

Simulation of Tail Weight Distributions in Biological Year 1986–2006 Landings of Brown Shrimp, *Farfantepenaeus aztecus*, from the Northern Gulf of Mexico Fishery

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Introduction

Size distribution within reported landings is an important aspect of northern Gulf of Mexico penaeid shrimp stock assessments. It reflects population characteristics such as numerical abundance of various sizes, age structure, and vital rates (e.g. recruitment, growth, and mortality), as well as effects of fishing, fishing power, fishing practices, sampling, size-grading, etc. (Kutkuhn, 1962; Neal,

1967; Rothschild and Brunenmeister, 1984; Nance et al., 1994; Diop et al., 2007; Caillouet et al., 2008; Nance et al., 2010; Parrack¹; Nichols²). Age of shrimp cannot be determined directly (Parrack, 1979; Rothschild and Brunenmeister, 1984; Neal and Maris, 1985). Therefore, age structure of shrimp in reported landings has been determined

indirectly by estimating numbers of shrimp from pounds allocated to marketing size categories, and transforming size into age using growth curves (Neal, 1967; Rothschild and Brunenmeister, 1984; Nance et al., 1994; Parrack¹; Nichols²).

Most but not all reported landings from northern Gulf of Mexico shrimp fisheries are size-graded. The usual measure of shrimp size in archived landings data is count (C), the number of shrimp tails (abdomen or edible portion) per pound (0.4536 kg). Shrimp are marketed and landings reported in pounds within tail count categories. Statistically, these count categories are count class intervals or bins with upper and lower limits expressed in C . The upper and lower limits of most count class intervals

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¹Parrack, M. L. 1981. Some aspects of brown shrimp exploitation in the northern Gulf of Mexico. Presented at the Workshop on the Scientific Basis for the Management of Penaeid Shrimp, Key West, Fla., Southeast Fisheries Science Center, National Marine Fisheries Service, NOAA, Miami, Fla. Unpubl. rep., 50 p.

²Nichols, S. 1984. Updated assessments of brown, white and pink shrimp in the U.S. Gulf of Mexico. Presented at the Workshop on Stock Assessment, Miami, Fla., Southeast Fisheries Science Center, National Marine Fisheries Service, NOAA, Miami, Fla. Unpubl. rep., 54 p.

ABSTRACT—Size distribution within reported landings is an important aspect of northern Gulf of Mexico penaeid shrimp stock assessments. It reflects shrimp population characteristics such as numerical abundance of various sizes, age structure, and vital rates (e.g. recruitment, growth, and mortality), as well as effects of fishing, fishing power, fishing practices, sampling, size-grading, etc.

The usual measure of shrimp size in archived landings data is count (C) the number of shrimp tails (abdomen or edible portion) per pound (0.4536 kg). Shrimp are marketed and landings reported in pounds within tail count categories. Statistically, these count categories are count class intervals or bins with upper and lower limits expressed in C . Count categories vary in width, overlap, and frequency of occurrence within the landings. The upper and lower limits of most count class intervals can be transformed to lower and upper limits (respectively) of class intervals

expressed in pounds per shrimp tail, w , the reciprocal of C (i.e. $w = 1/C$).

Age based stock assessments have relied on various algorithms to estimate numbers of shrimp from pounds landed within count categories. These algorithms required underlying explicit or implicit assumptions about the distribution of C or w . However, no attempts were made to assess the actual distribution of C or w . Therefore, validity of the algorithms and assumptions could not be determined. When different algorithms were applied to landings within the same size categories, they produced different estimates of numbers of shrimp.

This paper demonstrates a method of simulating the distribution of w in reported biological year landings of shrimp. We used, as examples, landings of brown shrimp, *Farfantepenaeus aztecus*, from the northern Gulf of Mexico fishery in biological years 1986–2006. Brown shrimp biological year, T_1 , is defined as beginning on 1 May of the same calendar year as T_1 and

ending on 30 April of the next calendar year, where subscript i is the place marker for biological year. Biological year landings encompass most if not all of the brown shrimp life cycle and life span. Simulated distributions of w reflect all factors influencing sizes of brown shrimp in the landings within a given biological year. Our method does not require a priori assumptions about the parent distributions of w or C , and it takes into account the variability in width, overlap, and frequency of occurrence of count categories within the landings. Simulated biological year distributions of w can be transformed to equivalent distributions of C .

Our method may be useful in future testing of previously applied algorithms and development of new estimators based on statistical estimation theory and the underlying distribution of w or C . We also examine some applications of biological year distributions of w , and additional variables derived from them.

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Age based stock assessments have relied on various algorithms to estimate numbers of shrimp from pounds landed within count categories (e.g. Neal, 1967; Rothschild and Brunenmeister, 1984; Nance et al., 1994; Diop et al., 2007; Parrack¹; Nichols²). These algorithms required underlying explicit or implicit assumptions about the distribution of C or w . However, no attempts were made to assess the actual distributions of C and w . Therefore, validity of the algorithms and assumptions could not be determined. When different algorithms were applied to landings within the same size categories (e.g. Parrack¹ vs. Nichols²), they produced different estimates of numbers of shrimp (Caillouet, 2003).

Estimating numbers of shrimp from pounds landed within size categories is statistically challenging for additional reasons. Some count categories representing the largest shrimp have an implied lower limit of zero (e.g. < 15 count), and some representing the smallest shrimp have an implied upper limit of ∞ (e.g. > 67 count). Neither zero nor ∞ can be transformed to real values of w . Count categories also exhibit considerable variability in width, overlap, and frequency of occurrence within the landings. Certain count categories dominate the landings, reflecting what are referred to as standard count categories: <15, 15–20, 21–25, 26–30, 31–40, 41–50, 51–67, and > 67 count (Caillouet et al., 2008).

This paper demonstrates a method of simulating the distribution of w in reported biological year landings of shrimp, as a basis for further investigation and evaluation of previously used algorithms and development of new ones. We used, as examples, landings of brown shrimp, *Farfantepenaeus aztecus*, from the northern Gulf of Mexico fishery in biological years 1986–2006. Neal (1967) defined brown shrimp biological year, T_i , as beginning 1 May of the same calendar year as T_i and ending 30 April of the next calen-

Table 1.—Symbols and descriptions of variables used in analyses of biological year reported landings of brown shrimp from the northern Gulf of Mexico fishery. These apply only to size-graded landings in legitimate count categories; i.e. data selected by filtering, editing, and removing residual outliers from archived landings data.

Symbols	Descriptions of variables
T_i	biological year, from 1 May of a given calendar year through 30 April of the next calendar year, where $i = 0, \dots, 20$ is the place marker for biological years 1986–2006
C_{ij}	the j^{th} lower limit of a legitimate count (number per pound) category in landings data from the i^{th} biological year, where $j = 0, \dots, m_i$
m_i	the total number of C_{ij} in landings data from the i^{th} biological year
w_{ij}	the j^{th} upper limit of a pounds per shrimp tail category, where $w_{ij} = 1/C_{ij}$, in landings data from the i^{th} biological year
P_{ij}	the j^{th} cumulative proportion of pounds landed at w_{ij} in i^{th} biological year
q_{ij}	the j^{th} weighting factor for the P_{ij} and w_{ij} data pairs in the i^{th} biological year. This weighting factor, q_{ij} , is the sum of observations over all count categories having C_j as their lower limit (or w_j as their upper limit), regardless of the recorded upper limits of these count categories
w'_k	the k^{th} simulated value of weight per shrimp tail, where $0.005155 \text{ lb} \leq w'_k \leq 0.111111 \text{ lb}$, $k = 0, \dots, 999$, and the interval between the w'_k is 0.000106
P'_{ik}	the k^{th} cumulative proportion of pounds landed at w'_k in the i^{th} biological year, which is simulated from the modified Richards function fitted to P_{ij} on w_{ij} in the i^{th} biological year
a_i	the parameter, estimated from the modified Richard's function fitted to P_{ij} on w_{ij} in the i^{th} biological year, which allows the w'_k at which $P'_{ik} = P_{\text{max}}/2$ to vary among biological years
b_i	the parameter, estimated from the modified Richard's function fitted to P_{ij} on w_{ij} in the i^{th} biological year, which represents the maximum intrinsic rate of increase in P'_{ik} per unit w'_k at the inflection point of the curve
c_i	the parameter, estimated from the modified Richard's function fitted to P_{ij} on w_{ij} in the i^{th} biological year, which allows the sigmoid shape of the curve to vary (symmetrical or asymmetrical) among biological years
p'_{ik}	the k^{th} simulated proportion of pounds landed at w'_k in the i^{th} biological year
Y_i	the i^{th} biological year yield, which includes pounds of brown shrimp tails landed in legitimate count categories and in the unknown size category combined
f'_k	the k^{th} simulated number of shrimp tails at w'_k , where $0.005155 \text{ lb} \leq w'_k \leq 0.111111 \text{ lb}$, in the i^{th} biological year
N_i	the simulated total number of shrimp tails landed in the i^{th} biological year
$w50_i$	the simulated pounds per shrimp tail at which half of Y_i is harvested in the i^{th} biological year
N_i/Y_i	the simulated mean count of brown shrimp in the landings from the i^{th} biological year
Y_i/N_i	the simulated mean pounds per shrimp tail of brown shrimp in the landings from the i^{th} biological year

dar year, where subscript i is the place marker for biological year (Table 1). Most landings in the i^{th} biological year are assumed to be produced from cohorts recruited to the fishery within that same biological year. In other words, a biological year encompasses most of the cycle and life span of brown shrimp within this intensive fishery.

Our approach does not require a priori assumptions about the parent distributions of w or C , and it takes into account the variability in width, overlap, and frequency of occurrence of count categories within the landings. Simulated biological year distributions of w can easily be transformed to equivalent distributions of C . Our method may be useful in future testing of previously applied algorithms and development of new estimators based on statistical estimation theory and the underlying distribution of w or C . We also examine some applications of biological year distributions of w and additional variables derived from them.

Materials and Methods

Fishery

The brown shrimp fishery of the northern Gulf of Mexico is bounded by statistical subareas 10–21, and comprises inshore (estuarine) and offshore (Gulf of Mexico) territorial waters of Texas, Louisiana, Mississippi, Alabama, and a portion of Northwestern Florida, as well as adjoining Federal waters landward of the 50 fm depth contour within the U.S. Exclusive Economic Zone (EEZ) (Fig. 1). Brown shrimp produce annual crops (Neal and Maris, 1985), with recruitment to the fishery occurring in May–July (Rothschild and Brunenmeister, 1984). Although life span is 20–27 mo (Baxter, 1971), most brown shrimp are harvested within 6 mo of age.³ Neal (1967)

³Fishery Management Plan for the Shrimp Fishery of the Gulf of Mexico, United States Waters. Gulf of Mexico Fishery Management Council, Tampa, Fla., Nov. 1981 (online at <http://www.gulfcouncil.org>).

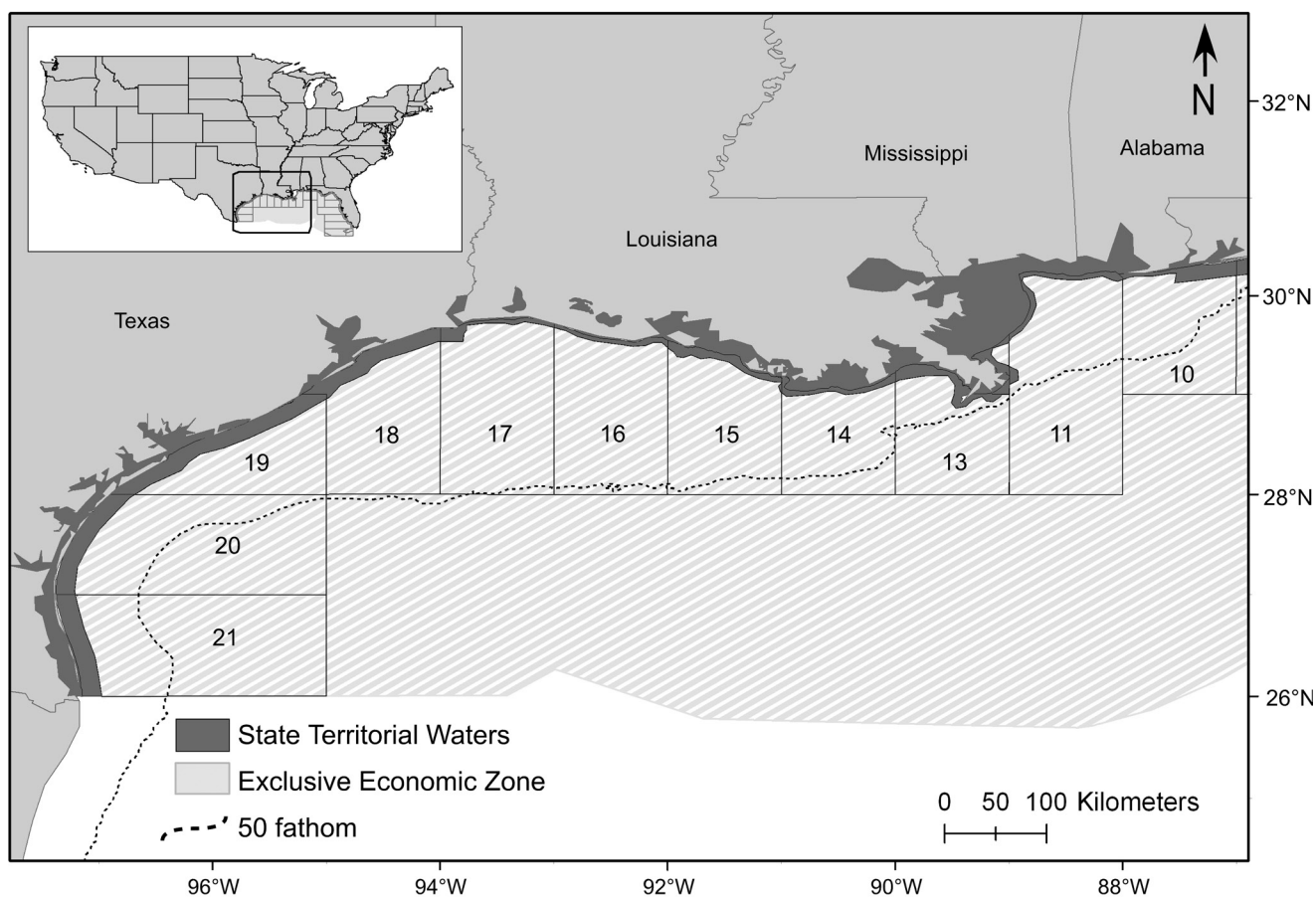


Figure 1.— Shrimp Statistical Subareas 10-21, encompassing the brown shrimp fishery within inshore (estuarine) and offshore (Gulf of Mexico) state territorial waters, and part (within the 50 fm depth contour) of the adjoining Federal EEZ in the northern Gulf of Mexico.

conducted virtual population analyses of brown shrimp in statistical subareas 18 and 19 (Fig. 1), and found that estimated numbers of brown shrimp in reported landings during biological year 1964 represented 97.7% of the total virtual population over a 17-mo period. This finding indicated that only 2.3% (by number) of the shrimp recruited as new cohorts in biological year 1964 contributed to the landings in biological year 1965. If shrimp landed in a given biological year within our time series (1986–2006) included survivors from cohorts recruited in preceding biological years, this could have affected our biological year simulations of w and other variables derived from them. However, such a carryover would be small, because it would involve only the larger sizes of shrimp which are

lowest in pounds and fewest in numbers within the landings.

Landings Data

Brown shrimp landings data are archived by the National Marine Fisheries Service (NMFS) Galveston Laboratory, Texas. Statistically, reported landings are fishery-dependent samples taken without replacement from the brown shrimp population. They are multitudinous but have limitations (Kutkuhn, 1962; Snow, 1969; Prytherch, 1980; Parrack¹; Nichols²; Poffenberger⁴) which may bias not

⁴Poffenberger, J. R. 1991. An overview of the data collection procedures for the shrimp fisheries in the Gulf of Mexico, National Marine Fisheries Service, Southeast Fisheries Science Center, Miami, Fla. (online at <http://www.sefsc.noaa.gov/gssprogram.jsp>).

only our simulated distributions of w and additional variables derived from them, but also may have biased previous estimates of numbers of shrimp from pounds landed within count categories. Not all brown shrimp that are caught are landed, and not all that are landed are reported (Kutkuhn, 1962; Berry and Benton, 1969; Baxter, 1973; Snow, 1969; Prytherch, 1980; Nance et al., 1991; Caillouet et al., 2008; Poffenberger⁴). Nonreported catch includes shrimp marketed directly to consumers, marketed as fishing bait (not all, but some), discarded for various reasons, kept for personal use by shrimpers, or otherwise not reported. Thus, reported landings are less than the actual catches, and also represent incomplete samples of the actual landings (Caillouet et al., 2008).

Reported shrimp landings data are recorded by calendar year, month, statistical subarea (Fig. 1), depth zone, shrimping trip, and count category or unknown size category, along with other information (Kutkuhn, 1962; Snow, 1969; Prytherch, 1980; Poffenberger⁴). We treated the unknown size category as a catch-all category. In selecting records for a working file of size-graded landings data for our simulations, we excluded all landings originally reported in the unknown size category, as well as landings added to the unknown size category after we judged their count categories to be outliers (see Data Selection and Preparation below). The resultant unknown size category contained landings that were:

- 1) not size-graded,
- 2) size-graded incorrectly or size limits not recorded,
- 3) not assigned to a count category for other reasons (e.g. pieces of shrimp tails), or
- 4) size-graded but reported in count categories we judged to be outliers.

Previous investigators (e.g. Rothschild and Brunenmeister, 1984; Parack¹; Nichols²) also excluded certain landings from their analyses for various reasons. Two methods of grading shrimp, box-grading and machine grading, were described by Kutkuhn (1962), Snow (1969), Prytherch (1980), and Poffenberger⁴. Differences between these grading methods and variations in their relative contributions to size-graded landings over time may have biased our simulated distributions of w and variables derived from them, but they may also have biased previous estimates of numbers of shrimp within count categories.

Data Selection and Preparation

Our final working file contained archived landings records selected from biological years 1986–2006, but only those we considered to have legitimate count class limits. We initially consulted NMFS port agents (who collect landings data) to obtain their opinions about the true range in size of brown shrimp tails

in the landings. It was agreed that the maximum C (smallest shrimp) for brown shrimp in the landings was around 250 tails per pound (equivalent to $w = 0.004$ lb, or 1.8 g), and minimum C (largest shrimp) around 9 tails per pound (equivalent to $w \approx 0.111$ lb, or ≈ 50.3 g).

Preparation of the working file involved filtering and editing a copy of archived data from biological years 1986–2006 as follows:

- 1) If a record was originally coded as belonging to the unknown category, it was excluded.
- 2) If an upper or lower limit of a count category was not recorded (i.e. left blank), the record was excluded.
- 3) If a recorded lower limit exceeded the recorded upper limit of a count category, the limits were assumed to have been inadvertently transposed at data entry, and the record was retained in the working file after being recoded by interchanging its count category limits.
- 4) If recorded upper and lower limits of a count category were both $C = 0$, the record was excluded.
- 5) If the recorded upper limit of a count category was $0 < C < 9$, both the lower and upper limits were recoded as $C = 9$, and the record was retained in the working file.
- 6) If only the recorded lower limit of a count category fell within $C < 9$, but the recorded upper limit was ≥ 9 , the lower limit was recoded as $C = 9$, and the record was retained in the working file.
- 7) If the recorded lower and upper limits of a count category were $C > 250$, the record was excluded.
- 8) If the recorded upper limit of a count category was $C > 250$, but the recorded lower limit was $C \leq 250$, the upper limit was recoded as $C = 250$, and the record was retained in the working file.
- 9) All other archived records were retained in the working file.

We then performed statistical analyses of the working file to identify and remove records having count class

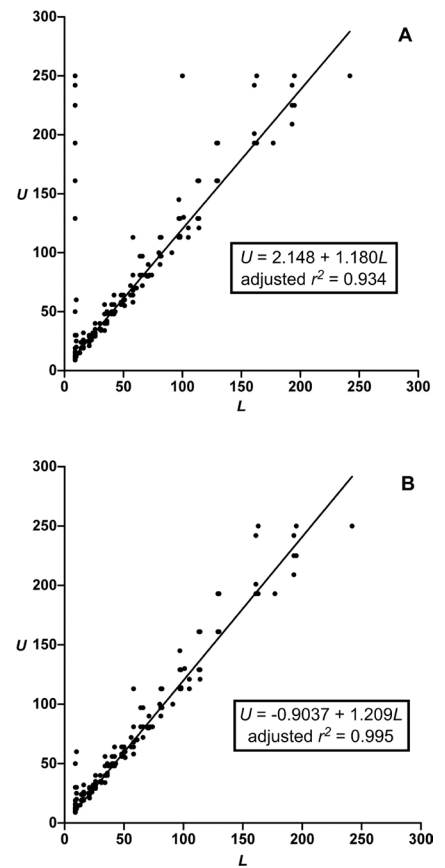


Figure 2.—Upper vs. lower limits of brown shrimp count categories in filtered and edited landings in biological year 2006; (A) before residual outlier records were removed and (B) after residual outlier records were removed. Lines were fitted by weighted linear regression, where the weighting factor was number of shrimping trips associated with each unique count category.

limits we judged to be outliers. For each biological year, we used SYSTAT⁵ to fit preliminary weighted linear regressions of upper limits on lower limits of the count categories, where the weighting factor was the number of observations (i.e. shrimping trips) associated with each unique count category (i.e. unique combination of upper and lower limits). Figure 2A is an example of a preliminary regression and data plot for biological

⁵Mention of trade names or commercial firms does not imply endorsement by the National Marine Fisheries Service, NOAA.

year 2006. Statistical weighting by number of shrimping trips was our way of dealing with variability in frequency of occurrence of count categories in the working file. Records removed from the working file by filtering, editing, and identification of residual outlier count categories represented a higher percentage of observations than percentage of pounds landed (Table 2); i.e. they contained relatively low pounds per observation.

We fitted final weighted linear regressions of upper limits on lower limits within the final working file for each biological year (Table 3). The weighting factor for these regressions was the number of observations (i.e. shrimping trips) associated with each unique count category remaining in the final working file. These final regressions characterized the relationship between legitimate count category upper and lower limits for each biological year. Figure 2B is an example final regression and data plot for biological year 2006. Slopes and intercepts of the final linear regressions (Table 3) for each biological year were examined for trends, using polynomial regression. Coded biological year ($T_i - 1996$) was substituted for T_i in these polynomial regressions, to avoid problems that otherwise might have been caused by correlations among powers of T_i (Sokal and Rohlf, 2000).

Aggregation and Cumulation of Landings

Landings from the final working file were aggregated (summed) by biological year and count category lower limits, C_{ij} , where j is the place marker for the C_{ij} within a biological year; $j = 0, \dots, m_i$, where m_i is the total number of C_{ij} in each biological year (Table 1). Upper limits of count categories (equivalent to lower limits of class intervals of w) were ignored. Because the number of unique count categories in the final working file varied among biological years, the number of C_{ij} also varied among biological years, as did m_i . Summing the landings by biological year and C_{ij} produced a subset of data with much lower spatial-temporal resolution than that of more detailed data sets used

Table 2.—Number of observations (shrimping trips) and pounds landed in the NMFS-archived records, compared to those remaining after filtering, editing, and removal of residual outlier count categories, for brown shrimp landings in the northern Gulf of Mexico fishery in biological years 1986–2006.

Records	Observations (shrimping trips)	Pounds (tails)
Archived	2,425,373 100.0%	1,682,806,769 100.0%
After filtering and editing	2,319,554 95.64%	1,668,305,100 99.14%
After residual outlier removal	2,308,674 95.19%	1,664,449,467 98.91%

in previous, bottom-up approaches to estimating numbers of shrimp within count categories (Neal, 1967; Rothschild and Brunenmeister, 1984; Nance et al., 1994; Diop et al., 2007; Parrack¹; Nichols²).

Biological year summations of landings combined all spatial-temporal influences (statistical subarea, depth zone, and month) on size of brown shrimp in the landings. These influences included sex ratio, recruitment, growth, mortality, fishing effort, fishing power of shrimp trawlers, experience of captains and crews, gear selectivity, discarding, data collection procedures, grading methods, and possibly other factors that affect count category landings within a biological year. Spatial influences were collapsed to the level of the entire fishery, and temporal influences to the level of biological years. Summation of landings by C_{ij} combined landings within count categories having C_{ij} as their lower limit. The simple hypothetical example below depicts this process:

Count Category	Observations	Pounds landed
9–12	2	500
9–15	3	1,200
9–20	1	40
Total	6	1,740

The sum of observations over all count categories having C_{ij} as their lower limit became the weighting factor, q_{ij} , for each C_{ij} and the sum of pounds associated with it. In the hypothetical example above, $C_{ij} = 9$, $q_{ij} = 6$, and both are associated with 1,740 lb landed.

Table 3.—Final weighted linear regressions of upper (U) on lower (L) limits of count categories in brown shrimp landings data selected by filtering, editing, and removal of residual outliers from the NMFS-archived landings data. The weighting factor was the number of shrimping trips associated with each unique count category (i.e. unique U and L data pair) in the landings data selected from each biological year. Sample size was the sum of these weighting factors for each biological year (see Fig. 2).

Biological year, T_i	Intercept _{<i>i</i>}	Slope _{<i>i</i>}	Sample size	Adjusted r^2
1986	-0.0609420	1.172878	141,523	0.988
1987	-0.7467180	1.189633	159,010	0.988
1988	0.4795405	1.160103	158,733	0.992
1989	0.0479237	1.171595	147,315	0.992
1990	0.6609263	1.157006	137,647	0.993
1991	-0.1564869	1.180132	122,065	0.992
1992	0.1879885	1.169404	117,633	0.991
1993	-0.5079871	1.187639	105,907	0.989
1994	0.0533226	1.173414	111,968	0.992
1995	-0.1264397	1.179191	102,643	0.993
1996	-0.7873293	1.195364	97,111	0.989
1997	-1.4422680	1.210573	98,415	0.987
1998	-1.0609130	1.199557	91,378	0.988
1999	-1.2453040	1.204808	92,638	0.985
2000	-0.3466840	1.179789	95,775	0.990
2001	0.1584804	1.169094	89,022	0.992
2002	-0.2696291	1.187891	122,160	0.992
2003	-1.0236740	1.206445	103,013	0.993
2004	-1.1202140	1.208740	82,006	0.993
2005	-0.3479283	1.192750	69,662	0.994
2006	-0.9036610	1.208740	63,050	0.995

Examples of variation in C_{ij} and q_{ij} for biological years 1986, 1996, and 2006 are shown in Figure 3. Dominant C_{ij} were conspicuous as indicated by their q_{ij} , and many were identical or close to the C_{ij} of standard count categories, as expected.

Within each biological year, the pounds associated with C_{ij} were cumulated over the observed range of C_{ij} , from the highest to the lowest C_{ij} (i.e. from the smallest to largest shrimp tails). These cumulative pounds were then converted to proportions of cumulative pounds landed, P_{ij} (Table 1), from the highest to the lowest C_{ij} . Figure 4A is an example of the stair-stepped relationship between P_{ij} and C_{ij} for biological year 2006, and Figure 4B is the equivalent stair-stepped relationship between P_{ij} and w_{ij} , where $w_{ij} = 1/C_{ij}$.

Modified Richards Function

We searched for an asymptotic, asymmetrical sigmoid regression model to convert the stair-stepped relationship between P_{ij} and w_{ij} to a smooth curve for each biological year. The regression model we chose was a simplified form of the Richards function (Richards, 1959):

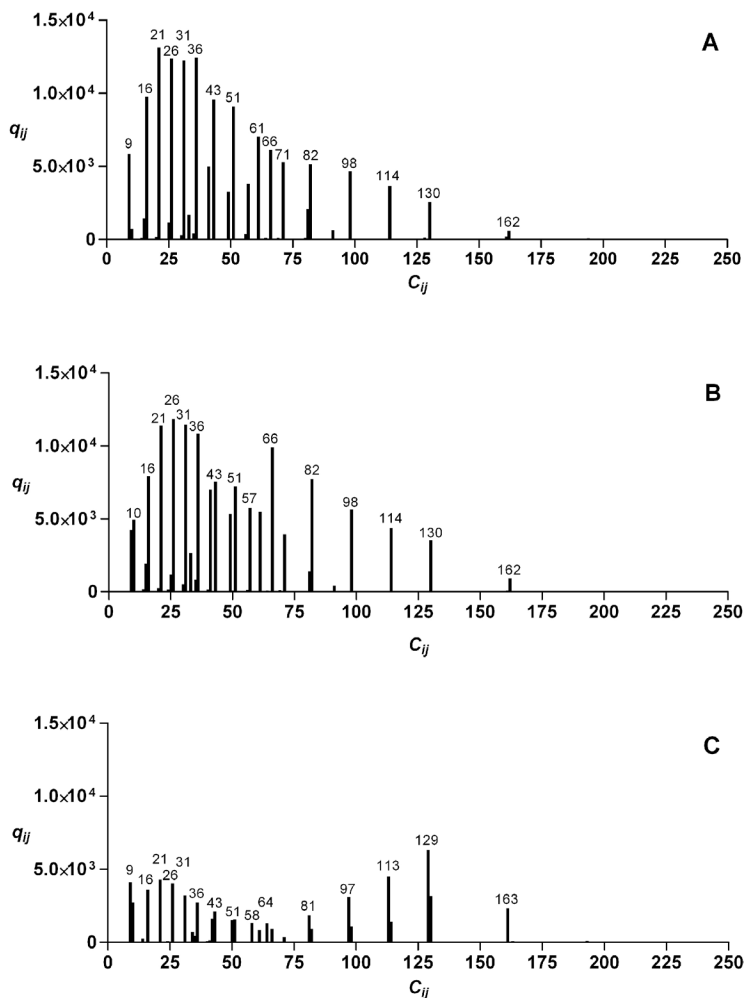


Figure 3.—Weighting factors, q_{ij} (i.e. shrimping trips) vs. legitimate brown shrimp count category lower limits, C_{ij} , for biological years (A) 1986, (B) 1996, and (C) 2006. Dominant C_{ij} are marked by the numbers above vertical bars representing their q_{ij} .

$$P = P_{\max} (1 - e^{-a-bw})^c \quad (1)$$

where, P is the cumulative proportion of pounds landed at w ,

w is shrimp tail weight in pounds, over the observed range from minimum to maximum w ,

P_{\max} is the upper asymptote,

a is the parameter which allows w at which $P = P_{\max}/2$ to vary,

b is the parameter which represents the maximum intrinsic rate of increase in P per unit w , which occurs at the inflection point on the curve,

c is the parameter that allows the sigmoid shape of the curve to vary (symmetrical or asymmetrical), and e is the base of natural logarithms.

Because we constrained P_{\max} to equal 1 in fitting all the regressions, Eq. (1) was simplified into the following regression model:

$$P = (1 - e^{-a-bw})^c. \quad (2)$$

For each biological year, we used GraphPad Prism (version 5.02) to fit Eq. (2) to P_{ij} on w_{ij} by weighted non-linear regression, where the weighting

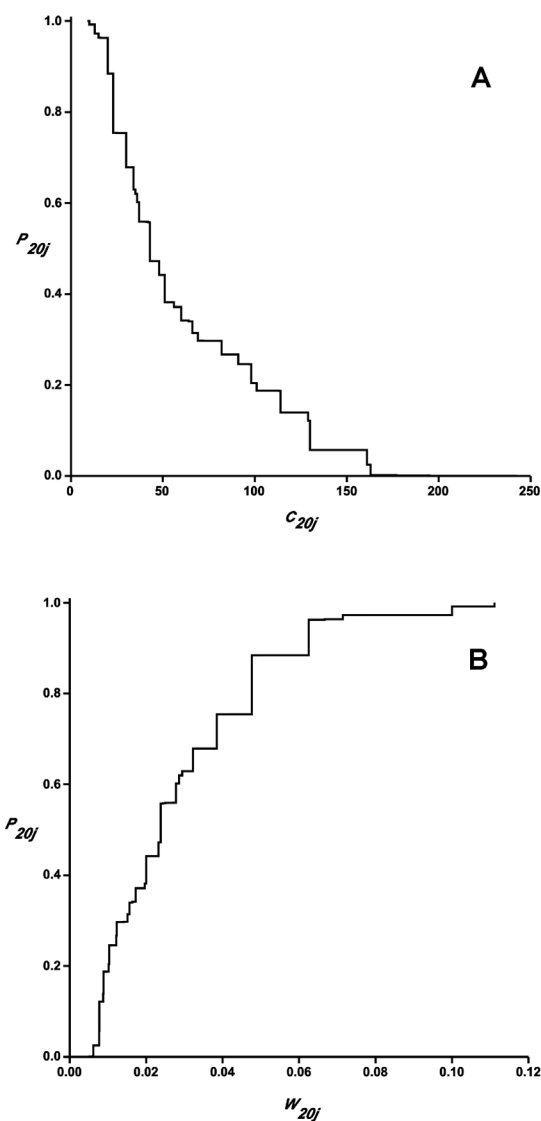


Figure 4.—Biological year 2006 (A) cumulative proportion of pounds landed, P_{20j} , vs. lower limit, C_{20j} , of count categories of filtered and edited brown shrimp landings from which residual outlier records were removed and (B) relationship between P_{20j} and w_{20j} , where $w_{20j} = 1/C_{20j}$.

factor was q_{ij} . In this way, parameters a_i , b_i , and c_i (Table 1) were estimated for each biological year (Table 4). The lower case parameter c_i should not be confused with the upper case count C_{ij} . We tried fitting a number of other asymmetrical sigmoid functions available in GraphPad Prism, but Eq. (2) was the best fitting of those we examined. While

we recognize that additional curve fitting methods and models could have been tested, Eq. (2) was adequate for purposes of demonstrating our simulation approach. By fitting Eq. (2), we smoothed the relationship between P_{ij} on w_{ij} , and obtained an equation representing this relationship for each biological year (Table 4). We also calculated the adjusted r^2 as an approximation of how well Eq. (2) fit the data points for each biological year (Table 4), but recognize it is not strictly applicable to nonlinear regression.

Simulating Biological Year Distribution of Tail Weight

The next step toward simulating the distribution of w was to generate a new set of data pairs for each biological year, using the fitted equations in Table 4. First, we generated equally spaced values of w'_k (Table 1), from a minimum, w'_0 ($= 0.005155$ lb), to a maximum, w'_{999} ($= 0.111111$ lb), where the k^{th} place marker for the w'_k was $k = 0, \dots, 999$ (Table 1). The increment, g , between the w'_k was then calculated as

$$g = (w'_{999} - w'_0) / 999 = 0.000106 \text{ lb.}$$

The w'_k were generated by

$$w'_k = k(g) + w'_0.$$

We then generated values of P'_{ik} for each w'_k for each biological year, using the following equation and estimates of parameters a_i , b_i , and c_i from Table 4:

$$P'_{ik} = (1 - e^{-a_i - b_i w'_k})^{c_i}. \quad (3)$$

Three reasons for applying $w'_0 = 0.005155$ lb (derived from $1/194$) as the minimum shrimp tail weight for all biological year simulations were:

- 1) The lowest maximum C_{ij} observed (in the working file) among all biological years was 194 count, the reciprocal of the highest minimum w_{ij} .
- 2) Imaginary numbers were generated by Eq. (3) for the minimum P'_{ik} in some biological years when the actual minimum w_{ij} observed

Table 4.—Biological year yield (Y_i), parameter estimates, and other statistics for weighted nonlinear regressions (modified Richards function, Eq. (2)) of cumulative proportions of pounds landed, P_{ij} , on pounds per shrimp tail, w_{ij} , in brown shrimp landings data selected by filtering, editing, and removal of residual outliers from the NMFS-archived landings data. The weighting factor was the number of shrimping trips, q_{ij} , associated with each data pair, P_{ij} and w_{ij} , in the selected landings data. For a given biological year, the number of data points analyzed (total sample size) was the sum of these weighting factors, $\sum_{j=0}^{m_i} q_{ij}$.

Biological year, T_i	Yield Y_i , pounds	Estimated parameters			Total sample size $\sum_{j=0}^{m_i} q_{ij}$	Adjusted r_i^2
		a_i	b_i	c_i		
1986	94,738,424	0.2770842	53.75477	1.016937	141,523	0.994
1987	89,394,421	0.3177634	61.64658	1.074417	159,010	0.997
1988	79,859,436	0.2713897	57.15293	1.286208	158,733	0.997
1989	94,170,525	0.2802385	58.89570	1.243767	147,315	0.996
1990	105,121,282	0.2865627	55.59358	0.968642	137,647	0.992
1991	85,602,708	0.1627544	47.08183	1.095961	122,065	0.993
1992	68,425,417	0.2294646	55.76027	1.150789	117,633	0.995
1993	66,431,237	0.2427682	55.10823	0.865503	105,907	0.989
1994	67,049,354	0.2126820	51.46945	1.107677	111,968	0.996
1995	75,859,021	0.2123855	48.21137	0.829590	102,643	0.991
1996	73,500,416	0.2459783	55.83692	0.888528	97,111	0.991
1997	65,389,618	0.2837078	55.32308	0.761172	98,415	0.994
1998	80,514,861	0.2723718	61.82822	0.975136	91,378	0.992
1999	81,035,496	0.2308989	56.10879	0.836558	92,638	0.987
2000	94,463,851	0.3038908	59.25881	1.084649	95,775	0.995
2001	87,660,251	0.3287329	74.62214	1.352838	99,022	0.987
2002	73,180,653	0.3917993	81.88587	1.447248	122,160	0.988
2003	82,309,001	0.3194503	79.86258	1.376817	103,013	0.986
2004	74,233,767	0.2973424	57.98183	0.884165	82,006	0.981
2005	58,819,403	0.2349768	56.86499	1.169126	69,662	0.981
2006	85,047,627	0.0818478	51.68214	1.640370	63,050	0.991

in those years was applied (this probably was due in part to the fact that Eq. (3) did not fit the data points representing very small shrimp tails closely in those years).

- 3) It was consistent to constrain w'_k to be the same for all biological years.

The first derivative of Eq. (3), $\delta P'_{ik} / \delta w'_k$, was

$$\delta P'_{ik} / \delta w'_k = b_i c_i (1 - e^{-a_i - b_i w'_k})^{c_i - 1} (e^{-a_i - b_i w'_k}). \quad (4)$$

For each biological year, we used Eq. (4) to generate first derivatives for each w'_k . To transform these first derivatives (Eq. (4)) into p'_{ik} (Table 1), which was the proportion of pounds landed at w'_k for each biological year, we divided them by the sum of all first derivatives over the range in w'_k , for each biological year. This sum was calculated as

$$\sum_{k=0}^{999} (\delta P'_{ik} / \delta w'_k).$$

In other words, for each biological year, p'_{ik} at each w'_k was calculated as

$$p'_{ik} = (\delta P'_{ik} / \delta w'_k) / \sum_{k=0}^{999} (\delta P'_{ik} / \delta w'_k).$$

Biological year yield, Y_i , encompassed all landings within a biological year, including those retained in our final working file as well as those that had been excluded from it. For each biological year, number of shrimp tails, f'_{ik} (Table 1), at each w'_k was calculated by

$$f'_{ik} = Y_i (p'_{ik}) / w'_k. \quad (5)$$

Equation 5, describing the relationship between f'_{ik} and w'_k , is the simulated distribution of w for the i^{th} biological year.

We would have been able to exclude some steps in our simulation sequence had the final working file represented total reported landings from each biological year (i.e. Y_i). However, the final working file was a subset of size-graded landings selected from the archived landings, and it did not contain landings we excluded (i.e. those relegated to the unknown category), whereas Y_i contained all landings for each biological year. Therefore, Eq. (5) applied the subset of proportions p'_{ik} to

the total yield Y_i to estimate f'_{ik} for each biological year.

We recognize that relative distributions of w for each biological year, and their corresponding cumulative relative distributions, also could have been derived from our simulated distributions of w . They might be of interest in some applications of our approach, but they were not essential to the purpose of our paper. They can easily be calculated from the information provided in this paper. However, the concept of cumulative relative distribution of w in biological year landings of brown shrimp is important in that it would estimate the probability of occurrence of tail weight $\leq w$; i.e. it would be an approximation of the cumulative distribution function (CDF) for w . This is the major part of the explanation of why we chose lower limits, C_{ij} (equivalent to upper limits of w_{ij}), for aggregating and cumulating landings, and then transformed C_{ij} to w_{ij} in preparation for fitting Eq. (2). Because a simulated distribution of w can be used to calculate the relative distribution of w and cumulative relative distribution of w , it is relevant to future testing of past algorithms and development of new ones to estimate numbers of shrimp from pounds landed within class intervals of w or C in the landings. Although we excluded certain landings (unknown size category) and ignored upper limits of legitimate count categories in simulating biological year p'_{ik} , our simulations of f'_{ik} included all biological year landings (Y_i); i.e. all biological year landings contributed to simulation of biological year distributions of w .

**Biological Year
Total Number of Shrimp
Tails (N_i), Mean C_i ,
and Mean w_i**

The total number of shrimp tails, N_i , in the landings from a biological year T_i was simulated by

$$N_i = \sum_{k=0}^{999} f'_{ik}. \quad (6)$$

Crude estimates of biological year mean count (N_i/Y_i) and its equivalent

mean tail weight (Y_i/N_i) were calculated. We examined trends in both of these means via polynomial regression, where coded years ($T_i - 1996$) were substituted for T_i .

Tail Weight at Half of Y_i

Given that a fitted equation representing the relationship between P_{ij} and w_{ij} was available for each biological year (Table 4), we estimated tail weight, $w50_i$, at which half of the annual yield, $Y_i/2$, was harvested in each biological year (note that when P_{max} is constrained to equal 1, $w50_i = P_{max}/2 = 0.5$). Each equation (Table 4) was solved for $w50_i$ as follows:

$$w50_i = \left[a_i - \ln(1 - 0.5^{1/c_i}) \right] / b_i.$$

This statistic is similar in concept to $LD50$, the estimated lethal dose (concentration) of a toxic substance at which 50% mortality occurs in exposed subjects. In our application, it is a potentially useful index of the relationship between brown shrimp size and yield (see Caillouet et al., 2008). We examined $w50_i$ via polynomial regression, where coded years ($T_i - 1996$) were substituted for T_i .

Results

Polynomial Regressions

We recognize that polynomial regression is an empirical approach to fitting a curve to a time series of data, and that the resulting polynomial terms have no structural meaning (Sokal and Rohlf, 2000). We applied it only to detect possible trends in the variables we simulated, and to demonstrate possible applications of our simulated distributions of w . Obviously, many other curve fitting approaches could have been used to examine the time series for each variable. Causes and effects within this brown shrimp fishery could have influenced the detected polynomial trends, despite variability (deviations from regression) caused by fluctuations in annual recruitment and other factors which are typical in shrimp populations (Caillouet et al., 2008).

**Weighted Linear Regressions
of Upper vs. Lower
Limits of Count Categories**

Data plots and preliminary weighted linear regressions of upper on lower limits of unique count categories in each biological year (see the example for the year 2006 in Fig. 2A) showed that count category outliers remained in the data after filtering and editing. In the year 2006 example, the outliers were concentrated near the minimum lower limit count of 9 (largest shrimp), which elevated the intercept of the fitted line in Figure 2A as compared to the intercept of the fitted line in Figure 2B, in which residual outliers had been removed. The unusually wide class intervals of outlier count categories could lead to serious biases in estimating numbers of shrimp within such count categories. Also, we emphasize that each data pair (upper and lower limits) was weighted, so the actual numbers of residual outliers are much higher than the number of data points representing outliers in Figure 2A (Table 2).

As expected, final weighted linear regressions of upper limits on lower limits of count categories were close fitting in all biological years as shown by high adjusted r^2 (Table 3, Fig. 2B). These final regressions characterized the relationship between upper and lower limits of what we considered to be legitimate count categories in each biological year. All slopes of these final regressions were slightly greater than 1 (Table 3), indicating that count class intervals in the working file widened as their lower limits increased. Trends in slopes and intercepts of these regressions are shown in Figures 5A and 5B, respectively.

**Biological Year m_i
and Weighted
Regressions of P_{ij} on w_{ij}**

The biological year total number, m_i , of C_{ij} exhibited a concave quadratic (parabolic) trend (Fig. 6); m_i dropped from 77 in 1986 to 35 in 1995, then increased but not to its earlier highest level. This trend in m_i reflected changes in the total number of legitimate

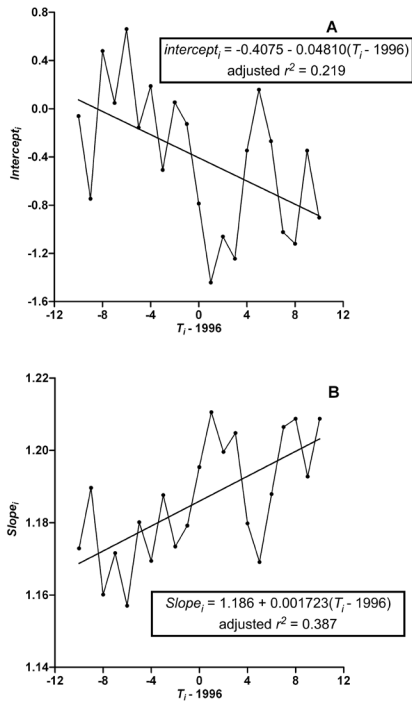


Figure 5.—Linear trends in (A) intercepts and (B) slopes of final weighted linear regressions of upper limits on lower limits of legitimate brown shrimp count categories, over coded biological years ($T_i - 1996$).

count categories over the biological years. However, the total number of count categories in biological year T_i exceeds m_i , because upper limits of count categories were ignored in our simulations; i.e. landings at the count category level were combined at the count category lower limit level, C_{ij} (see Aggregation and Cumulation of Landings). Wide variation in biological year numbers of count categories and the consequential quadratic trend in m_i (Fig. 6) are interesting and worthy of further investigation. They could reflect changes in size-related marketing strategies, recruitment, and perhaps other influences on choices of count categories in the landings.

Weighted nonlinear regressions of P_{ij} on w_{ij} for all biological years were close fitting, as indicated by very high adjusted r_i^2 (Table 4). Over all biological years, adjusted r_i^2 equaled or exceeded 0.981. Examples of plotted data points P_{ij} vs. w_{ij} and fitted curves for 1986,

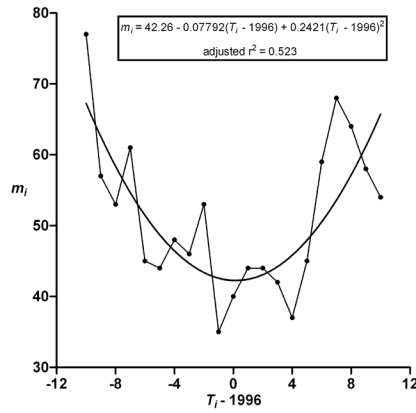


Figure 6.—Quadratic trend in total number, m_i , of count category lower limits, C_{ij} , for filtered and edited brown shrimp landings from which residual outlier count categories were removed, over coded biological years ($T_i - 1996$).

1996, and 2006 are shown in Figures 7A–C, respectively. Inflection points of the regressions were far to the lower left in such plots (Fig. 7A–C), suggesting that brown shrimp were fully recruited to the landings at very small sizes, which is a very important finding.

The total sample size,

$$\sum_{j=0}^{m_i} q_{ij}$$

(Fig. 8), for each biological year regression (Table 4), and the adjusted r_i^2 (Fig. 9) for these regressions, declined over biological years. In other words, adjusted r_i^2 and total sample size were dependent, as expected (Fig. 10); i.e. the larger the sample size the higher the adjusted r_i^2 . We emphasize that the total sample size (Fig. 8) used in fitting the regressions of P_{ij} on w_{ij} for each biological year was less than the actual number of shrimping trips in the archived data for each biological year, because landings from some trips were initially in the unknown category or later placed there by filtering, editing, and outlier removal from the working file. Therefore, the data points and trend in Figure 8 should not be taken to represent total shrimping trips in the biological years.

As is common in fitting models containing more than one parameter,

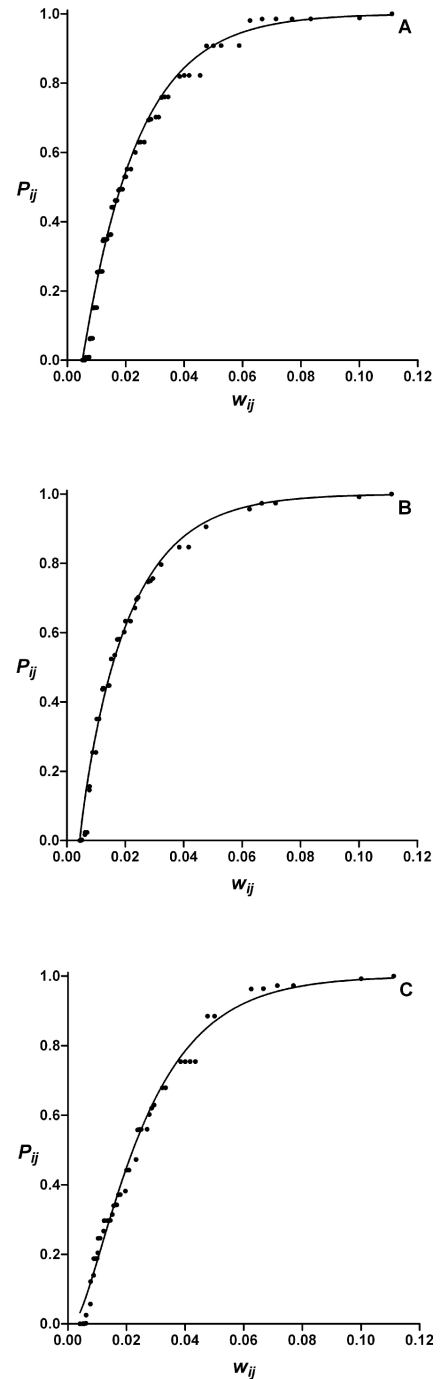


Figure 7.—Weighted nonlinear regressions (modified Richards function) fitted to cumulative proportion of brown shrimp landings, P_{ij} , vs. pounds per shrimp tail, w_{ij} , for biological years (A) 1986, (B) 1996, and (C) 2006.

the parameter estimates often are not independent (i.e. orthogonal). Graph-

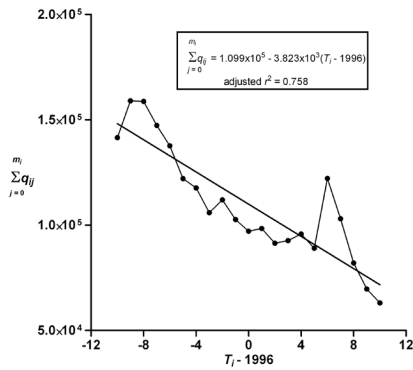


Figure 8.—Linear trend in annual shrimping trips,

$$\sum_{j=0}^{m_i} q_{ij},$$

over coded biological years ($T_i - 1996$), for brown shrimp landings in legitimate count categories.

Pad Prism provided estimates of dependency of estimated parameters a_i , b_i , and c_i within each biological year regression (dependency = 1 represents complete dependency, and dependency = 0 indicates orthogonality). Over biological years, dependency was 0.865–0.985 for parameter a_i , 0.968–0.981 for parameter b_i , and 0.978–0.994 for parameter c_i . Not only did all these parameters show strong dependency within each biological year regression, but they also appeared related to each other over biological years (Fig. 11A–C).

Simulated Distributions of w

Example distributions of w'_k for biological years 1986, 1996, and 2006 are shown in Figures 12A–C. All were strongly skewed to the right. Their most striking feature was their likeness to negative exponential curves. Therefore, we plotted them in the form of $\ln(f'_{ik})$ vs. w'_k for all biological years (Fig. 13). Straight lines for $\ln(f'_{ik})$ vs. w'_k would have indicated that these simulated distributions of w followed a negative exponential pattern, once full recruitment to the landings was reached at very small sizes (Fig. 13). Only slight concavity was evident in all the curves.

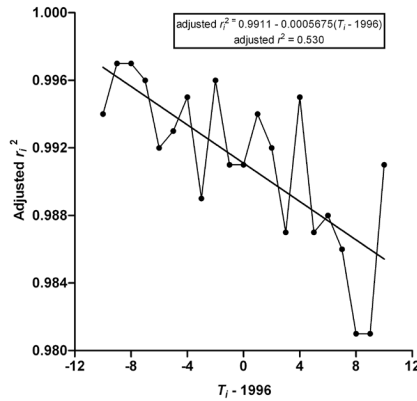


Figure 9.—Linear trend in adjusted r_i^2 for weighted nonlinear regressions (modified Richards function) fitted to cumulative proportion of brown shrimp landings, P_{ij} , vs. pounds per shrimp tail, w_{ij} , over coded biological years ($T_i - 1996$).

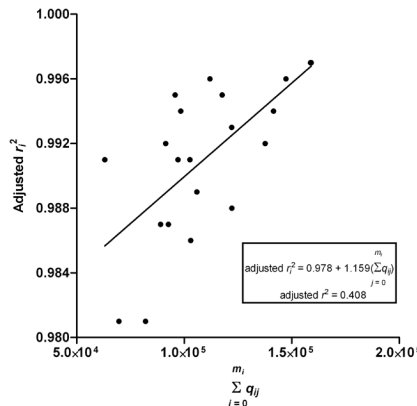


Figure 10.—Linear relationship between adjusted r_i^2 and annual shrimping trips,

$$\sum_{j=0}^{m_i} q_{ij},$$

for weighted nonlinear regressions (modified Richards function) fitted to cumulative proportion of brown shrimp landings, P_{ij} , vs. pounds per shrimp tail, w_{ij} .

Biological Year Total Number of Shrimp Tails and Yield

Interestingly, although the biological year total number of shrimp tails, N_i (Fig. 14), and yield, Y_i (Fig. 15), showed hints of declines, they exhibited no significant trends over biological years, because of wide year to year variation. A close linear relationship between N_i and

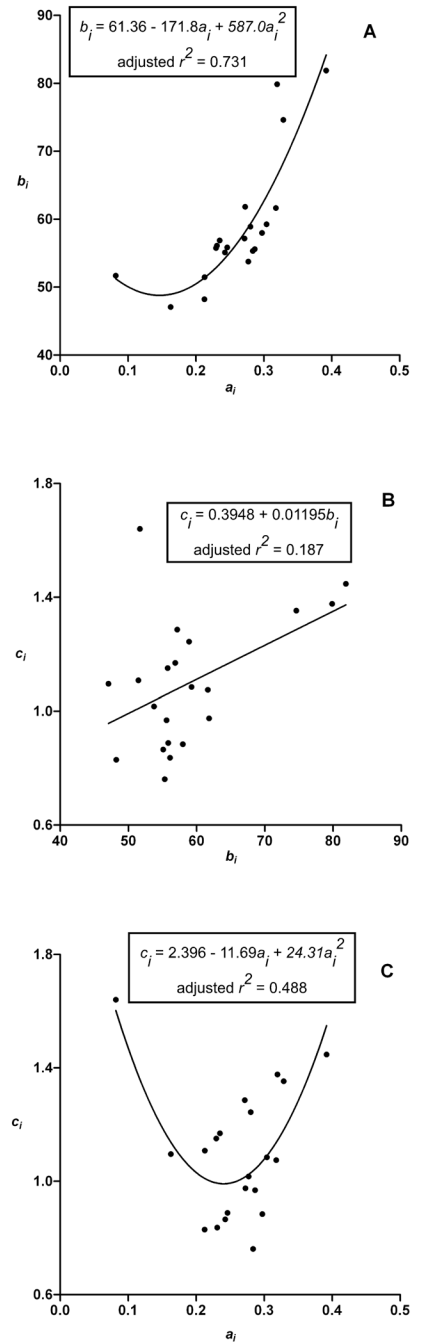


Figure 11.—(A) Quadratic relationship between parameters b_i and a_i , (B) linear relationship between parameters c_i and b_i , and (C) quadratic relationship between parameters c_i and a_i , for weighted nonlinear regressions (modified Richards function) fitted to cumulative proportion of brown shrimp landings, P_{ij} , vs. pounds per shrimp tail, w_{ij} .

Y_i (Fig. 16) was expected; i.e. the more pounds landed the greater the number of shrimp tails in the landings, and vice versa. However, biological year mean count, N_i/Y_i (Fig. 17A) was not constant, because simulated distributions of w and Y_i were not constant over biological years (Fig. 12A–C, Fig. 13). Biological year mean tail weight (Y_i/N_i) also was not constant (Fig. 17B). We emphasize that N_i/Y_i and Y_i/N_i are crude estimates of mean count and mean tail weight, respectively, and do not represent biological year central tendency of C and w in the landings very well. Trends in N_i/Y_i (Fig. 17A) and Y_i/N_i (Fig. 17B) were cubic (sigmoid), mirroring each other as expected.

Tail Weight at Which Half of the Biological Year Yield was Harvested

The cubic trend in $w50_i$ is shown in Figure 18. As expected, it is similar in shape to that of Y_i/N_i (Fig. 17B). However, the two trends (Fig. 17B, Fig. 18) were not parallel, because the slope of the regression of $w50_i$ on Y_i/N_i did not equal 1 (Fig. 19). Instead, $w50_i$ was 1.459 times Y_i/N_i . Although significantly different from zero, the intercept of the regression of $w50_i$ on Y_i/N_i was very small (i.e. near the origin).

Discussion

It is clear that brown shrimp landings data should be filtered, edited, and residual records representing outlier count categories removed before distributions of shrimp tail weight are simulated. The same should be (and in most cases have been) done before numbers of shrimp are estimated from landings within count categories, regardless of the algorithm used to estimate numbers of shrimp within count categories, unless the algorithms are based on actual sampling of size distributions within count categories (Ehrhardt and Legault, 1996). The problem of unreported landings and other limitations of reported landings data affect not only our simulations, but all other uses of reported landings to estimate numbers of shrimp within count categories. These data problems cannot be rectified

retroactively, but should be addressed in the future.

Our simulated biological year distributions of brown shrimp tail weight could be biased to unknown degrees by many factors. This is true of all estimates of numbers of shrimp derived from landings within count categories, whether at the highest possible level of data resolution (i.e. an individual shrimping trip within a statistical sub-area, depth zone, and month), or at lower levels of data resolution represented by various spatial-temporal aggregations of landings data, including ours. Our simulated distributions of shrimp tail weight should not be taken as equivalent to distributions of brown shrimp tail weight in the population of the northern Gulf of Mexico. However, our simulated distributions of w in biological year landings no doubt have some yet undetermined relationship to actual distributions of shrimp tail weight in the brown shrimp population in biological years. This relationship cannot be determined retroactively due to lack of or paucity of required data. Unreported landings are much less than reported landings, but our simulated distributions of shrimp tail weight only represent landings that were reported and archived.

Despite landings data deficiencies, our simulated distributions of w , and other fishery-dependent statistics derived from them, can be useful in examining changes in the brown shrimp fishery over biological years. Their relationships to other important fishery-dependent and fishery-independent variables could be examined in attempts to explain causes and effects.

Our method could be applicable to fisheries of other penaeid shrimp species for which landings are recorded within size categories expressed in C or w . It might also be applicable to finfish fisheries in which landings are reported within size categories expressed in number of fish per unit weight or in weight per fish. The method may also be applicable to shrimp landings aggregated at spatial-temporal levels lower (i.e. higher resolution) than that of an entire fishery and biological year.

Our results suggest that brown shrimp were fully recruited to the fishery at

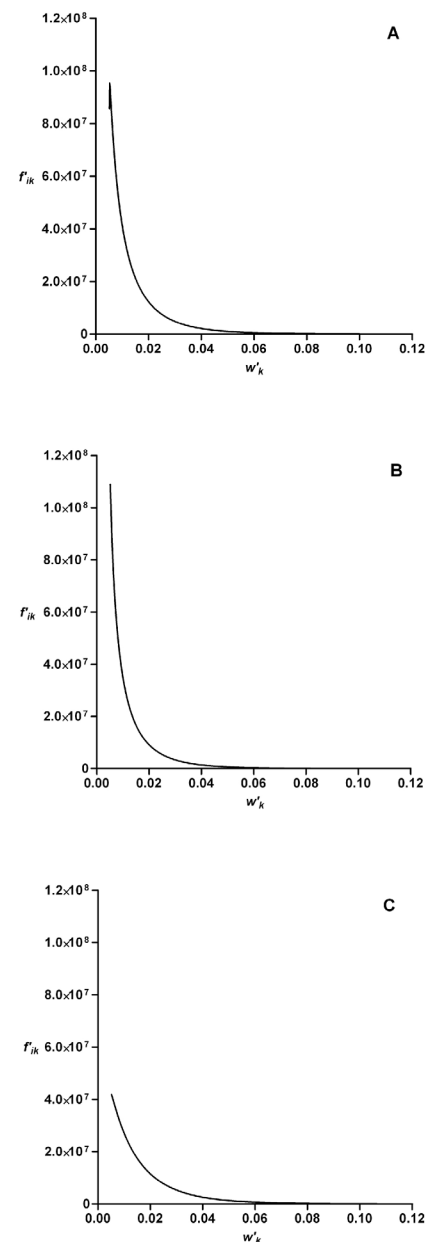


Figure 12.—Simulated distributions of w (i.e. the relationship between f'_{ik} and w'_k) for brown shrimp in biological years (A) 1986, (B) 1996, and (C) 2006.

small sizes in each biological year, then declined in number with w in a pattern similar but not identical to that of a negative exponential curve. In a study of distributions of growth rates of shrimp in captivity, Banks et al. (2009) examined effects of bin width, sample size, and sampling frequency on distributions

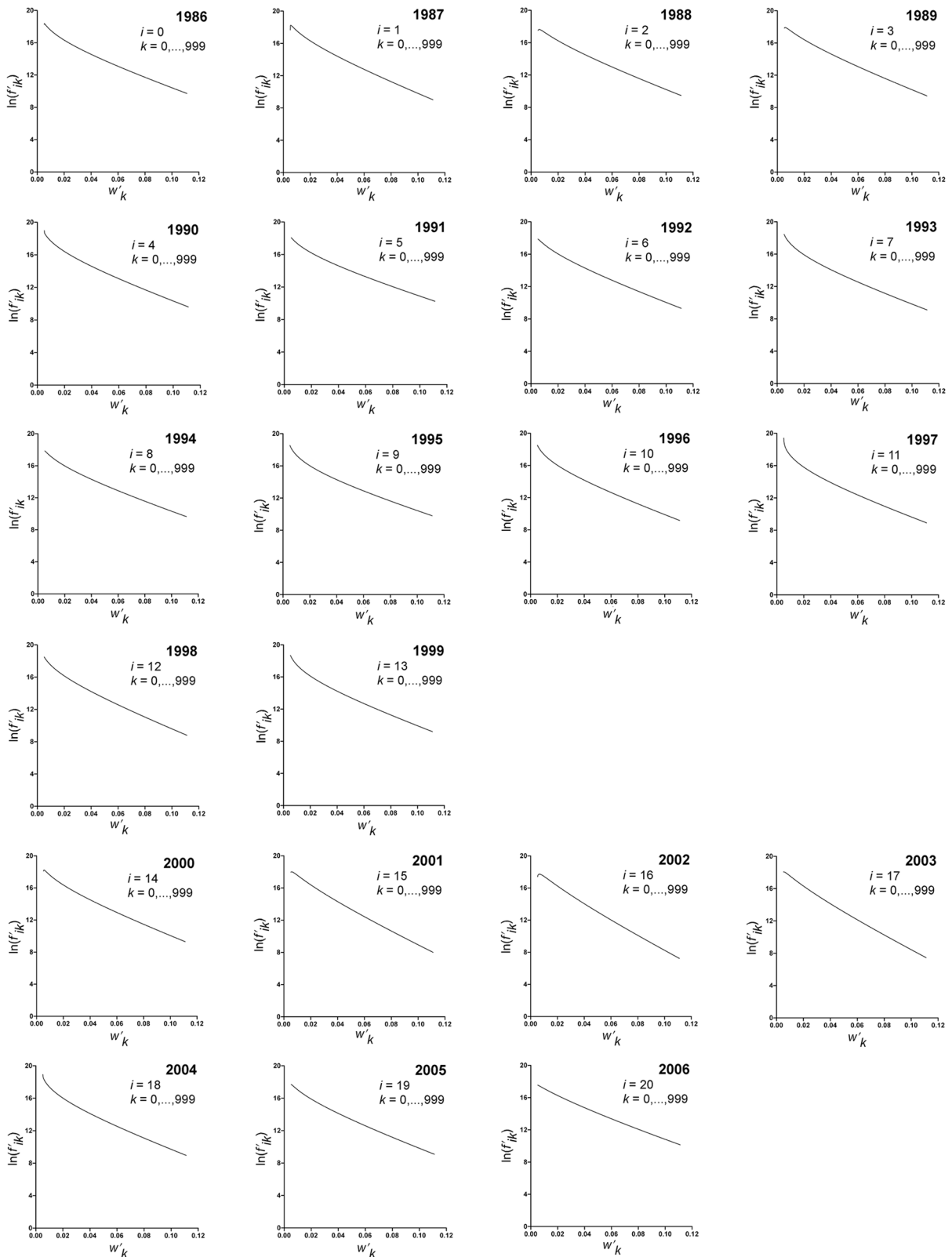


Figure 13.—Simulated distributions of w for brown shrimp in biological years 1986–2006, as shown with the ordinate in natural logarithmic scale (i.e. the relationship between $\ln(f'_{ik})$ and w'_k).

of weight per shrimp. Interestingly, the shapes of their distributions of weight per shrimp were similar to those of our

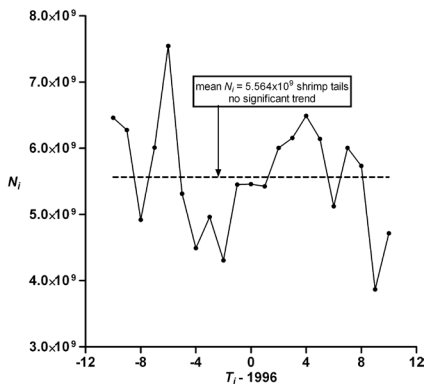


Figure 14.—Simulated biological year total number of brown shrimp tails, N_i , vs. coded biological year ($T_i - 1996$).

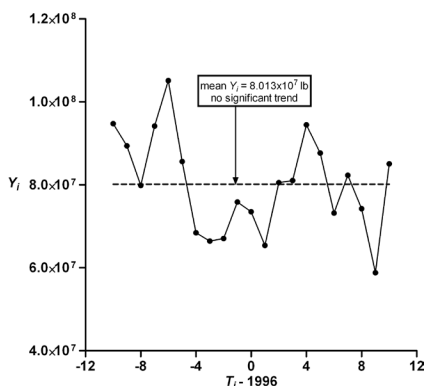


Figure 15.—Brown shrimp yield, Y_i , vs. coded biological year ($T_i - 1996$).

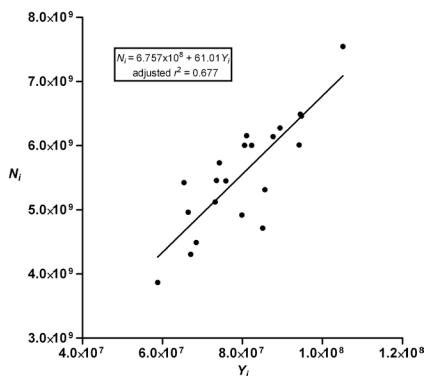


Figure 16.—Linear relationship between simulated total number of brown shrimp tails, N_i , and yield, Y_i .

simulated distributions of w . Although we did not simulate relative distributions of w or corresponding cumulative relative distributions of w , we noted that they could be simulated from our approach, and they too might be of interest and use in shrimp stock assessments.

Simulated biological year distributions of w could be used to estimate numbers of parents and recruits, for purposes of determining parent-recruit relationships (Rothschild and Brunenmeister, 1984; Gracia, 1991; Ehrhardt and Legault, 1996; Parrack¹; Nichols²). Numbers of parents or recruits could be extracted from curves representing distributions of w by integrating them over the size ranges of parents and recruits. However, estimates or assumptions about size at maturity and growth patterns of males and females would

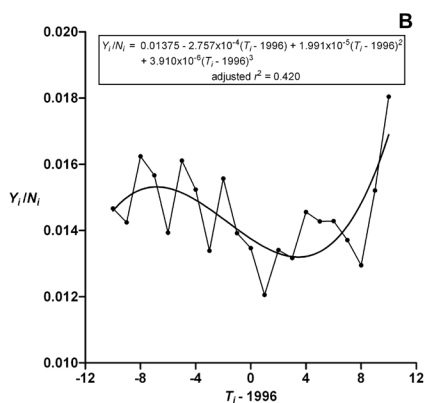
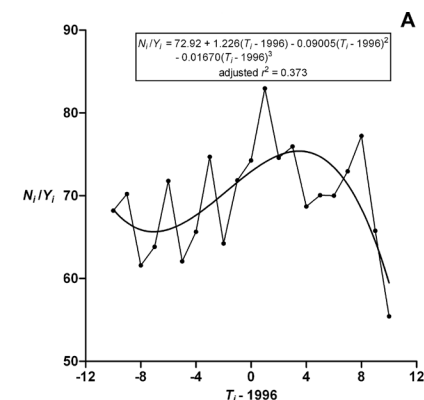


Figure 17.—Cubic trends in (A) biological year mean count, N_i/Y_i , vs. coded biological year ($T_i - 1996$), and in (B) biological year mean pounds per shrimp tail, Y_i/N_i , vs. coded biological year ($T_i - 1996$).

be required, as well as estimates of size-specific sex ratios in the landings (Gracia, 1991; Ehrhardt and Legault, 1996 Parrack¹, Nichols²).

It may be possible to estimate instantaneous total mortality rate (Z) from simulated biological year distributions of w by transforming them to bounded length distributions and applying length-based models similar to those of Ehrhardt and Ault (1992) (Ehrhardt⁶).

⁶Ehrhardt, N. M. Rosentiel School of Marine and Atmospheric Science, Miami, FL. Personal commun., August 2010.

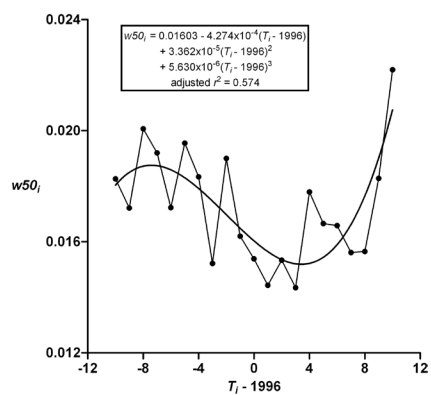


Figure 18.—Cubic trend in $w50_i$, the simulated pounds per shrimp tail at which half of the brown shrimp biological year yield, Y_i , was harvested, vs. coded biological year ($T_i - 1996$).

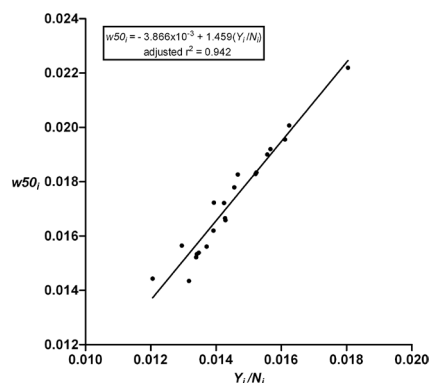


Figure 19.—Linear relationship between $w50_i$, the simulated pounds per shrimp tail at which half of the brown shrimp biological year yield, Y_i , was harvested, and biological year mean pounds per shrimp tail, Y_i/N_i .

Alternatively, the length-based models used by Ehrhardt and Ault (1992) might be reformulated for direct application to biological year distributions of w for purposes of estimating Z (Ehrhardt⁶). Biological year distributions of w could also be transformed to age-frequencies for age-structured stock assessments. This would require conversion of tail weight to age using sex-specific growth curves and knowledge of size-specific sex ratios in the landings (Parrack, 1979; Rothschild and Brunenmeister, 1984; Gracia, 1991; Ehrhardt and Legault, 1996; Parrack¹; Nichols²).

Simulated distributions of w of brown shrimp in biological year reported landings are linked, by definition and calculations, to biological year yield. Fishing effort influences size-composition of the landings and therefore influences yield, although environmental variables affecting recruitment also affect yield (Caillouet et al., 2008; Nance et al., 2010). Numbers of shrimp estimated from landings within count categories have been used in evaluating the influence of environmental factors on abundance, growth, and survival (Diop et al., 2007).

Our method provides an alternate way to estimate abundance of shrimp in reported annual landings, as compared to algorithms used by previous investigators. However, the relationship between abundance of shrimp in the landings and in the population remains undetermined. Our simulated distributions of w provide examples for comparison with explicit or implicit assumptions made by previous investigators about the distributions of C and w . They also provide information of potential use in developing new estimators of number of shrimp from landings data, based on statistical estimation theory and the underlying distribution of w or C . Finally, there may be other useful applications of our approach and results that we have not realized or anticipated.

Acknowledgments

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report shrimp fishery statistics, and to all shrimp industry participants whose cooperation made it possible over the years. We are especially grateful to those who perpetuated and improved this system, and to those who collected, processed, and archived shrimp fishery statistics, making them available for analyses such as ours. Joseph H. Kutkuhn's comprehensive statistical examination and evaluation provided an early and important understanding of the usefulness and limitations of landings data in shrimp stock assessment. Succeeding investigators expanded and improved this understanding, for which we are grateful. We greatly appreciated reviews of our manuscript by Nelson M. Ehrhardt, and three anonymous reviewers. We thank Jo Anne Williams for assistance in drafting the figures; James Primrose and John Cole for assistance in landings data compilation; and Brian Linton, James A. Bailey, and Stephen A. Bailey for assistance with derivatives of the modified Richards function, and in solving this function for $w50$. This paper is dedicated to the memory of the senior author's parents, Charles W. Caillouet, Sr. (1908–1971) and Elida P. Millet Caillouet (1906–2004).

Literature Cited

Banks, H. T., J. L. Davis, S. L. Ernstberger, S. Hu, E. Artimovich, and A. K. Dhar. 2009. Experimental design and estimation of growth rate distributions in size-structured shrimp populations. *Inverse Problems* 25(9):95003–95030.

Baxter, K. N. 1971. Brown shrimp live longer than many biologists believe. *Commer. Fish. Rev.* 33(3):2.

_____. 1973. Shrimp discarding by the commercial fishery in the western Gulf of Mexico. *Mar. Fish. Rev.* 35(9):26.

Berry, R. J., and R. C. Benton. 1969. Discarding practices in the Gulf of Mexico shrimp fishery. *FAO Fish. Rep.* 57:983–999.

Caillouet, Jr., C. W. 2003. Improved assessments and management of shrimp stocks could benefit sea turtle populations, shrimp stocks and shrimp fisheries. *Mar. Turtle Newsl.* 100:22–27.

_____, R. A. Hart, and J. M. Nance. 2008. Growth overfishing in the brown shrimp fishery of Texas, Louisiana, and adjoining Gulf of Mexico EEZ. *Fish. Res.* 92:289–302.

Diop, H., W. R. Keithly, Jr., R. F. Kazmierczak, Jr., and R. F. Shaw. 2007. Predicting the abundance of white shrimp (*Litopenaeus setiferus*) from environmental parameters and previous life stages. *Fish. Res.* 86:31–41.

Ehrhardt, N. M., and C. M. Legault. 1996. Tuned length-based cohort analysis (TLCA) computer program. Crustacean stock assessment techniques incorporating uncertainty. Report of the CFRAMP/FAO/DANIDA Stock Assessment Workshop on the Shrimp and Groundfish Fisheries of the Guiana-Brazil Shelf, Port of Spain, Trinidad. *FAO Fish. Rep.* 544, Suppl. 111–131.

_____, and J. S. Ault. 1992. Analysis of two length-based mortality models applied to bounded catch length frequencies. *Trans. Am. Fish. Soc.* 121:115–122.

Gracia, A. 1991. Spawning stock-recruitment relationships of white shrimp in the southwestern Gulf of Mexico. *Trans. Am. Fish. Soc.* 120:519–527.

Kutkuhn, J. H. 1962. Gulf of Mexico commercial shrimp populations—trends and characteristics, 1956–59. *Fish. Bull.* 62:343–402.

Nance, J. M., N. Garfield, and J. A. Paredes. 1991. A demographic profile of participants in two Gulf of Mexico inshore shrimp fisheries and their response to the Texas Closure. *Mar. Fish. Rev.* 53(1):10–18.

_____, E. X. Martinez, and E. F. Klima. 1994. Feasibility of improving the economic return from the Gulf of Mexico brown shrimp fishery. *N. Am. J. Fish. Manage.* 14:522–536.

_____, C. W. Caillouet, Jr., and R. A. Hart. 2010. Size-composition of annual landings in the white shrimp fishery of the northern Gulf of Mexico, 1960–2006: its trend and relationships with other fishery-dependent variables. *Mar. Fish. Rev.* 72(2):1–13.

Neal, R. A. 1967. An application of the virtual population technique to penaeid shrimp. *Proc. 21st Annu. Conf. Southeast Assoc. Game Fish Comm.* 21:264–271.

_____, and R. C. Maris. 1985. Fisheries biology of shrimps and shrimplike animals. Chapter 1: Fisheries biology of shrimps and shrimplike animals. *In* A. J. Provenzano, Jr. (Editor), *Economic aspects: fisheries and culture*, vol. 10, *The biology of crustacea*, p. 1–110. Acad. Press, Inc., N.Y.

Parrack, M. L. 1979. Aspects of brown shrimp, *Penaeus aztecus*, growth in the northern Gulf of Mexico. *Fish. Bull.* 76:827–836.

Prytherch, H. F. 1980. A directory of fishery data collection activities conducted by the statistical surveys division in the southeast region of the United States. U.S. Dep. Commer., NOAA Tech. Memo. NMFS-SEFC-16, 91 p.

Richards, F. J. 1959. A flexible growth function for empirical use. *J. Exper. Bot.* 10:290–300.

Rothschild, B. J., and S. L. Brunenmeister. 1984. The dynamics and management of shrimp in the northern Gulf of Mexico. *In* J. A. Gulland and B. J. Rothschild (Editors), *Penaeid shrimps—their biology and management*, p. 145–172. Fish. News Books Ltd., Farnham, Surrey, Engl.

Snow, G. W. 1969. E/54 Detailed shrimp statistical program in the Gulf states. *In* M. N. Mistakidis (Editor), *Proceeding of the World Science Conference on the Biology and Culture of Shrimps and Prawns*, FAO Fish. Rep. R57, Vol. 3, p. 947–956. (online at <http://www.fao.org/docrep/005/ac741t/AC741T25.htm>).

Sokal, R. R., and F. J. Rohlf. 2000. *Biometry, the principles and practice of statistics in biological research*, 3rd edition, W. H. Freeman and Co., N.Y., 887 p.