 miles north of Point Pleasant. Samples from the Hudson River were taken weekly at Malden and Kingston, N.Y., beginning May 8 for a period of 4 weeks and at Old Saybrook and Rocky Hill, Conn., on the Connecticut River, for a 5-week period beginning May 16. Table 1 lists the number of shad sampled at each sampling location.

TABLE 1.—Number of shad (*Alosa sapidissima*) sampled, by weeks, on New York-New Jersey coast and in Hudson and Connecticut Rivers, spring 1956

Sample week	New York-New Jersey coast			Hudson River		Connecticut River		Total
	Beach Haven, N.J.	Point Pleasant, N.J.	Staten Island, N.Y.	Kingston, N.Y.	Malden, N.Y.	Old Saybrook, Conn.	Rocky Hill, Conn.	
March 31- April 6.....		20	35					55
April 7-13.....			25					25
April 14-20.....	15	20	20					55
April 21-27.....	23	20	11					54
April 28-May 4.....	30	40						70
May 5-11.....		40	30	20	20			110
May 12-18.....	8	40	24	20	20	20	20	152
May 19-25.....	9	10	10	20	20	20	20	109
May 26-June 1.....				20	20	20	20	80
June 2-8.....						20	20	40
June 9-15.....						20	20	40
Total.....	85	190	155	80	80	100	100	790

Meristic counts, fork length, weight, sex, and a scale sample were taken from each of the 790 shad studied. The age of each shad was determined from its scales using methods outlined by Cating (1953). The meristic counts were defined as follows:

*Anterior scutes:* All scutes from the most anterior scute just reaching the branchiostegals, counted posteriorly up to and including the scute between the ventral (pelvic) fins. The embedded portion of the last anterior scute is anterior to the origin of the ventral fins.

*Posterior scutes:* All scutes posterior to the ventral fins. The exposed surface of the first posterior scute in adult fish is usually longer than that of the last anterior scute.

*Pectoral-fin rays:* All rays in the left pectoral fin were counted.

*Dorsal-fin rays:* All rays including rudimentary and well-developed spinous rays (at the

anterior edge of the fin) were included in the total fin-ray count.

*Anal-fin rays:* Same as in the dorsal rays.

## REPRESENTATIVENESS OF SAMPLES

The shad samples from the Hudson and Connecticut Rivers were taken in large-mesh gill nets (averaging 5½ inches stretch measure), and samples from the New York-New Jersey coast were taken in pound nets. Because gill nets tend to select the larger shad while pound nets are regarded as nonselective, there was a possibility that the river-sampled shad did not represent all size classes in each population, and that in their meristic count the samples taken were not representative of the exploited population in each river. To determine whether the sampled data were representative, the meristic counts were analyzed in the following manner with the results indicated:

1. Analysis of variance for linear regression of each meristic count on length in the samples obtained from each location in the Hudson and Connecticut Rivers showed no significant relation. Therefore, the counts can be regarded as varying independently of length.

2. Since none of the individual meristic counts varied significantly with length, the five meristic counts from each shad were added. These sums were used after grouping the samples by river, sex, and age group to test for any cumulative meristic variation with fish length. Of the 17 regression analyses, only one group of fish, 6-year-old males from the Hudson River, showed a significant regression (5-percent level) between total count and length. This single relation will be disregarded because significance of this nature can be expected to occur by chance in 1 of 20 similar statistical tests.

3. Analysis of variance tests, using data from all shad collected at each location in the rivers, indicated no significant differences in meristic counts between males and females of the same age group or of different age groups (4-, 5-, 6-, and 7-year-old fish).

4. There were no significant differences in total meristic count between weeks at each location, between locations, or between weeks at different locations in each river. The interaction of weeks with location was not significant in either river.

From the results obtained, it can be concluded that there was no significant relation between individual meristic counts or between total meristic counts and length. Also, no significant differences in total meristic count between the age groups in each river, between males and females of any age group, or between individual samples taken in each river were found, although the samples were obtained over a period of several weeks and at two locations in each of the rivers.

Therefore, as regards meristic counts, samples of shad taken in the Hudson and Connecticut Rivers in size-selective gill nets will be considered representative of the shad population in each river subject to exploitation by the commercial fishery and will be referred to collectively as the Hudson River sample (160 fish) and the Connecticut River sample (200 fish).

The meaning of population as used in this paper is synonymous with local population as defined by Mayr, Linsley, and Usinger (1953) as follows: "The individuals of a given locality which potentially form a single interbreeding community." A population can differ from another population in the mean values of various quantitative characters and also may differ to some degree in gene makeup or frequency.

### ANALYSES OF MERISTIC DATA

To determine whether there were significant differences in meristic counts between shad from the Hudson and Connecticut Rivers, analyses of variance of the five meristic counts from samples taken in each river were calculated. These analyses are summarized in table 2. Three of the five counts showed a difference between rivers at the 1-percent level of significance and one count showed a difference at the 5-percent level. Those characters showing a difference at the 5-percent level or higher were the posterior scutes, dorsal-fin rays, pectoral-fin rays, and anal-fin rays. These four characters were of most value in separating a mixed sample of fish native to the two rivers. Linear discriminant function analysis applied to the meristic data was used to ascertain the best separation of a mixed sample of Hudson and Connecticut River shad. A simple discriminant function and a more complicated calculated discriminant function were both presented as a means of separating a mixed population. It was

shown that the calculated function distinguished Hudson from Connecticut River shad in a mixed sample with a higher degree of accuracy than the simpler function.

TABLE 2.—Analyses of variance for the five meristic characters to test for difference between rivers in 1956

[All fish (360) sampled in the Hudson and Connecticut Rivers were used in these tests]

Source of variation	df	Sum of squares	Mean square	Variance ratio <i>F</i>
Anterior scutes:				
Means.....	1	1.742	1.742	3.3
Within group.....	358	190.455	.532	
Total.....	359	192.197		
Posterior scutes:				
Means.....	1	22.894	22.894	**34.5
Within group.....	358	237.770	.664	
Total.....	359	260.664		
Dorsal-fin rays:				
Means.....	1	3.209	3.209	*5.4
Within group.....	358	212.055	.592	
Total.....	359	215.264		
Pectoral-fin rays:				
Means.....	1	23.461	23.461	**55.5
Within group.....	358	151.514	.423	
Total.....	359	174.975		
Anal-fin rays:				
Means.....	1	22.445	22.445	**24.2
Within group.....	358	332.655	.929	
Total.....	359	355.100		

\*Significant at 5-percent level.

\*\*Significant at 1-percent level.

### SIMPLE DISCRIMINANT FUNCTION

Following the methods developed by Ginsburg (1938) and used by Raney and de Sylva (1953), the sum of the five meristic counts for each fish was determined for all of the shad sampled from the Hudson and Connecticut Rivers in 1956. Actually, in summing the five counts, use is made of a simple linear discriminant function,  $Z = X_1 + X_2 + X_3 + X_4 + X_5$ . In this function,  $X_1$  = anterior scutes;  $X_2$  = posterior scutes;  $X_3$  = dorsal-fin rays;  $X_4$  = pectoral-fin rays;  $X_5$  = anal-fin rays; and  $Z$  = sum of the five counts. From the sums or "character indices" of all the fish in the samples, the frequency distributions of the counts from each river were tabulated (table 3).

The overall bias, or percentage of misclassification, of shad native to the Connecticut and Hudson Rivers is lowest when the distinction between the two populations is made between counts of 92 and 93. The number of shad sampled from the Connecticut River with a total count above 92 is 35, or 17.5 percent of the sample. The number of shad sampled from the Hudson River with a total count below 93 is 73, or 45.6 percent of the



TABLE 3.—Frequency distributions of simple linear discriminant function applied to meristic data from Hudson and Connecticut River shad.

$$[Z = X_1 + X_2 + X_3 + X_4 + X_5]$$

Z	Connecticut River	Hudson River
84	1	
85	1	
86	3	
87	7	
88	15	2
89	16	7
90	38	8
91	39	20
92	45	36
93	14	32
94	13	28
95	8	19
96		5
97		3
Total	200	160
Mean	90.9	92.7

sample. The average percentage of shad sampled from the Connecticut River with a total count higher than 92 and of shad from the Hudson River with a total count lower than 93 is 31.5 percent. If shad with a total count above 92 are considered as being native to the Hudson River, and shad with a total count below 93 are considered as being native to the Connecticut River, the overall error of classification will be 31.5 percent. Conversely, an average of 68.5 percent of a mixed sample of Hudson and Connecticut River shad will be correctly classified. The method used to determine the percentage of shad correctly or incorrectly classified is given by Ginsburg (1938).

**CALCULATED DISCRIMINANT FUNCTION**

Rao (1952), Johnson (1950), and Hill (1959) show a method of finding the best linear discriminant function for two multivariate normal populations. This method gives emphasis to the large differences that occur in posterior scute counts, pectoral-fin ray counts, and anal-fin ray counts between shad from the Hudson and Connecticut Rivers, and also makes use of the smaller differences (not necessarily significant) that occur in anterior scute counts and in dorsal-fin ray counts. This discriminant function takes the form  $Y = aX_1 + bX_2 + cX_3 + dX_4 + eX_5$ , in which  $X_1$  through  $X_5$  represent the same meristic counts as previously defined, and the coefficients ( $a$  through  $e$ ) are derived constants.

To obtain the discriminant function, the pooled "within group" sums of squares and sums of prod-

ucts for the meristic data from shad of the Hudson and Connecticut Rivers were divided by the number of degrees of freedom (358) which gave the variances and covariances that appear in table 4 in the form of a 5 by 5 matrix. The variances are in table 4 under  $X_1 \times X_1, X_2 \times X_2, \dots, X_5 \times X_5$ , and the covariances under the various combinations  $X_1 \times X_2, X_1 \times X_3, \dots, X_4 \times X_5$ . Following Rao (1952) and using the pivotal condensation method, the best linear discriminant function, using all five meristic characters, was obtained from the 5 by 5 matrix. The calculated function which will best discriminate between Hudson and Connecticut River shad is  $Y = 0.1053X_1 + 0.8014X_2 + 0.0292X_3 + 1.1978X_4 + 0.5173X_5$ . The method used to determine the discriminant function is illustrated in the appendix.

TABLE 4.—Variances and covariances of the five meristic counts in the samples of Hudson and Connecticut River shad, 1956

[Based on 300 shad]

Meristic counts	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	Difference in mean
$X_1$	0.5320	0.0060	0.0243	0.0200	0.1054	0.1400
$X_2$	.0060	.6642	.0235	-.0088	-.0290	.5080
$X_3$	.0243	.0235	.5923	.0709	.1283	.1900
$X_4$	.0200	-.0088	.0709	.4204	.0257	.5140
$X_5$	.1054	-.0290	.1283	.0257	.9292	.5030

The mean values of the meristic counts obtained in samples of shad from the Hudson and Connecticut Rivers are shown in table 5. By substituting these values into the calculated discriminant function,  $Y = 0.1053X_1 + 0.8014X_2 + 0.0292X_3 + 1.1978X_4 + 0.5173X_5$ , the mean  $Y$  values for shad from the Hudson (45.00) and Connecticut Rivers (43.70) are obtained. The mean  $Y$  value for the Hudson River sample is separated from the mean  $Y$  value for the Connecticut River sample by 1.30 units. This difference is also the variance of the discriminant function and is termed  $D^2$  (Rao 1952). If  $D^2$  is the variance of the discriminant function, the normal deviate is  $\frac{D}{2}$  with mean 0 and a standard deviation of 1. The probability of obtaining a normal deviate equal to  $\frac{D}{2}$  is equal to the probability that a fish from one of the two rivers will be classified correctly when the derived discriminant function is used. The probability of a normal deviate,  $\frac{D}{2}$  or 0.571, is equal to 1 minus

the probability of a deviate falling outside the range of 0.571 in a 1-tailed normal distribution. This probability equals 1 minus 0.284 (Fisher and Yates, table IX, 1953) or 0.716. Thus, the calculated discriminant function will correctly classify 71.6 percent of the fish in a mixed sample of Hudson and Connecticut River shad.

TABLE 5.—Means of the five meristic counts in the samples of Hudson and Connecticut River shad, 1956

Meristic count	Hudson River <sup>1</sup>	Connecticut River <sup>2</sup>
Anterior scutes, $X_1$ .....	21.87	21.73
Posterior scutes, $X_2$ .....	15.56	15.05
Dorsal-fin rays, $X_3$ .....	18.17	17.99
Pectoral-fin rays, $X_4$ .....	15.44	14.93
Anal-fin rays, $X_5$ .....	21.66	21.16
Sum.....	92.7	90.9

<sup>1</sup> Based on 160 shad.

<sup>2</sup> Based on 200 shad.

Straight addition of the counts and the use of these "character indices" would correctly classify 68.5 percent of the shad in a mixed sample from both rivers, while the more complex calculated discriminant function will correctly classify 71.6 percent of these fish as to their native river. Rao (1952) gives a test to determine if the more complicated discriminant function is better than the simpler character-index type of function when the theoretical midpoint between the two populations is the basis of separation. Applying this test to the two functions, an  $F$  value of 7.45 was obtained, which is significant at the 1-percent level. Therefore, the calculated discriminant function was significantly better than the simpler function.

Three assumptions must be satisfied before the preceding analyses are valid: that the samples approximate multivariate normal populations; that they have equal variances and covariances; and that they are large enough to be representative of the shad population in their respective rivers. Each of the five meristic characters in the samples used in the calculations approximated normal distributions. Tests for the homogeneity of variance (Snedecor 1956) of each meristic count in shad from both rivers indicated equality. Earlier it was shown that there was no significant correlation between meristic count and length of the shad. Using methods given by Snedecor (1956), the range of the variates in the meristic samples, the estimated size of the 1956 shad population in

each river (Nichols 1958), and the tests applied, the number of fish in each sample was known to be large enough for the sample to be considered representative of each population. It was concluded, therefore, that the discriminant function was developed from sufficient data and that the samples from the two rivers were representative and did approximate multivariate normal populations with equal variances and covariances.

#### REDUCING ERROR IN CLASSIFICATION

The distance between the mean  $\bar{Y}$  values of shad in samples from the Hudson and Connecticut Rivers after application of the calculated discriminant function is 1.30. Dividing this figure by 2 and adding the quotient to the mean for fish from the Connecticut River, the value 44.35 is obtained. All shad having a greater value than this are considered of Hudson River origin, and those below this value of Connecticut River origin. This function will classify correctly, as previously stated, 71.6 percent of the fish in a mixture of Hudson and Connecticut River shad, and incorrectly 28.4 percent. A reduction in this error of classification would be desirable.

In figure 2, two theoretical normal curves are shown representing the  $\bar{Y}$  values for samples of equal size from the Connecticut and Hudson Rivers, and with the line of discrimination intersecting the line of  $\bar{Y}$  values at 44.35. The error of classification (28.4 percent), using 44.35 as the separation point, is indicated by the dotted and vertical line areas. If every shad with a  $\bar{Y}$  value above 45.00 is classified as a Hudson River fish, and if every shad with a  $\bar{Y}$  value below 43.70 is classified as a Connecticut River fish, the maximum error of classification of fish from either the Connecticut or the Hudson River is equal to the probability of a deviate falling outside the range of the normal deviate  $\frac{1.30}{1.14}$ , or an error of 12.7 percent (Fisher and Yates, table IX, 1953). This error is represented by the vertical line area in figure 2. The unclassified portion of the sample fish with their total counts, after application of the discriminant function, ranging from 43.70 to 45.00 (the dotted and cross-hatched areas of figure 2) will be an expected 37.3 percent (50 minus 12.7). Of those

fish classified (62.7 percent),  $\frac{50.0}{62.7} \times 100$ , or 79.7 percent will be correctly classified, and 20.3 percent will be incorrectly classified. Therefore, an increased reliability of classification has been obtained.

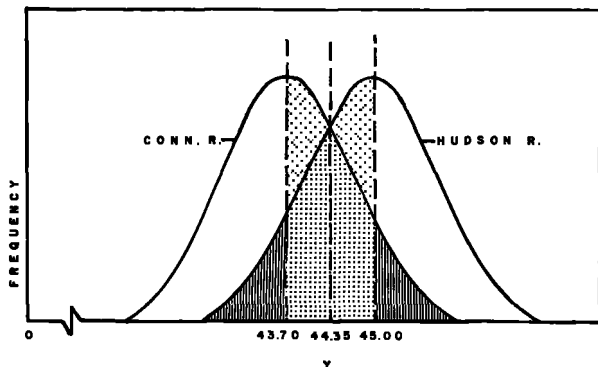


FIGURE 2.—Theoretical normal curves of the  $Y$  values for samples of shad of equal size from the Hudson and Connecticut Rivers.

### APPLYING THE CALCULATED DISCRIMINANT FUNCTION

The results of applying the calculated discriminant function,  $Y = 0.1053X_1 + 0.8014X_2 + 0.0292X_3 + 1.1978X_4 + 0.5173X_5$ , to samples of shad from Beach Haven, Point Pleasant, and Staten Island are summarized in table 6. Those shad having a  $Y$  value above 45.00 were classified as Hudson River shad, and those having a  $Y$  value below 43.70 were classified as Connecticut River shad for a total of 265 fish. Shad with a  $Y$  value between 43.70 and 45.00 were not classified, which amounted to 165, or 38.4 percent of all the sampled shad. This is in close agreement with the expected 37.3 percent unclassified, as given in the previous section.

TABLE 6.—Classification of 265 shad from the New York-New Jersey coast as Hudson and Connecticut River fish by applying calculated discriminant function  
[38.4 percent of shad in samples not classified]

Sampling location	From Hudson River		From Connecticut River		Total classified
	Number	Percent	Number	Percent	
Beach Haven.....	41	73	15	27	56
Point Pleasant.....	45	39	70	61	115
Staten Island.....	74	79	20	21	94

It was necessary to assume when applying the calculated function that the sampled shad from the New York-New Jersey coast were native to either the Hudson or Connecticut River. Therefore, the following percentages include shad correctly and incorrectly classified as from the Hudson and Connecticut Rivers and may also include shad native to other rivers along the coast. If shad native to other rivers in addition to the Hudson and Connecticut River were present in the samples obtained on the New York-New Jersey coast, an unknown bias in the percentage of Hudson River to Connecticut River shad would be introduced. This bias or error would be significant if the shad from the other rivers had a distribution of meristic counts more closely resembling those of the Hudson River than the Connecticut River, or vice versa. A further discussion of the effects of sampling shad on the New York-New Jersey coast native to neither of the two rivers will be given in a later section.

From table 6 it can be seen that 73 percent of the shad in the Beach Haven sample were classified as native to the Hudson River and 27 percent to the Connecticut River. The results at Point Pleasant were 39 percent Hudson River shad and 61 percent Connecticut River shad. At Staten Island, 79 percent of the fish in the sample were classified to the Hudson River and 21 percent to the Connecticut River. The proportion of Hudson to Connecticut River shad classified at Point Pleasant, which is located between Beach Haven and Staten Island, was almost directly opposite to that found at each of the other two stations. To determine the reason for this reversal, the best linear discriminant function for each age group was applied to the various age groups at each coastal sampling location. Although the numbers of shad in the samples differed and the meristic counts were lower in the age groups at Point Pleasant than at the other locations on the coast, the more precise  $Y$  values obtained could not account for the reversal in proportion of Hudson to Connecticut River shad in the Point Pleasant samples.

A tagging program conducted on the New Jersey coast in the spring of 1956 revealed that the shad catch from the New York-New Jersey coast was composed of 76 percent fish from the Hudson River, 13 percent from the Connecticut

River, and 11 percent from other rivers along the coast from Chesapeake Bay to the St. Lawrence River (Nichols 1958). The meristic and tagging studies gave similar results for the proportion of Hudson and Connecticut River shad in the samples obtained at Beach Haven and Staten Island, but differed in the Point Pleasant sample. Because the data obtained at Point Pleasant during the meristic study were not in agreement with those obtained at Beach Haven and Staten Island, or with those of the tagging study, it was assumed that a sampling or counting error occurred at Point Pleasant. Therefore, only the Beach Haven and Staten Island meristic data were used to estimate the proportion of Hudson and Connecticut River shad caught on the New York–New Jersey coast. Averaging the data from these two stations; the estimated percentage of shad classified to the Hudson and Connecticut Rivers that were taken off the New York–New Jersey coast in 1956 was 77 percent and 23 percent, respectively.

Discriminant function analysis will not completely discriminate a mixed sample of shad from the two rivers because 28.4 percent of the  $Y$  value distribution of each river overlaps that of the other river. Therefore, shad with  $Y$  values in the area of greatest overlap (between  $Y$  values 43.7 and 45.0 in fig. 2) were not classified. This amounted to 37.3 percent of the shad sampled in the Hudson and Connecticut Rivers, and 38.4 percent of the shad sampled on the New York–New Jersey coast. Refusing to classify 38.4 percent of all the shad sampled on the coast reduced the error in classification from 28.4 percent to 20.3 percent. This is the percentage of fish native to one river that is incorrectly classified as being native to the other river. If the coastal sample was composed of an equal number of fish from both rivers, the 20.3 percent error would cancel out. Since the percentage of shad native to the two rivers was not the same (77 percent versus 23 percent), a correction must be made to remove the 20.3 percent error and thereby obtain the best estimate of the percentage of Hudson and Connecticut River shad taken on the New York–New Jersey coast.

Of the 150 shad in the Staten Island and Beach Haven samples that were classified (table 6), 115 were classified as native to the Hudson River and 35 were classified as native to the Connecticut

River. The 115 shad classified as native to the Hudson River contained shad native to the Connecticut River, and the 35 shad classified as native to the Connecticut River contained shad native to the Hudson River. The best estimate of the number of shad native to each river was determined by solving the following pair of simultaneous equations:

$$\begin{aligned} H + 0.203 C &= 115 \\ C + 0.203 H &= 35 \end{aligned}$$

In these equations  $H$  equals the number of shad classified as Hudson River shad that were Hudson shad;  $0.203 C$  equals the number of shad classified as Hudson River shad that were Connecticut River shad;  $C$  equals the number of shad classified as Connecticut River shad that were Connecticut River shad; and  $0.203 H$  equals the number of shad classified as Connecticut River shad that were Hudson River shad. The number of shad sampled on the New York–New Jersey coast and assigned to the Hudson and Connecticut Rivers was 135 ( $112 + 23$ , or  $H + 0.203 H$ ) and 15 ( $12 + 3$ , or  $C + 0.203 C$ ), respectively. Therefore, it was concluded from this meristic study that the proportion of shad landed on the New York–New Jersey coast classified as Hudson River or Connecticut River shad was 90 percent and 10 percent, respectively.

## DISCUSSION

In the analysis of meristic data, it was assumed that the catch of shad along the New York–New Jersey coast was composed only of shad native to the Hudson and Connecticut Rivers. The tagging study, which was conducted concurrently with the meristic study on the New York–New Jersey coast (Nichols 1958), revealed that 11 percent of the shad caught here in 1956 were native to streams other than the Hudson and Connecticut Rivers, from Chesapeake Bay to the St. Lawrence River. If, as estimated from tag returns, 11 percent of the shad taken on the New York–New Jersey coast were not native to the Hudson and Connecticut Rivers, there may be an error of as much as 11 percent in the proportion of shad found native to both of these rivers (90 percent Hudson, 10 percent Connecticut). The effect of shad native to other rivers on the determination of the proportion of Hudson and Connecticut River shad taken on the coast would depend on the

meristic-count distribution of these shad. If most of the fish native to other streams had total meristic counts of 45.00 and above, after application of the calculated discriminant function they would be classified as Hudson River fish; and conversely, if most of the fish had total meristic counts of 43.70 or less, they would be classified as Connecticut River fish.

Cable, in years previous to this study, collected meristic data<sup>1</sup> from shad caught in many shad-producing areas from Chesapeake Bay to Maine. The average meristic counts obtained by Cable for shad from areas other than the Hudson and Connecticut Rivers were generally in the range between the average 1956 counts for the shad sampled in the two rivers. Therefore, the error introduced by classifying coastal-caught shad native to other streams as Hudson River or Connecticut River fish was considered negligible.

From the tagging study which was conducted concurrently with the meristic study, it was concluded that the shad catch on the New York-New Jersey coast was composed of 76 percent Hudson River fish, 13 percent Connecticut River fish, and 11 percent fish from other areas (Nichols 1958). The proportion of Hudson River to Connecticut River fish in the New York-New Jersey coastal shad catch was determined from these data. Seventy-six percent of the shad caught on the coast were considered to be native to the Hudson River, and 13 percent were considered to be native to the Connecticut River. Consequently, the portion of Hudson River fish in a ratio of Hudson River to Connecticut River shad caught on the coast was  $\frac{76}{76+13}$ , or 85 percent. The portion of shad considered to be native to the Connecticut River was  $\frac{13}{76+13}$ , or 15 percent. Therefore, as determined from the tagging study, the best estimate of the proportion of shad landed on the coast native to the Hudson River and Connecticut River was 85 percent and 15 percent, respectively. From the meristic study it was calculated that the coastal shad catch was composed of 90 percent Hudson River and 10 percent Connecticut River fish. The proportion of shad taken on the coast native to the Hudson River and Connecticut River as deter-

mined by meristic data and tagging studies compares favorably. This favorable comparison indicates that the proportion of shad native to the Hudson River and Connecticut River taken on the coast, as calculated from the sampled meristic data, was not appreciably affected by shad native to rivers other than the Hudson and Connecticut.

In the meristic study it was assumed that the coastal catch was composed of only Hudson and Connecticut River fish; however, the tagging study revealed that approximately 11 percent of this catch was composed of shad from other areas. Since this percentage was small, its effect on the meristic determination of the proportion of Hudson River to Connecticut River shad caught on the coast would be negligible, and for practical purposes could be disregarded.

It was estimated that the cost of the meristic study was approximately one-tenth that of the tagging program. Therefore, when a meristic study is practical to separate populations, this method should be considered since it may yield information comparable to that obtained from a tagging study at only a fraction of the cost.

## SUMMARY

Meristic data obtained from shad sampled on the New York-New Jersey coast and in the Hudson and Connecticut Rivers were analyzed to determine the proportion of the 1956 coastal catch native to the two rivers.

Meristic counts obtained from shad sampled in the Hudson and Connecticut Rivers were found to be representative of each shad population. Five meristic characters were used to derive a simple discriminant function that correctly classified 68.5 percent of the fish in a mixed sample of Hudson and Connecticut River shad. The calculated best linear discriminant function, which gave emphasis to the larger differences between certain meristic characters of Hudson and Connecticut River shad, correctly classified 71.6 percent of a mixed sample of Hudson and Connecticut River shad. The percentage correctly classified can be increased if the fish in the region of greatest overlap in meristic counts are not classified. Therefore, when 62.7 percent of the fish in the sample are classified, the percentage correctly classified is increased to 79.7 percent. The error in classification (20.3 percent) is the percent of

<sup>1</sup> Unpublished data. U.S. Bureau of Commercial Fisheries, Biological Laboratory, Beaufort, N.C.

either population classified as being native to the other population.

The calculated best linear discriminant function obtained from the Hudson and Connecticut River shad meristic data was applied to the meristic data from shad samples obtained at three locations on the New York–New Jersey coast—Beach Haven and Point Pleasant, N.J., and Staten Island, N.Y. Assuming that only shad from the Hudson and Connecticut Rivers were present in the coastal samples, the percentages of shad assigned to each river were Beach Haven, 73 percent Hudson River, 27 percent Connecticut River; Point Pleasant, 39 percent Hudson River, 61 percent Connecticut River; Staten Island, 79 percent Hudson River, 21 percent Connecticut River. The meristic data obtained at Point Pleasant were not used since they did not agree with the findings at Beach Haven or at Staten Island or with the results of the tagging program which was conducted concurrently with the meristic study.

Analysis of meristic data from shad sampled at Beach Haven and Staten Island revealed that the ratio of Hudson-to-Connecticut shad in the New York–New Jersey coast catch was 77 percent and 23 percent, respectively. After correction of these results for the 20.3 percent error in classification of shad native to either river, the percentages of Hudson and Connecticut shad in the New York–New Jersey coast shad catch in 1956 were estimated to be 90 and 10 percent.

A tagging study conducted concurrently with the meristic study on the New York–New Jersey coast revealed that the coastal shad catch was composed of 11 percent fish native to rivers other

than the Hudson and Connecticut. If these shad had meristic-count distributions similar to either Hudson or Connecticut River shad, a bias would be introduced into the determination of the proportion of Hudson to Connecticut River fish taken on the coast as determined from the meristic study. Previous studies indicate that the average meristic counts for shad caught in many shad producing areas from Maine to Chesapeake Bay are generally in the range of the meristic counts found for the shad sampled in the Hudson and Connecticut Rivers. Therefore, the error introduced into the determination of the proportion of Hudson River to Connecticut River fish in the coastal catch was considered negligible.

It was concluded from the tagging study conducted concurrently with the present investigation that the coastal shad catch was composed of 76 percent Hudson River fish, 13 percent Connecticut River fish, and 11 percent fish from other areas. The proportion of Hudson to Connecticut River shad in the coastal shad catch was therefore 85 percent and 15 percent, respectively. These results compare favorably with those obtained from the meristic study where it was determined that the coastal catch was 90 percent Hudson River fish to 10 percent Connecticut River fish.

The cost of the meristic study was approximately one-tenth that of the tagging study. Therefore, when a meristic study is practical to separate populations, it should be considered since it may yield comparable information to that obtained from a tagging study at only a fraction of the cost.

## LITERATURE CITED

- CATING, JAMES P.  
1953. Determining age of Atlantic shad from their scales. U.S. Fish and Wildlife Service, Fish. Bull., vol. 54, No. 85, pp. 187-199.
- FISHER, R. A., and FRANK YATES.  
1953. Statistical tables for biological, agricultural, and medical research. 126 pp., Hafner Publishing Co., Inc., New York.
- FREDIN, REYNOLD A.  
1954. Causes of fluctuations in abundance of Connecticut River shad. U.S. Fish and Wildlife Service, Fish. Bull., vol. 54, No. 88, pp. 247-259.
- GINSBURG, I.  
1938. Arithmetical definition of the species, subspecies, and race concept, with a proposal for a modified nomenclature. Zoologica, vol. 23, Part 3, pp. 253-286.
- HILL, DONALD R.  
1959. Some uses of statistical analysis in classifying races of American shad (*Alosa sapidissima*). U.S. Fish and Wildlife Service, Fish. Bull., vol. 59, No. 147, pp. 269-286.
- JOHNSON, P. O.  
1950. Statistical methods in research. 377 pp., Prentice Hall, Inc., New York.
- MAYR, E., E. G. LINSLEY, and R. L. USINGER.  
1953. Methods and principles of systematic zoology. 328 pp., McGraw-Hill Book Co., Inc., New York.
- NICHOLS, PAUL R.  
1958. Effect of New Jersey-New York pound net catch on shad runs of Hudson and Connecticut Rivers. U.S. Fish and Wildlife Service, Fish. Bull., vol. 58, No. 143, pp. 491-500.
- RAO, C. RADHAKRISHNA.  
1952. Advanced statistical methods in biometric research. 390 pp., John Wiley and Sons, New York.
- RANEY, EDWARD C., and DONALD P. DE SYLVA.  
1953. Racial investigations of the striped bass, *Morone saxatilis* (Walbaum). Jour. Wildlife Mgt., vol. 17, No. 4, pp. 495-509.
- SNEDECOR, GEORGE W.  
1956. Statistical methods. 5th ed., 534 pp., The Iowa State College Press, Ames, Iowa.
- TALBOT, GERALD B.  
1954. Factors associated with fluctuations in abundance of Hudson River shad. U.S. Fish and Wildlife Service, Fish. Bull., vol. 56, No. 101, pp. 373-413.
- TALBOT, GERALD B., and JAMES E. SYKES.  
1958. Atlantic coast migrations of American shad. U.S. Fish and Wildlife Service, Fish. Bull., vol. 58, No. 142, pp. 473-490.

APPENDIX

CALCULATING LINEAR DISCRIMINANT FUNCTION

Rao (1952) and Johnson (1950) outlined the method that was used to calculate the discriminant function. In this supplement, calculation of the discriminant function is presented in a simple but detailed manner. The method can be applied to any number of characters, but to simplify the process, three characters from each fish have been used. The three sets of characters from each of two groups of fish are denoted as *a*, *b*, and *c*. Group 1 consists of 30 fish and group 2 of 40 fish (table A-1). The steps in calculating the discriminant function are as follows:

*Step 1.*—Calculate the values of the characters as listed in table A-2.

TABLE A-1.—Values of three selected characters from two groups of shad

Group 1 <sup>1</sup>			Group 2 <sup>2</sup>		
<i>a</i> <sub>1</sub>	<i>b</i> <sub>1</sub>	<i>c</i> <sub>1</sub>	<i>a</i> <sub>2</sub>	<i>b</i> <sub>2</sub>	<i>c</i> <sub>2</sub>
23	16	17	22	13	17
23	15	17	22	14	17
22	14	20	22	15	17
21	16	17	22	15	18
21	16	18	21	15	18
21	16	18	22	15	18
22	16	17	22	13	18
22	17	17	21	15	18
22	16	18	22	15	18
22	16	17	22	14	18
21	15	18	21	15	18
21	17	17	21	16	17
23	16	18	22	15	17
21	15	18	21	16	18
22	14	17	22	15	17
23	17	17	22	15	18
22	16	20	22	15	18
22	15	18	21	15	17
22	14	18	22	15	17
23	17	19	22	15	19
22	15	19	23	17	18
22	15	18	22	16	17
21	16	18	22	14	17
21	17	19	22	15	18
22	15	19	21	13	18
21	16	18	21	15	18
21	15	19	20	14	17
22	16	17	22	16	19
23	14	18	23	15	19
22	15	18	21	15	18
			21	15	19
			21	15	19
			23	13	17
			22	15	18
			22	16	18
			21	15	19
			23	15	18
			23	14	18
			21	14	19
			21	15	17

<sup>1</sup> Based on 30 shad.  
<sup>2</sup> Based on 40 shad.

*Step 2.*—Calculate the pooled within-groups sums of squares and sums of products. This is shown in detail in table A-3 and summarized in table A-4.

*Step 3.*—The variances and covariances shown in table A-5 were determined by dividing the within-groups values in table A-4 by the number of degrees of freedom (*n*<sub>1</sub> + *n*<sub>2</sub> - 2), which in this case is 68. The "difference in means" column in table A-5 is determined as follows:

$$\bar{a}_1 - \bar{a}_2, \bar{b}_1 - \bar{b}_2, \text{ and } \bar{c}_1 - \bar{c}_2.$$

TABLE A-2.—Sum, mean, sum of squares, and sum of products for the three characters in the two groups of shad

Characters	Sum	Mean	Sum of squares	Sum of products
Group 1 ( <i>n</i> <sub>1</sub> =30 fish):				
<i>a</i> .....	Σ <i>a</i> <sub>1</sub> =656	$\bar{a}_1=21.8667$	Σ <i>a</i> <sub>1</sub> <sup>2</sup> =14,360	Σ <i>a</i> <sub>1</sub> <i>b</i> <sub>1</sub> =10,232
<i>b</i> .....	Σ <i>b</i> <sub>1</sub> =468	$\bar{b}_1=15.6000$	Σ <i>b</i> <sub>1</sub> <sup>2</sup> =7,326	Σ <i>a</i> <sub>1</sub> <i>c</i> <sub>1</sub> =11,784
<i>c</i> .....	Σ <i>c</i> <sub>1</sub> =539	$\bar{c}_1=17.9667$	Σ <i>c</i> <sub>1</sub> <sup>2</sup> =9,707	Σ <i>b</i> <sub>1</sub> <i>c</i> <sub>1</sub> =8,403
Group 2 ( <i>n</i> <sub>2</sub> =40 fish):				
<i>a</i> .....	Σ <i>a</i> <sub>2</sub> =868	$\bar{a}_2=21.7000$	Σ <i>a</i> <sub>2</sub> <sup>2</sup> =18,854	Σ <i>a</i> <sub>2</sub> <i>b</i> <sub>2</sub> =12,869
<i>b</i> .....	Σ <i>b</i> <sub>2</sub> =593	$\bar{b}_2=14.8250$	Σ <i>b</i> <sub>2</sub> <sup>2</sup> =8,821	Σ <i>a</i> <sub>2</sub> <i>c</i> <sub>2</sub> =15,537
<i>c</i> .....	Σ <i>c</i> <sub>2</sub> =716	$\bar{c}_2=17.9000$	Σ <i>c</i> <sub>2</sub> <sup>2</sup> =12,836	Σ <i>b</i> <sub>2</sub> <i>c</i> <sub>2</sub> =10,617

TABLE A-3.—Calculation of pooled total and group sums of squares and sums of products for groups 1 and 2

[*n*<sub>1</sub>=30 fish; *n*<sub>2</sub>=40 fish]

Characters	Calculation for groups 1 and 2
<i>a</i> <sub>1</sub> , <i>a</i> <sub>2</sub> :	
Total.....	Σ <i>a</i> <sub>1</sub> <sup>2</sup> + Σ <i>a</i> <sub>2</sub> <sup>2</sup> = 33,214.0000
Group sums.....	$\frac{(\Sigma a_1)^2}{n_1} + \frac{(\Sigma a_2)^2}{n_2} = \frac{(656)^2}{30} + \frac{(868)^2}{40} = 33,180.1333$
<i>a</i> <sub>1</sub> , <i>b</i> <sub>1</sub> , <i>a</i> <sub>2</sub> , <i>b</i> <sub>2</sub> :	
Total.....	Σ <i>a</i> <sub>1</sub> <i>b</i> <sub>1</sub> + Σ <i>a</i> <sub>2</sub> <i>b</i> <sub>2</sub> = 23,101.0000
Group sums.....	$\frac{(\Sigma a_1)(\Sigma b_1)}{n_1} + \frac{(\Sigma a_2)(\Sigma b_2)}{n_2} = \frac{(656)(468)}{30} + \frac{(868)(593)}{40} = 23,101.7000$
<i>a</i> <sub>1</sub> , <i>c</i> <sub>1</sub> , <i>a</i> <sub>2</sub> , <i>c</i> <sub>2</sub> :	
Total.....	Σ <i>a</i> <sub>1</sub> <i>c</i> <sub>1</sub> + Σ <i>a</i> <sub>2</sub> <i>c</i> <sub>2</sub> = 27,321.0000
Group sums.....	$\frac{(\Sigma a_1)(\Sigma c_1)}{n_1} + \frac{(\Sigma a_2)(\Sigma c_2)}{n_2} = \frac{(656)(539)}{30} + \frac{(868)(716)}{40} = 27,323.3333$
<i>b</i> <sub>1</sub> , <i>b</i> <sub>2</sub> :	
Total.....	Σ <i>b</i> <sub>1</sub> <sup>2</sup> + Σ <i>b</i> <sub>2</sub> <sup>2</sup> = 16,147.0000
Group sums.....	$\frac{(\Sigma b_1)^2}{n_1} + \frac{(\Sigma b_2)^2}{n_2} = \frac{(468)^2}{30} + \frac{(593)^2}{40} = 16,092.0250$
<i>b</i> <sub>1</sub> , <i>c</i> <sub>1</sub> , <i>b</i> <sub>2</sub> , <i>c</i> <sub>2</sub> :	
Total.....	Σ <i>b</i> <sub>1</sub> <i>c</i> <sub>1</sub> + Σ <i>b</i> <sub>2</sub> <i>c</i> <sub>2</sub> = 19,020.0000
Group sums.....	$\frac{(\Sigma b_1)(\Sigma c_1)}{n_1} + \frac{(\Sigma b_2)(\Sigma c_2)}{n_2} = \frac{(468)(539)}{30} + \frac{(593)(716)}{40} = 19,023.1000$
<i>c</i> <sub>1</sub> , <i>c</i> <sub>2</sub> :	
Total.....	Σ <i>c</i> <sub>1</sub> <sup>2</sup> + Σ <i>c</i> <sub>2</sub> <sup>2</sup> = 22,543.0000
Group sums.....	$\frac{(\Sigma c_1)^2}{n_1} + \frac{(\Sigma c_2)^2}{n_2} = \frac{(539)^2}{30} + \frac{(716)^2}{40} = 22,500.4333$



TABLE A-4.—Calculation of within-groups sum of squares and sum of products for the three characters within the two groups, 1 and 2

	a	b	c
a	Total=33,214.0000 Groups=33,180.1333 Within groups= 33.8667	Total=23,101.0000 Groups=23,101.7000 Within groups= -.7000	Total=27,321.0000 Groups=27,323.3333 Within groups= -2.3333
b		Total=16,147.0000 Groups=16,092.0250 Within groups= 54.9750	Total=19,020.0000 Groups=19,023.1000 Within groups= -.1000
c			Total=22,543.0000 Groups=22,500.4333 Within groups= 42.5667

TABLE A-5.—Variance and covariance based on (n<sub>1</sub>+n<sub>2</sub>-2) degrees of freedom

	a	b	c	Difference in means	Sum including indented
a	0.4980	-0.0103	-0.0343	0.1667	0.6201
b		.8085	-.0456	.7750	1.5276
c			.6260	.0667	.6128

TABLE A-6.—Pivotal condensation of 3 by 3 matrix to obtain successive best discriminant functions

[Numerical values]

	I	II	III	IV	V	VI
				Difference in means	Sum including indented	Check excluding indented
01	0.4980	-0.0103	-0.0343	0.1667	0.6201	
02		.8085	-.0456	.7750	1.5276	
03			.6260	.0667	.6128	
10	1.0000	-.0207	-.0689	.3347	1.0084	1.2451
11	-.0207	.8083	-.0463	.7784	1.5197	1.5404
12	-.0689		.6236	.0782	.5866	.6555
13	.3347			-.0558	1.1355	.8008
20	-.0256	1.0000	-.0573	.9630		1.8901
21	-.0701	-.0573	.6209	.1228	.6163	.6736
22	.3546	.9630		-.8054	.6350	-.3280
30	-.1129	-.0923	1.0000	.1978		.9926
31	.3685	.9743	.1978	-.8297	.7109	.5131

<sup>1</sup> Sum of difference.

Step 4.—Lines 01, 02, and 03 in table A-6 are the same as lines a, b, and c in table A-5. By applying the pivotal condensation method to the 3 x 3 matrix in table A-5, successive discriminant functions are obtained using one, two, and then three characters (table A-6, lines 13, 22, 31). In

table A-7, letters are used to illustrate the pivotal condensation method for obtaining the values shown in table A-6. For example, the value 0.6236 shown in line 12, column III was calculated by the formula  $K - \left( G \cdot \frac{G}{E} \right)$  (table A-7, line 12, column III) as follows: 0.6260 - (-0.0343) (.0689) = 0.6236. Column VI of table A-6 is used to check on the mathematical computations as one proceeds with the pivotal condensation of the matrix.

Line 31 of table A-6 is the best linear discriminant function calculated from the three sets of measured characters. This function takes the form  $Y = .3685a + .9743b + .1978c$ .

### CALCULATING PERCENTAGE OF MISCLASSIFICATION

If the values for  $\bar{a}$ ,  $\bar{b}$ , and  $\bar{c}$ , in group 1 of table A-2 are substituted in the evolved discriminant function, the Y value for group 1 is 0.3685(21.8667) + 0.9743(15.6000) + 0.1978(17.9667) = 26.8108. When the mean Y value for group 2, 0.3685 (21.7000) + 0.9743 (14.8250) + 0.1978 (17.9000) = 25.9811, is subtracted from the Y value for group 1, the difference is 0.8297. This value is the same as line 31, column IV, table A-6, and is equal to the variance of the derived function.

TABLE A-7.—Pivotal condensation of a 3 by 3 matrix to obtain successive best discriminant functions

[Coded letter values]

	I	II	III	IV	V	VI
				Difference in means	Sum including indented	Check excluding indented
01.....	$E$	$F$	$G$	$d_a$	$(E+F+G+d_a)=L$ $(F+H+J+d_b)=M$ $(G+K+d_c)=N$ $(d_a+d_b+d_c)=T$	
02.....		$H$	$J$	$d_b$		
03.....			$K$	$d_c$		
10.....	$\frac{E}{E}$	$\frac{F}{E}$	$\frac{G}{E}$	$\frac{d_a}{E}$		$\frac{E+F+G+d_a}{E}=S$
11.....	$\frac{F}{E}$	$H - \left( F \cdot \frac{F}{E} \right) = P$	$J - \left( F \cdot \frac{G}{E} \right) = Q$	$d_b - \left( F \cdot \frac{d_b}{E} \right) = R$	Sum of line 11 = $U$	$M - (F.S)$ or $U - \frac{F}{E}$
12.....	$\frac{G}{E}$		$K - \left( G \cdot \frac{G}{E} \right) = V$	$d_c - \left( G \cdot \frac{d_c}{E} \right) = W$	Sum of line 12 + $Q = X$	$N - (G.S)$ or $X - \frac{G}{E}$
13.....	$\frac{d_a}{E}$			$0 - \left( d_a \cdot \frac{d_a}{E} \right) = Y$	Sum of line 13 + $R + W = Z$	$T - (d_a.S)$ or $Z - \frac{d_a}{E}$
20.....	$\frac{F}{E} + P = c$	$\frac{P}{P}$	$\frac{Q}{P}$	$\frac{R}{P}$		$\frac{F}{E} + P + Q + R$ $\frac{\quad}{P} = g$
21.....	$\frac{G}{E} = (c.Q) = m$	$\frac{Q}{P}$	$V - \left( Q \cdot \frac{Q}{P} \right) = h$	$W - \left( Q \cdot \frac{R}{P} \right) = j$	Sum of line 21 = $f$	$X - (g.Q)$ or $f - \frac{Q}{P}$
22.....	$\frac{d_a}{E} - (c.R) = r$	$\frac{R}{P}$		$Y - \left( R \cdot \frac{R}{P} \right) = t$	Sum of line 22 + $j = z$	$Z - (G.R)$ or $z - \frac{R}{P}$
30.....	$\frac{m}{h}$	$\frac{Q}{Ph} = w$	$\frac{h}{h}$	$\frac{j}{h}$		Sum of line 30 = $q$
31.....	$r - \left( \frac{m}{h} \cdot j \right)$	$\frac{R}{P} - (w.j)$	$\frac{j}{h}$	$t - \left( j \cdot \frac{j}{h} \right)$	Sum of line 31 = $v$	$v - \frac{j}{h}$ or $z - q.j$

When 0.8297 is divided by two and this quotient (0.4148) is added to 25.9811 or subtracted from 26.8108, the value 26.3960 is obtained. If the  $a$ ,  $b$ , and  $c$  values for any unclassified fish belonging to group 1 or group 2 are substituted in the discriminant function, any fish with a  $Y$  value above 26.3960 will be classified as group 1, and any fish with a  $Y$  value of less than 26.3960 will be classified as group 2.

The error of classification will be equal to 1 minus the probability of the normal deviate  $\frac{0.4148}{\sqrt{0.8297}} = \frac{0.4148}{0.9110} = 0.46$ . The probability of this normal deviate is 0.68 (Fisher and Yates, table IX, 1953). Therefore, the error of classification for group 1 fish or group 2 fish is 32 percent. When classifying a mixed sample containing shad belonging to either of the two groups, 32 percent of the sample will be incorrectly classified.