

Abstract—Stock-rebuilding time isopleths relate constant levels of fishing mortality (F), stock biomass, and management goals to rebuilding times for overfished stocks. We used simulation models with uncertainty about F_{MSY} and variability in annual intrinsic growth rates (r_y) to calculate rebuilding time isopleths for Georges Bank yellow-tail flounder, *Limanda ferruginea*, and cowcod rockfish, *Sebastes levis*, in the Southern California Bight. Stock-rebuilding time distributions from stochastic models were variable and right-skewed, indicating that rebuilding may take less or substantially more time than expected. The probability of long rebuilding times increased with lower biomass, higher F , uncertainty about F_{MSY} , and autocorrelation in r_y values. Uncertainty about F_{MSY} had the greatest effect on rebuilding times. Median recovery times from simulations were insensitive to model assumptions about uncertainty and variability, suggesting that median recovery times should be considered in rebuilding plans. Isopleths calculated in previous studies by deterministic models approximate median, rather than mean, rebuilding times. Stochastic models allow managers to specify and evaluate the risk (measured as a probability) of not achieving a rebuilding goal according to schedule. Rebuilding time isopleths can be used for stocks with a range of life histories and can be based on any type of population dynamics model. They are directly applicable with constant F rebuilding plans but are also useful in other cases. We used new algorithms for simulating autocorrelated process errors from a gamma distribution and evaluated sensitivity to statistical distributions assumed for r_y . Uncertainty about current biomass and fishing mortality rates can be considered with rebuilding time isopleths in evaluating and designing constant- F rebuilding plans.

Stock-rebuilding time isopleths and constant- F stock-rebuilding plans for overfished stocks

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Stock-rebuilding plans proposed for overfished stocks are best evaluated by stock-specific simulation analysis (e.g. PFM¹). However, general approaches are also valuable because many stocks are overfished (NMFS, 1999) and default or generic rebuilding plans can be used without extensive analyses for each species (e.g. PFM²; Applegate et al.³). In this article we show how stock-rebuilding time isopleths can be used to design, evaluate, and monitor progress of “constant F ” and other types of rebuilding plans. Constant- F stock-rebuilding plans maintain fishing mortality at a fixed level until the stock is rebuilt, and are relatively simple and easy to analyze. The isopleth approach is easy to use as both a general and stock-specific tool.

The U.S. Sustainable Fisheries Act (SFA) mandates rebuilding plans for overfished stocks (DOC, 1996, 1998). Federally managed stocks are considered overfished when stock biomass is less than the biomass threshold ($B_{Threshold}$) defined in the Fishery Management Plan (FMP). National Standard 1 (DOC, 1998) for the SFA indicates that $B_{Threshold}$ should be the greater of one-half of B_{MSY} (the theoretical biomass level for maximum sustained yield, MSY) or the minimum biomass from which rebuilding to B_{MSY} could be expected to occur within ten years if the stock is exploited at $F_{Threshold}$. Typically, $F_{Threshold} = F_{MSY}$ (the theoretical fishing mortality rate for MSY) when current biomass is at or above $B_{Threshold}$, and $F_{Threshold} < F_{MSY}$ at lower biomass levels. A common approach

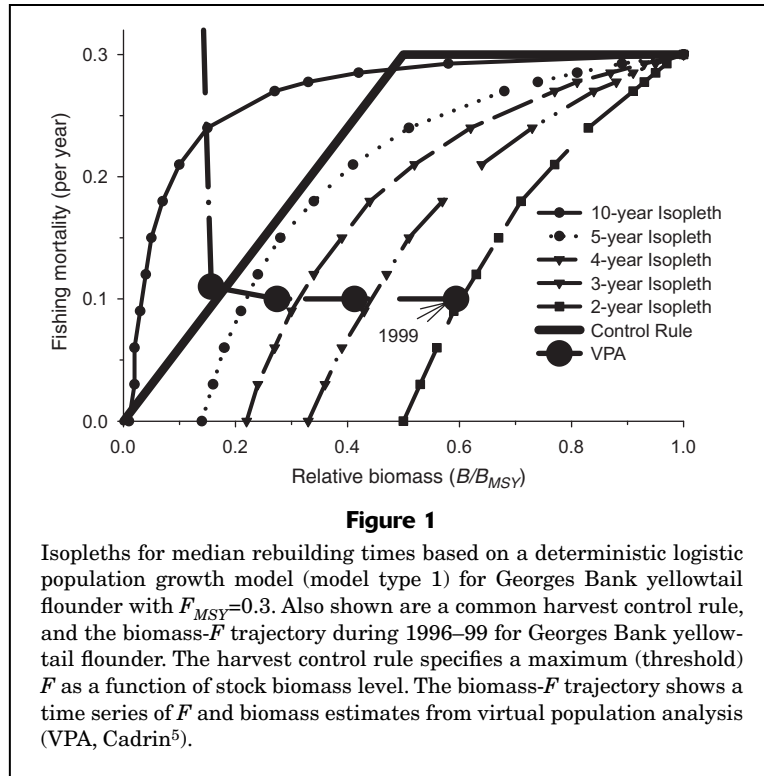
(Fig. 1, and Thompson, 1999) reduces $F_{Threshold}$ from the F_{MSY} level linearly to zero as biomass declines from $B_{Threshold}$. In cases where B_{MSY} and F_{MSY} can not be estimated, reasonable proxy values (e.g. one-half unfished biomass or $F_{0.1}$) are typically used instead.

The goal for most rebuilding plans under the SFA is to achieve the target biomass level (B_{MSY} or an acceptable proxy level) in ten years or less. Even with zero fishing mortality, ten years may not be sufficient to rebuild some overfished stocks. In such cases, the Guidelines for National Standard 1 allow a rebuilding time period no longer than one mean generation time (Restrepo et al., 1998) plus the expected time to recovery in the absence of fishing mortality (DOC, 1998).

¹ PFM (Pacific Fishery Management Council). 1999. The coastal pelagic species fishery management plan, Amendment 8, 405 p. Pacific Fishery Management Council, 7700 NE Ambassador Place, Portland, OR, 97220-1384.

² PFM (Pacific Fishery Management Council). 1999. Status of the Pacific Coast groundfish fishery through 1999 and recommended acceptable biological catch for 2000 stock assessment and fishery evaluation, 230 p. Pacific Fishery Management Council, 7700 NE Ambassador Place, Portland, OR, 97220-1384.

³ Applegate, A., S. Cadrin, J. Hoenig, C. Moore, S. Murawski, and E. Pikitch. 1998. Evaluation of existing overfishing definitions and recommendations for new overfishing definitions to comply with the Sustainable Fisheries Act, 179 p. New England Fishery Management Council, 50 Water Street, Mill 2, Newburyport, MA 01950.



Stock-rebuilding time isopleths

Cadrin (1999) calculated theoretical recovery times for Georges Bank yellowtail flounder (*Limanda ferruginea*) and used rebuilding time isopleths to depict trends in stock biomass in relation to fishing mortality. Calculations were based on a deterministic logistic population growth model with a range of constant annual fishing mortality rates (\tilde{F} =zero to F_{MSY}) and a range of initial biomass levels less than the target level (B_0 =zero to B_{MSY}). Recovery time was the number of years required for stock biomass to increase from an initial overfished biomass level ($B_0 < B_{Threshold}$) to the biomass target $B_{Target} = B_{MSY}$, assuming a constant annual fishing mortality rate. Rebuilding time isopleths were formed by connecting points of initial biomass and constant fishing mortality (B_0, \tilde{F}_{B_0}) with the same recovery time (Fig. 1). For example, beginning at the initial biomass level $B_0 < B_{Target}$, any constant fishing mortality rate \tilde{F}_{B_0} on the 10-year isopleth would theoretically rebuild the stock to B_{Target} in ten years. In contrast, any constant- F value $< \tilde{F}_{B_0}$ (below or to the right of the isopleth) would rebuild the stock sooner and any constant F values $> \tilde{F}_{B_0}$ (above or to the left of the isopleth) would rebuild the stock later. Rebuilding time isopleths were used to develop overfishing definition options for nine overfished New England groundfish stocks (Applegate et al.³).

In this article, we calculate stock-rebuilding time isopleths based on stochastic population dynamics models and characterize statistical distributions (mean, median, and percentiles) of stock-rebuilding times under different assumptions about uncertainty and process error (pro-

cess errors are uncertainty in population dynamics due to natural variability in growth, recruitment, and other biological factors, Hilborn and Walters, 1992). Like Cadrin (1999), we use logistic population growth models, but our analysis includes uncertainty about F_{MSY} and autocorrelated process errors in production. We analyze rebuilding times for two stocks (cowcod rockfish, *Sebastes levis*, and Georges Bank yellowtail flounder) with different life histories, levels of F_{MSY} , and autocorrelation in production process errors (the calculations are examples only and not for use by managers). We also describe how stock-rebuilding time isopleths from deterministic and stochastic models can be used to develop and evaluate rebuilding plans and to monitor their progress.

Materials and methods

Following Prager (1994), we used the continuous time version of the logistic population dynamic model in simulation calculations.⁴ In particular, for the logistic population growth parameter r_y (subscripted to represent the value in year y) carrying capacity K , $b_y = r_y/K$, and $a_y = r_y - F_y \neq 0$:

$$B_{y+1} = \frac{a_y B_y e^{a_y}}{a_y + b_y B_y (e^{a_y} - 1)} \quad (1)$$

⁴ SAS simulation program code available from the senior author.

Table 1

Six types of simulation models used to estimate rebuilding time isopleths. Model types include all of the meaningful combinations of uncertainty about F_{MSY} and variance and autocorrelation in production process errors.

Model type	CV for uncertainty in F_{MSY} (%)	Variance in process errors (%)	Autocorrelation in process errors (ρ)	Description
1	Zero	0	zero	No uncertainty about F_{MSY} ; deterministic production
2	20	0	zero	Uncertainty about F_{MSY} ; deterministic production
3	Zero	from stock assessment	zero	No uncertainty about F_{MSY} ; stochastic production; no autocorrelation
4	20	from stock assessment	zero	Uncertainty about F_{MSY} ; stochastic production; no autocorrelation
5	Zero	from stock assessment	from stock assessment	No uncertainty about F_{MSY} ; stochastic production; with autocorrelation
6	20	From stock assessment	from stock assessment	Uncertainty about F_{MSY} ; stochastic production; with autocorrelation

When fishing and the intrinsic rate of increase exactly balance ($\alpha_y = r_y - F_y = 0$):

$$B_{y+1} = \frac{B_y}{1 + b_y B_y} \quad (2)$$

Note that biomass declines ($B_{y+1} < B_y$) when $r_y - F_y = 0$ because r_y is a maximum value defined in the limit as biomass approaches zero (Eq. 1).

Following Beddington and May (1977) and May et al. (1978), natural variation in population growth rates was included in our analysis by adding process errors to simulated r_y values. We hypothesized that the intrinsic rate of population increase was more important than carrying capacity in simulating population growth rates at low biomass levels, and in rebuilding overfished stocks. We focused on stochastic variation in r_y because it likely varies annually (e.g. due to variation in recruitment and growth). We could hypothesize reasonable lower bounds which included negative values; and variability in r_y could be reasonably described in statistical terms (e.g. mean, variance, and autocorrelation) based on available data. Carrying capacity (K) was assumed constant over time because no information about potential covariance in r_y and K was available.

In the deterministic logistic population model, $F_{MSY} = r/2$ and $B_{MSY} = K/2$ (Schaefer, 1954). For the sake of simplicity, we assumed $K=1$ so that $B_{Target} = B_{MSY} = 0.5$ and biomass B_y was measured in relation to K (e.g. $B_y = K=1$ at carrying capacity). It was useful to express biomass in relation to K because estimates of ratios like $B_t/B_{MSY} (=2B_t/K)$ are often more precisely estimated than either biomass B_y or carrying capacity K (Prager, 1994) and because the approach makes results easier to apply to other stocks.

We used six types of logistic population growth models (Table 1) based on a wide range of initial biomass (B_0), two levels of uncertainty about F_{MSY} , variance in process errors (stock dependent), and autocorrelation in process errors (also stock dependent). The number of years (an integer) required for the stock to rebuild to $0.95B_{MSY}$ was recorded in each simulation run. Recovery in the simulation model was at $0.95B_{MSY}$, rather than B_{MSY} , because biomass in the deterministic logistic production model at $F = F_{MSY}$ approaches asymptotically (but never reaches) B_{MSY} (this convention had negligible effect on results). Stochastic simulation model results were derived from 2000 individual model runs (the maximum length of each run was 2000 years) starting from each point in a grid of 31 values of F (i.e. $0, F_{MSY}/30, 2F_{MSY}/30, \dots, 29F_{MSY}/30, F_{MSY}$) and 35 values of initial biomass (i.e. $B_0 = 0, \delta \times 10^{-3}, \delta \times 10^{-2}, \delta \times 10^{-1}, \delta, 2\delta, \dots, 30\delta$, where $\delta = 0.9999 \times 0.95 \times B_{MSY}/K/30$).

We calculated distributional statistics including the mean, median ($Q_{50\%}$), and various quantiles (e.g. $Q_{90\%}$ for the ninety-percent quantile) for recovery times from all runs at each point in the grid of F and initial biomass levels. We then plotted isopleths (contours) for the distributional statistics. For example, to produce 10-year median rebuilding time isopleths, we calculated median recovery times for each point in the grid of F and initial biomass, and then drew contours (isopleths) by connecting points with 10 year median rebuilding times to identify fishing mortality rates that, if held constant, would give a 50% probability of rebuilding from the initial biomass to the target in ten years. We smoothed the isopleths in plots by using LOESS (locally weighted regression smoothing) regression (Cleveland and Devlin, 1988; Cleveland et al., 1988) to remove variation caused by the contouring algorithm and coarse grid of fishing mortality and biomass starting points.

Uncertainty

Uncertainty in estimates of F_{MSY} is likely larger than typically measured by variance estimates in assessment models because uncertainties in catch, the assumed natural mortality rate, somatic growth, and other factors are generally not included in stock assessment model variance calculations. In simulation runs including uncertainty, we used

$$F_{MSY,s} = \hat{F}_{MSY} + \varepsilon_s, \quad (3)$$

where $F_{MSY,s}$ was used in simulation s , \hat{F}_{MSY} was the “best” estimate, and ε_s was drawn from a normal distribution with mean zero and variance $\sigma_\varepsilon^2 = CV^2 \hat{F}_{MSY}^2$. The CV (20%) assumed in our simulation runs implies that the true F_{MSY} is within $\pm 40\%$ of the best estimate with about 95% probability. We truncated $F_{MSY,r}$ values at $\pm 50\% \hat{F}_{MSY}$ to avoid implausibly small (including negative) or large $F_{MSY,r}$ values. These *ad hoc* bounds seemed reasonable because they were slightly larger than the 95% confidence interval implied by the CV for uncertainty ($\pm 40\%$).

Our assumptions about uncertainty in F_{MSY} are crude but seem reasonable based on our experience and by analogy to uncertainty about natural mortality rates (M), which are sometimes used as a proxy for F_{MSY} (Clark, 1991). In assessment work, a stock with an assumed natural mortality rate $M=0.2/\text{yr}$, for example, might have a “subjective” uncertainty range of about $\pm 40\%$ (i.e. 0.12–0.28/yr). It seems reasonable to assume that uncertainty about M and F_{MSY} would be similar.

Process errors

We modeled process errors as potentially autocorrelated random changes in the intrinsic population growth parameter (r_y). Previous analyses used independent or autocorrelated random errors in realized annual production rates (dB_y/dy , Sissenwine, 1977; Gleit, 1978; Shepherd and Horwood, 1980; Ludwig, 1981; Sissenwine et al., 1988), or independent random errors in recruitment (e.g. Getz, 1984), next year’s biomass (B_{t+1} , Ludwig et al., 1988), or surplus production (Doubleday, 1976). Production process errors may be independent in some cases but were autocorrelated for both of our example stocks (see “Results” section).

Our analysis, like Sissenwine et al.’s (1988), includes autocorrelated errors because they affected rebuilding times in preliminary model runs, are biologically plausible and widely recognized (favorable and unfavorable conditions for production seem to persist for more than one year in many stocks), and because correlated errors were obvious in production model fits for Georges Bank yellowtail flounder and cowcod rockfish. In contrast to previous studies, we estimated variances and autocorrelations for stochastic r_y values in our simulation models from available data. In addition, our simulation models used lower bounds for r_y based on the natural mortality rate.

We used the gamma distribution (Johnson et al., 1994, Appendix 1) to describe process errors in the produc-

tion model because it is flexible, asymmetrical (like our estimates of production process errors for cowcod rockfish), and (in the three-parameter form) accommodates negative r_y values. We devised a simple way to simulate autocorrelated process errors from a distribution nearly identical to a gamma distribution used to simulate uncorrelated process errors. This makes comparisons between runs with and without autocorrelation easier. Sissenwine et al. (1988) also used a gamma distribution for production process errors because simulated state variables in logistic models (with constant catch and Gaussian process errors on the realized production rate dB_y/dy) have distributions that resemble a gamma distribution (Dennis and Patil, 1984).

The first step in modeling production process errors was to obtain empirical estimates of variance and autocorrelation. Based on stock assessment results, surplus production in each year (P_y) was computed with the following equation:

$$P_{y+1} = B_{y+1} - B_y + C_y, \quad (4)$$

where C_y = catch data was catch; and

B_y = estimated biomass at the beginning of year y .

The discrete time version of our logistic model with process errors is

$$P_y = r_y B_y \left(1 - \frac{B_y}{K} \right). \quad (5)$$

Solving for r_y gives

$$r_y = \frac{P_y K}{B_y (K - B_y)}, \quad (6)$$

where B_y should be no larger than, say, 95% K to avoid unrealistic values of r_y that are calculated when positive production is observed in stock assessment results at biomass levels near or above estimates of K . As shown in the “Results” section, empirical estimates of variance σ^2 and autocorrelation (ρ) for r_y values were relatively insensitive to assumptions about K . The variance of observed r_y values includes both process and measurement errors and is an upper bound estimate for the variance due to process errors only. In other words, results of our simulation analyses may overstate the importance of production process errors in rebuilding overfished stocks because our variance estimates may be too large.

We used $-M$ (where M is the instantaneous natural mortality rate assumed in the stock assessment) as a lower bound on r_y in simulations. Negative r_y values are common in some stocks (e.g. 36% and 17% of years for anchovies [*Engraulis* spp.] and sardines [*Sardinops* and *Sardina* spp.], Jacobson et al., 2001) because stocks can decrease in biomass from one year to next with no fishing and because negative values are occasionally seen in real data sets (e.g. Myers et al., 1999). If process errors are ignored and M is constant, then

Table 2

Summary statistics and gamma distribution parameters for annual process errors in the intrinsic population growth rate (r_y) for Georges Bank yellowtail flounder (*Limanda ferruginea*), estimated from stock assessment results (Cadrin⁵ in the main text). Parameters α and β were estimated with fixed $\gamma = -M$ by maximum likelihood. According to Cadrin,⁵ the carrying capacity is $K=99,000$ metric tons (t) (95% confidence interval 84,500–103,000 t).

Parameter	$K=84,500$ t	$K=99,400$ t	$K=103,000$ t
Number B_y values $\leq 95\% K$	25	25	25
Sample mean	0.58	0.65	0.58
Sample variance	0.036	0.037	0.036
Autocorrelation ρ	0.34	0.33	0.34
α	15.7	17.5	15.7
β	0.0498	0.0488	0.0499
$\gamma = -M$	-0.2	-0.2	-0.2
Mean	0.58	0.65	0.58
Variance	0.039	0.039	0.039
Mode	0.53	0.61	0.53

$$r_y = G_y + R_y - M \quad (7)$$

where G_y = the instantaneous rate of somatic growth;
and

R_y = an instantaneous rate for recruitment (in units of biomass).

Somatic growth and recruitment may be density dependent but are usually positive ($G_y \geq 0$ and $R_y \geq 0$). Thus, $r_y = -M$ is possible in the extreme case of zero growth and zero recruitment.

Production process errors were simulated by drawing random numbers from a three-parameter gamma probability distribution (Johnson et al., 1994, Appendix 1). Runs with autocorrelated process errors used one of two algorithms based on gamma distributions with adjusted parameter estimates (Appendix 2).

Georges Bank yellowtail flounder

Cadrin⁵ used virtual population analysis (VPA, calibrated by using survey data) to estimate stock biomass for Georges Bank yellowtail flounder during 1973–98. In the same assessment, a surplus production model (ASPIC [stock-production model incorporating covariates], Prager, 1994) was used to estimate $K=93,700$ metric tons (t) (80% bootstrap confidence interval 87,700–97,000 t) and $F_{MSY}=0.30/\text{yr}$ (80% bootstrap confidence interval 0.27–0.32/yr).

Cadrin's stock assessment⁵ and our production calculations indicate that Georges Bank yellowtail flounder is a moderately long-lived (maximum observed age 14 yr,

assumed $M=0.2$ /yr), relatively productive ($\bar{r}=0.58$ –0.65) stock with some autocorrelation ($\rho=0.33$ –0.34) in production process errors (Table 2). Empirical, and gamma distributions fit by maximum likelihood and the method of moments had similar means and variances (Table 2). Surplus production and biomass are related for Georges Bank yellowtail flounder, with P_y reduced at the low B_y levels (Fig. 2A). Variability in estimated r_y values indicate autocorrelation in process errors (Fig. 2B). The distribution of r_y values (Fig. 2C) was skewed to the left and there were no negative values. Gamma distributions fitted by maximum likelihood and the method of moments (Appendix 1) were similar in shape (Fig. 2C). In simulations for yellowtail flounder, we used $F_{MSY}=0.30$ (from ASPIC) with $\sigma_{r_y}^2=0.037$ and $\rho=0.33$.

Cowcod rockfish

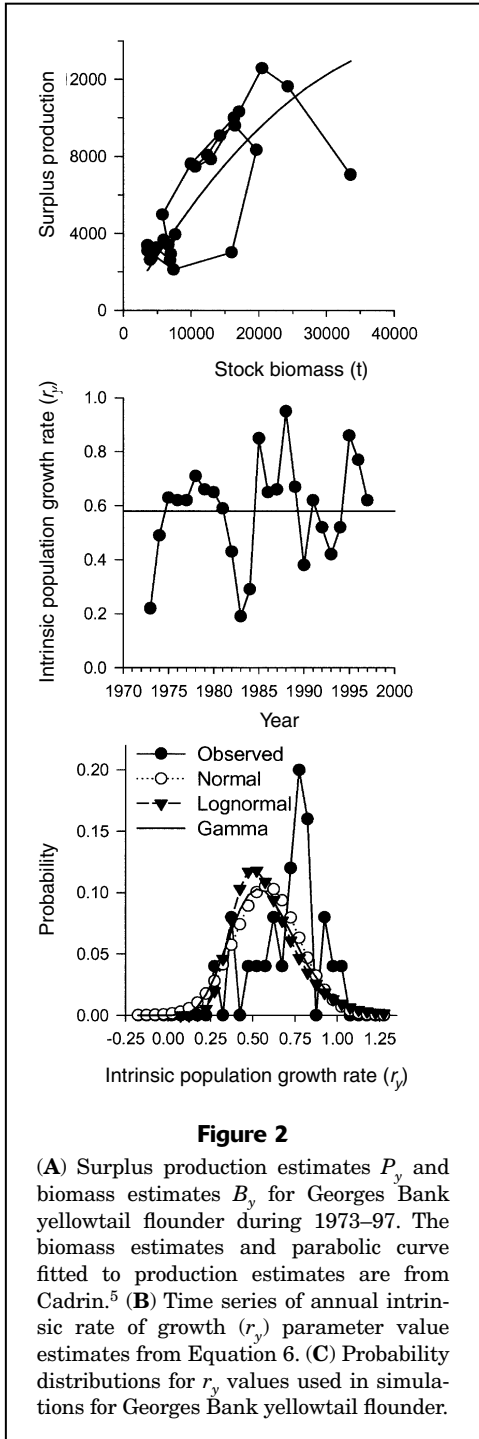
Butler et al.⁶ (see also Butler et al.⁷) estimated $K=3400$ t (95% CI 2800–4000 t) with a delay-difference biomass dynamic model for cowcod rockfish in the Southern California Bight. Annual biomass estimates from the same source were used to calculate surplus production during 1951–97 when the stock was fished down from about 3200 t to 240 t (about 7% of virgin biomass).

Butler et al.⁶ and our production calculations indicate that cowcod rockfish are a long-lived (maximum observed

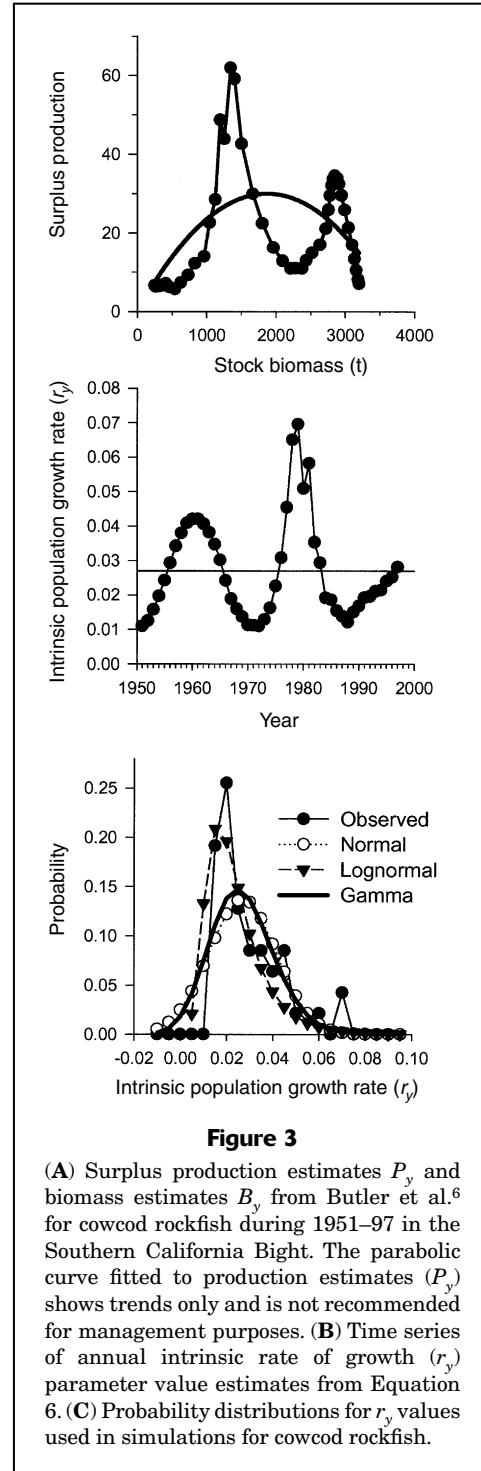
⁵ Cadrin, S. X. 2000. Georges Bank yellowtail flounder. In Northern demersal working group: assessment of 11 northeast groundfish stocks through 1999. In Northeast Fisheries Science Center Reference Document 00-05, p. 45–64. Northeast Fisheries Science Center 166 Water Street, Woods Hole, MA, 02543.

⁶ Butler, J. L., L. D. Jacobson, J. T. Barnes, and H. G. Moser. 2002. Manuscript in review. Biology and population dynamics of cowcod rockfish (*Sebastes levis*) in the southern California Bight.

⁷ Butler, J. L., L. D. Jacobson, J. T. Barnes, H. G. Moser, and R. Collins. 1999. Stock assessment of cowcod. In Appendix to the state of the Pacific Coast groundfish fishery through 1999 and recommended acceptable biological catch for 2000 stock assessment and fishery evaluation, p. Vi–113 (section 5). Pacific Fishery Management Council, 7700 NE Ambassador Place, Portland, OR, 97220-1384.



age 55 yr, assumed $M=0.055/\text{yr}$), relatively unproductive stock ($\bar{r}_y=0.027-0.037$). Cowcod rockfish are much less productive than Georges Bank yellowtail flounder because of their long lives and slower growth and because adult habitat is limited to steep rocky areas in relatively deep water (90–500 m, Butler et al.⁶). Production process errors show a high level of autocorrelation ($\rho=0.83-0.94$, Table 3). Means, variances and autocorrelations for r_y were not very sensitive to assumptions about K (Table 3). There was



no clear relationship between surplus production and biomass, but P_y was lowest at the highest and lowest B_y levels and autocorrelation in production process errors was obvious (Fig. 3A). The distribution of r_y values (Fig. 3B) was skewed to the right and there were no negative values. Gamma distributions fitted by maximum likelihood and the method of moments (Appendix 1) were similar in shape (Fig. 3).

Table 3

Summary statistics and gamma distribution parameters for annual process errors in the intrinsic population growth rate (r_y) for cowcod rockfish (*Sebastes levis*), estimated from stock assessment results (Butler et al.⁶). Parameters α and β , were estimated with fixed $\gamma = -M$ by maximum likelihood. K is the carrying capacity in the logistic population dynamics model. According to Butler et al.,⁶ $K=3400$ metric tons (t) (95% confidence interval 2800–4000 t).

Parameter	$K=84,500$ t	$K=99,400$ t	$K=103,000$ t
Number B_y values $\leq 95\% K$	31	47	47
Sample mean	0.037	0.039	0.027
Sample variance 0.00021	0.00059	0.00050	
Autocorrelation ρ	0.83	0.94	0.89
α	-0.055	-0.055	-0.055
β	17.7	19.0	36.1
$\gamma = -M$	0.0052	0.0049	0.0023
Mean	0.037	0.039	0.027
Variance 0.00019	0.00048	0.00047	
Mode	0.032	0.034	0.025

With $K=3400$ t (Table 3), $\bar{r} = 0.039$ which suggests $F_{MSY} = 0.0185/\text{yr}$. This crude estimate is less than implied by the simple calculation $r = 2F_{MSY} = 0.11$ based on the proxy $F_{MSY} = M = 0.055/\text{yr}$, possibly because the natural mortality rate overestimates F_{MSY} for cowcod (Deriso, 1982). ASPIC estimates were similar ($F_{MSY} = 0.018/\text{yr}$ with an 80% bootstrap confidence interval 0.00082–0.039/yr) to estimates based on \bar{r}_y . In simulations for cowcod, we used $\hat{F}_{MSY} = 0.018$ (from ASPIC), $\sigma_{r_y}^2 = 0.00050$ and $\rho = 0.9$.

Simulation model runs indicated that cowcod are very unlikely to rebuild to B_{MSY} in ten years. The mean generation time for cowcod is about 35 years (calculated as described by Restrepo et al., 1998). Simulations indicate that the mean time for rebuilding the stock with zero F is approximately 40 years (30–50 years, depending on model type). In accord with National Standard 1 Guidelines, it may be reasonable to develop plans with the goal of rebuilding the cowcod stock in 75 years or less. We therefore calculated and plotted 75-year, rather than 10-year rebuilding time isopleths, for cowcod rockfish.

Sensitivity analyses

We conducted three sensitivity analyses for each stock to determine if the choice of statistical distribution for stochastic $r_{s,y}$ values influenced rebuilding times in simulations. Sensitivity analyses used uncorrelated process errors and no uncertainty in F_{MSY} (model type 3). The first sensitivity analysis run for each stock was a non-parametric bootstrap (Efron, 1982) with $r_{s,y}$ values drawn randomly with replacement from the observed values (Eq. 6). The second and third sensitivity analyses run for each stock were parametric bootstraps with $r_{s,y}$ values drawn from a normal or lognormal distribution with the same mean (μ) and variance (σ^2) as the observed values (Tables 2–3, Figs. 2–3). To avoid bias in runs with the lognormal

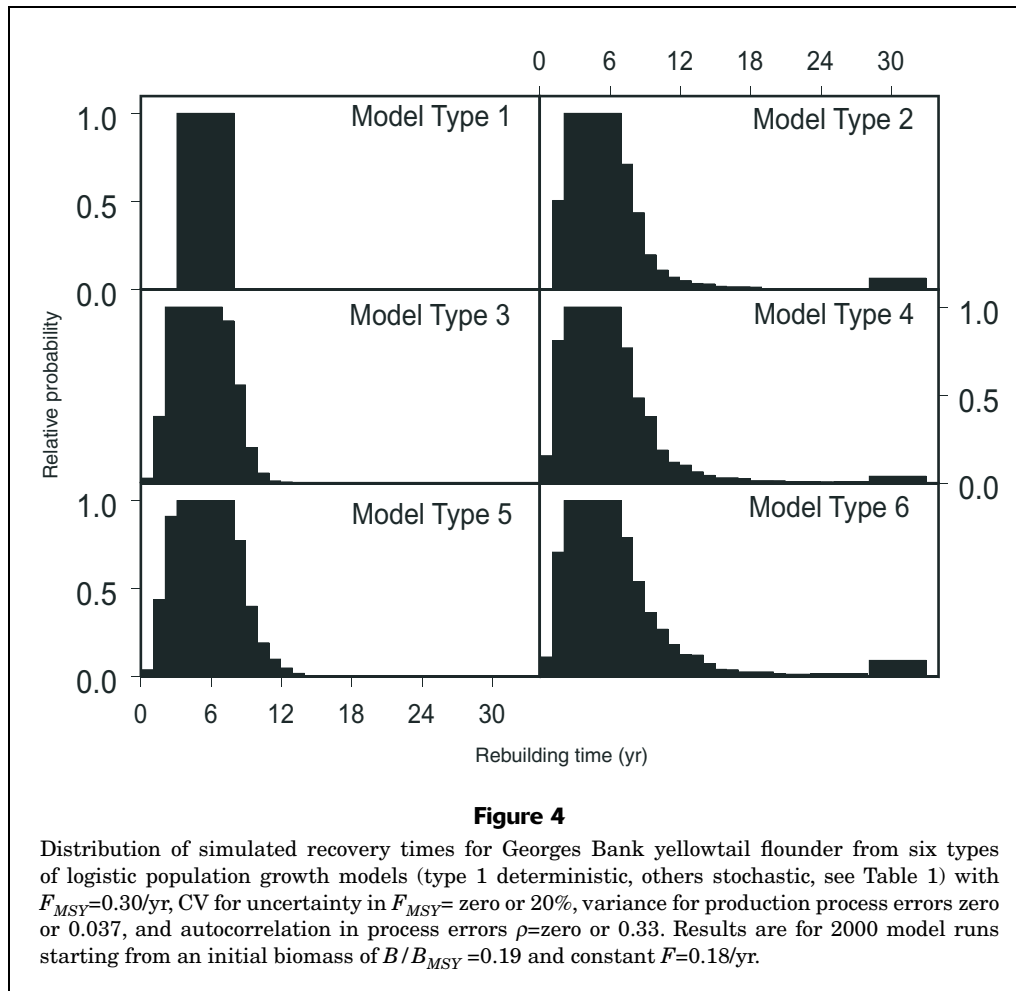
distribution (Beauchamp and Olson, 1973), log-transformed $r_{s,y}$ values were drawn from a normal distribution with mean $\ln(\mu) - \tau^2/2$ and variance $\tau^2 = \ln(\text{CV}^2 + 1)$, with $\text{CV} = \sigma/\mu$ (Jacobson et al., 1994). For convenience in programming, $r_{s,y}$ values in normal and lognormal runs were sampled with replacement from a fixed pool of 200 random numbers drawn from the proper statistical distribution at the outset of the simulation.

Results

Mean rebuilding times were longer than median rebuilding times in stochastic simulations (run types 2–6) for yellowtail flounder and cowcod rockfish because distributions of recovery times were skewed to the right (Figs. 4–5). Skewness (and the extent to which mean recovery times exceeded median recovery times) was more pronounced at lower starting biomass levels, at higher F levels, where there was more uncertainty about F_{MSY} and autocorrelation in process errors.

Skewed distributions for rebuilding times affected the shape of rebuilding time isopleths (Figs. 6–7). For all stochastic models of yellowtail flounder and most stochastic models of cowcod rockfish, isopleths for mean and median rebuilding times were widely separated. In addition, isopleths were asymmetrical. For example, the distance between median and $Q_{90\%}$ isopleths was greater than the distance between $Q_{10\%}$ and median isopleths. Separation of isopleths and asymmetry were greatest for model type 6 which had correlated process errors and uncertainty about F_{MSY} .

In simulations for both stocks, uncertainty about F_{MSY} had a greater effect than process error on the shape and separation between isopleths for $Q_{10\%}$, median, mean, and $Q_{90\%}$ rebuilding times. For example, in both stocks, there



was more asymmetry and separation between isopleths (Figs. 6–7) for model type 2 (no process errors and with uncertainty about F_{MSY}) than for model type 3 (uncorrelated process error with no uncertainty about F_{MSY}).

The fishing mortality rate that gave a median recovery time of ten years for yellowtail flounder (or 75 years for cowcod rockfish) was generally higher than the fishing mortality rate that gave a corresponding mean recovery time from the same initial biomass level (Figs. 8–9). Similarly, the F that gave a mean recovery time of ten or 75 years was generally higher than the F that gave the corresponding $Q_{90\%}$ recovery time. There were some exceptions for cowcod at high F and initial biomass levels in runs with uncertainty in F_{MSY} (run types 2, 4, and 6) due to a few runs with very long rebuilding times. The very long rebuilding times in runs for cowcod with uncertainty were due to F in excess of the simulated true F_{MSY} .

Isopleth shape may be important in interpreting simulation results. For example, isopleths for 10-year median rebuilding times in all models for yellowtail flounder were steep for F in the range 0–0.2/yr (Figs. 8–9). Therefore, according to example model results, the probability of recovery in ten years is at least 50% for Georges Bank

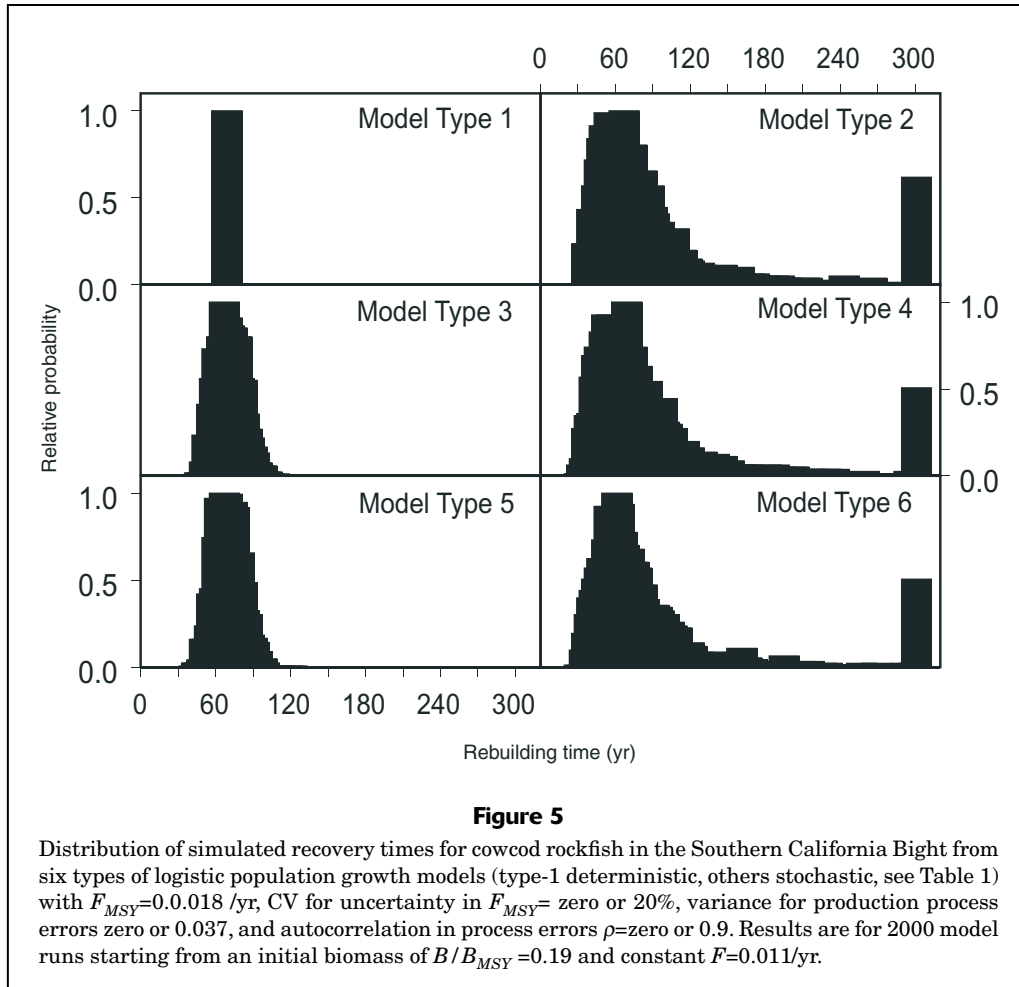
yellowtail flounder at biomass values $\geq 0.05 K$, as long as fishing mortality rates are less than 0.2/yr.

Seventy-five year stock-rebuilding time isopleths for cowcod rockfish were sensitive to assumptions about the distribution of production process errors but 10-year isopleths for yellowtail flounder were not (Figs. 10–11). We hypothesize that differences among statistical distributions r_y assumed in simulations were magnified for cowcod by long (e.g. 75 yr) rebuilding times (see “Discussion” section).

Rebuilding isopleths for cowcod based on r_y values from a gamma distribution had higher F , at a given biomass, than rebuilding isopleths based on the distribution of observed r_y values (Fig. 11). In other words, results based on the gamma distribution suggest a more productive cowcod stock, presumably because the distribution of r_y values for cowcod had more mass than the gamma distribution over low r_y values ($<0.03/\text{yr}$, Fig. 3).

Discussion

Managers should consider using median rebuilding time goals, in addition to mean or other quantiles, in develop-

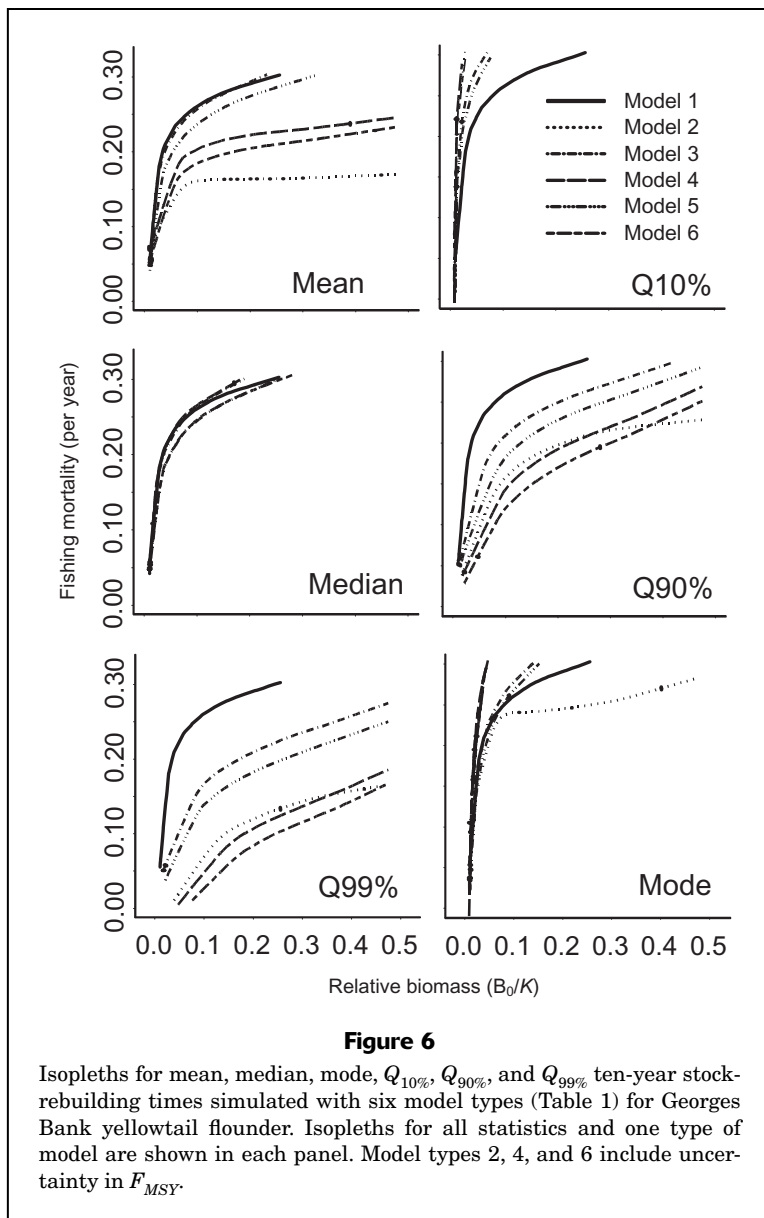


ing and evaluating rebuilding plans. This recommendation is based on a narrow technical consideration, i.e. that median recovery time calculations are less sensitive to model assumptions. A median rebuilding time plan is risk neutral in the sense that the probability of rebuilding times less than intended is the same as the probability of rebuilding times longer than intended (i.e. both 50%). Of course, rebuilding plans based on median rebuilding time goals will be more liberal in terms of short-term catch (i.e. have higher F) and have longer rebuilding times on average than plans based on mean or, for example, $Q_{90\%}$ rebuilding time goals. In summary, a manager who is willing to accept a 50% chance of rebuilding times greater than desired and who is concerned about model uncertainties, might choose a median rebuilding time approach.

A potential advantage in using percentiles other than the median is that managers can specify risk levels in trying to rebuild stocks. For example, managers could choose and evaluate rebuilding plans based on a $Q_{90\%}$ isopleth to insure at least a 90% chance of rebuilding in specified time period. However, $Q_{90\%}$ isopleths may be sensitive to model assumptions.

Approaches to using rebuilding time isopleths without relying on uncertain estimates of rebuilding time distributions (e.g. mean or $Q_{90\%}$), are an important area for future research. Cadrin (1999) used deterministic rebuilding time isopleths with the tenth percentile of the estimate for r . Isopleths for median rebuilding time might be used with lower bounds on confidence intervals for B_0 and B_0/K or upper bounds on current F , F_{MSY} , or F/F_{MSY} . Estimates of uncertainty in these parameters are often available (Prager, 1994) and can be incorporated in an *ad hoc* fashion. For example Butler et al.⁶ suggest that B_{1998}/K for cowcod rockfish is 7% (CV about 30%) and that the F in 1998 was 0.085/yr (CV 34%). A risk-averse manager might implement a rebuilding plan that reduces that $F_{Threshold}$ to a point on the 75-year rebuilding time isopleth that lies above the lower boundary of a 95% confidence interval for biomass. Similarly, a risk-averse manager might select a rebuilding plan that reduces $F_{Threshold}$ to account for the upper bound on uncertainty in estimating F .

Uncertainty in F_{MSY} is an important factor to consider in rebuilding plans as F increases from low levels towards F_{MSY} . Expected rebuilding times increase at high F because F may exceed true F_{MSY} (assumed known but with



error) in a high proportion of cases (Figs. 8–9). Fortunately, process errors and autocorrelation may reduce this problem because stock growth rates increase in some years so that the true F_{MSY} exceeds the manager's estimate.

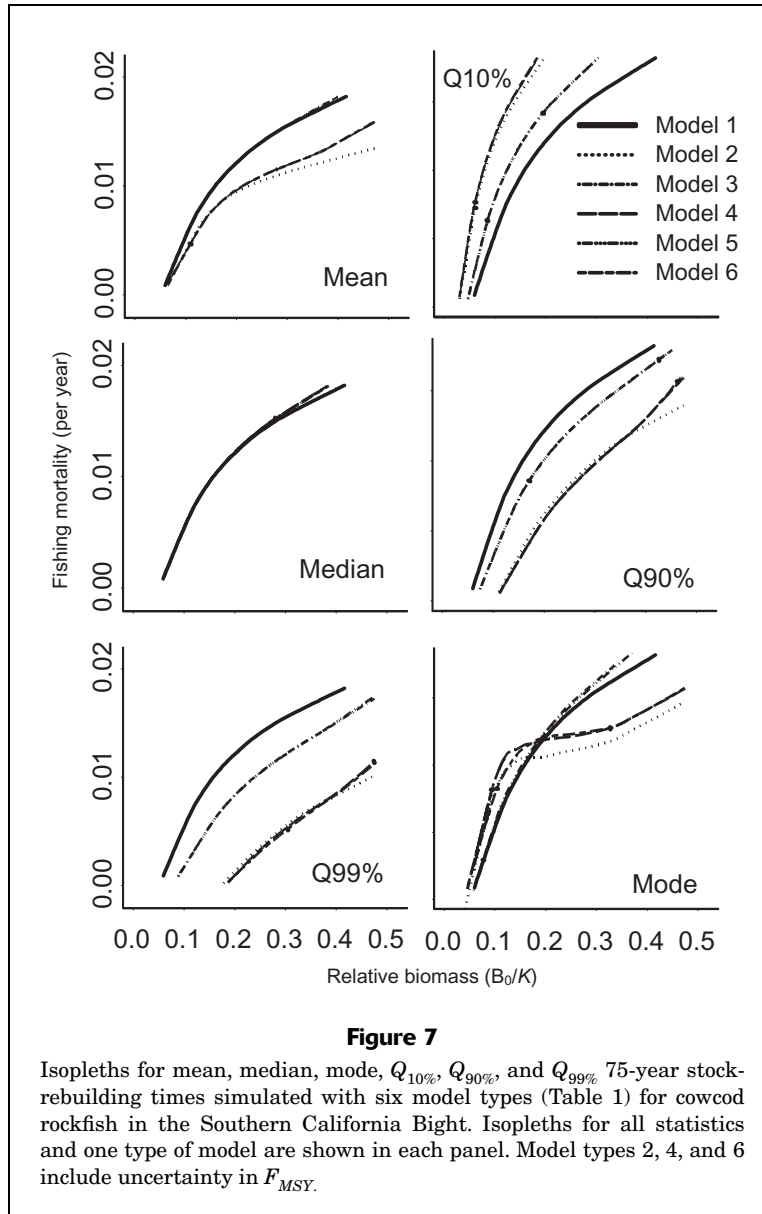
Stock-rebuilding times calculated with deterministic models approximate median rebuilding times from stochastic models. For example, rebuilding times in Cadrin (1999) calculated with a deterministic model for Georges Bank yellowtail flounder are very close to median rebuilding times from our stochastic models. From our results, we hypothesize that rebuilding time isopleths for other species (Applegate et al.³) based on Cadrin's (1999) deterministic model should also be viewed as approximations to isopleths for median rebuilding times.

Our simulation analyses indicate that rebuilding times for overfished stocks (with a range of life history charac-

teristics, initial biomass levels and fishing mortality rates) tend to be skewed and can be highly variable (Figs. 4–5). Hence, rebuilding in any specific case may be quicker or take much longer than expected, particularly if expectations are based on deterministic models that approximate median rebuilding times. For example, probabilities of rebuilding times twice as long as the goal for Georges Bank yellowtail flounder (10 yr) and cowcod (75 yr) were 4% and 8% and probabilities of rebuilding times half as long were 40% and 1%.

Modeling choices

Stochastic models are necessary when estimates of mean rebuilding times or quantiles other than the deterministic approximation to the median are needed. Rebuilding time



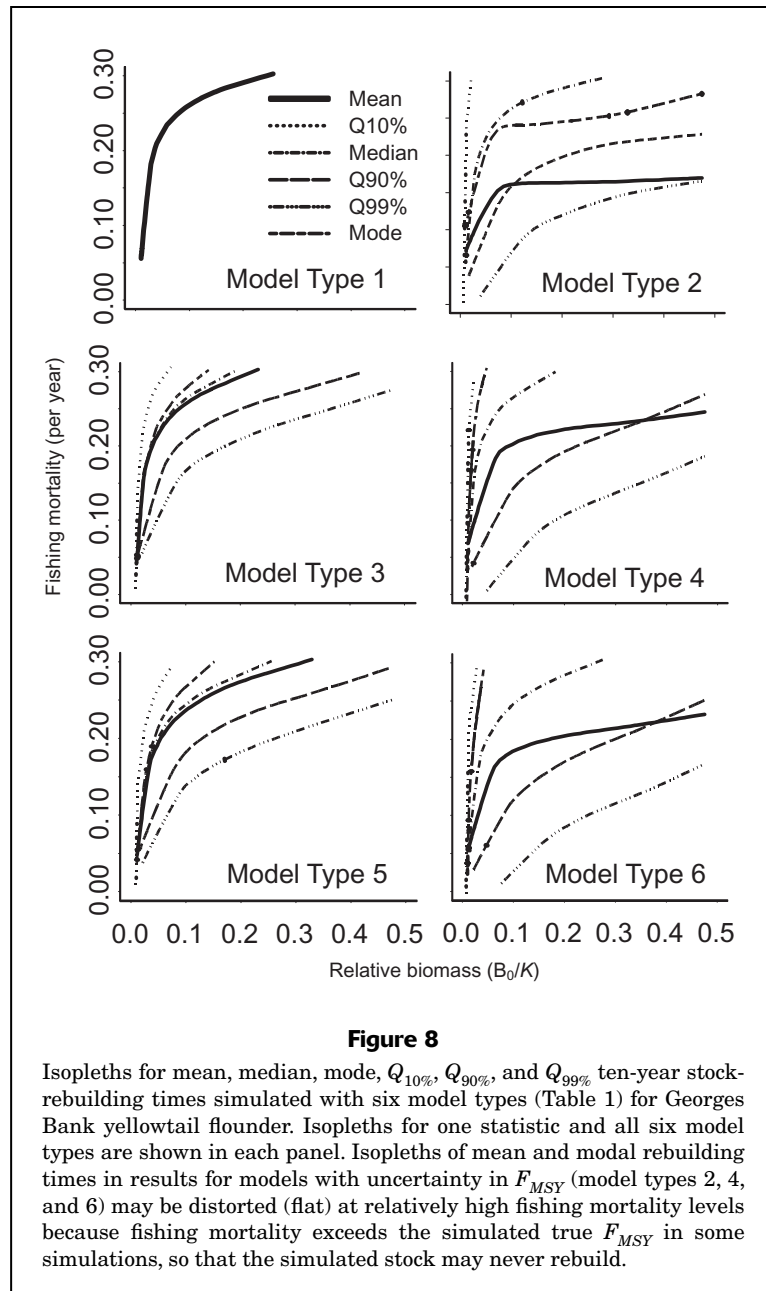
isopleths from deterministic model type 1 were quite different from isopleths (other than for median rebuilding times) from stochastic models.

Rebuilding time isopleths can be used with any type of population dynamics model. We used the logistic population growth model in this paper because it is clearly linked to B_{MSY} and F_{MSY} , incorporates density dependence, is easy to apply to a wide range of stocks (with varying amounts of information), and computationally efficient. However, rebuilding time isopleths could have been calculated by using Cadrin's⁵ age-structured model for Georges Bank yellowtail flounder or Butler et al.'s⁶ biomass dynamic model for cowcod rockfish. Age-structured models might be best for calculating rebuilding time isopleths if the rebuilding time frame is relatively short and estimates of abundance are available for several incoming year classes because

age-structured models account for transient conditions (e.g. recruitment and growth patterns) that are important in the short term.

Age-based projections were used to test the expected performance of rebuilding targets for six New England groundfish stocks (NDWG⁸). The original rebuilding targets were derived from five-year rebuilding time isopleths calculated with a deterministic logistic growth models, but the targets incorporated estimation uncertainty by assuming the tenth percentile of the estimate

⁸ NDWG (Northern Demersal Working Group). 2000. Assessment of 11 northeast groundfish stocks through 1999. Northeast Fisheries Science Center Reference Document 00-05, 175 p. Northeast Fisheries Science Center, 166 Water Street, Woods Hole, MA, 02543.



of r (Cadrin, 1999; Applegate et al.³). For the six stocks, starting biomass in 1999 ranged from 25% to 93% of B_{MSY} and estimates of F_{MSY} ranged from 0.5 to 0.8. Estimated rebuilding times averaged 3.5 years (ranging from 1 to 7 yr) for 50% probability of attaining B_{MSY} (NDWG⁸). Therefore, the age-based simulations indicated that isopleths based on deterministic biomass dynamic models generally performed well for overfished New England groundfish stocks. Brodziak et al. (2001) analyzed stock-recruit data and concluded that “(1) time horizons for rebuilding will be uncertain, owing to recruitment variability, (2) some productive stocks (haddock, yellowtail flounder) have serial correlation in recruitment and this may either enhance

or diminish chances for stock recovery.” Thus, results with surplus production models, age-structured models, and stock-recruit analyses highlight the fundamental similarities between a wide range of modeling approaches (Sissenwine and Shepherd, 1987).

The best choice of stochastic simulation model for developing and evaluating rebuilding plans will depend on the situation. Sainsbury (1993) concluded that models incorporating simple assumptions about population dynamics were more appropriate for evaluating performance of control rules than models with more complex assumptions. PFMC¹ used a simple model incorporating environmental effects on recruitment with useful results. However, Bell

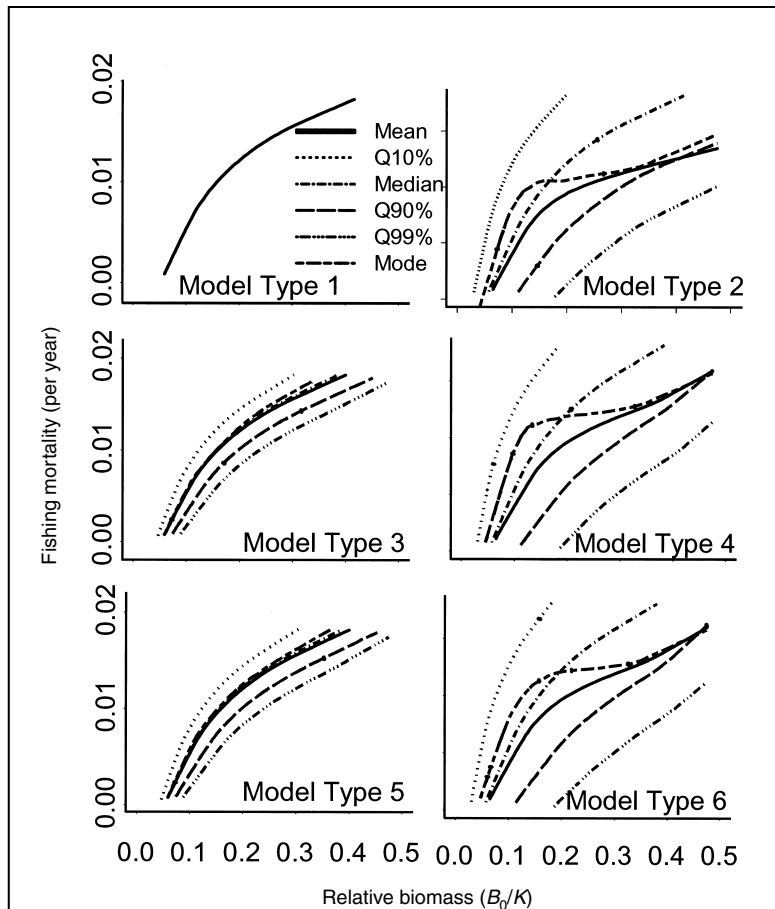


Figure 9

Isopleths for mean, median, mode, $Q_{10\%}$, $Q_{90\%}$, and $Q_{99\%}$ 75-year stock-rebuilding times simulated with six model types (Table 1) for cowcod rockfish in the Southern California Bight. Isopleths for one statistic and all six model types are shown in each panel. Model types 2, 4, and 6 include uncertainty in F_{MSY} . Isopleths of mean and modal rebuilding times in results for models with uncertainty in F_{MSY} (model types 2, 4, and 6) may be distorted (flat) at relatively high fishing mortality levels because fishing mortality exceeds the simulated true F_{MSY} in some simulations so that the simulated stock may never rebuild.

and Stefansson⁹ and Patterson (1999) used more complex simulation models with success.

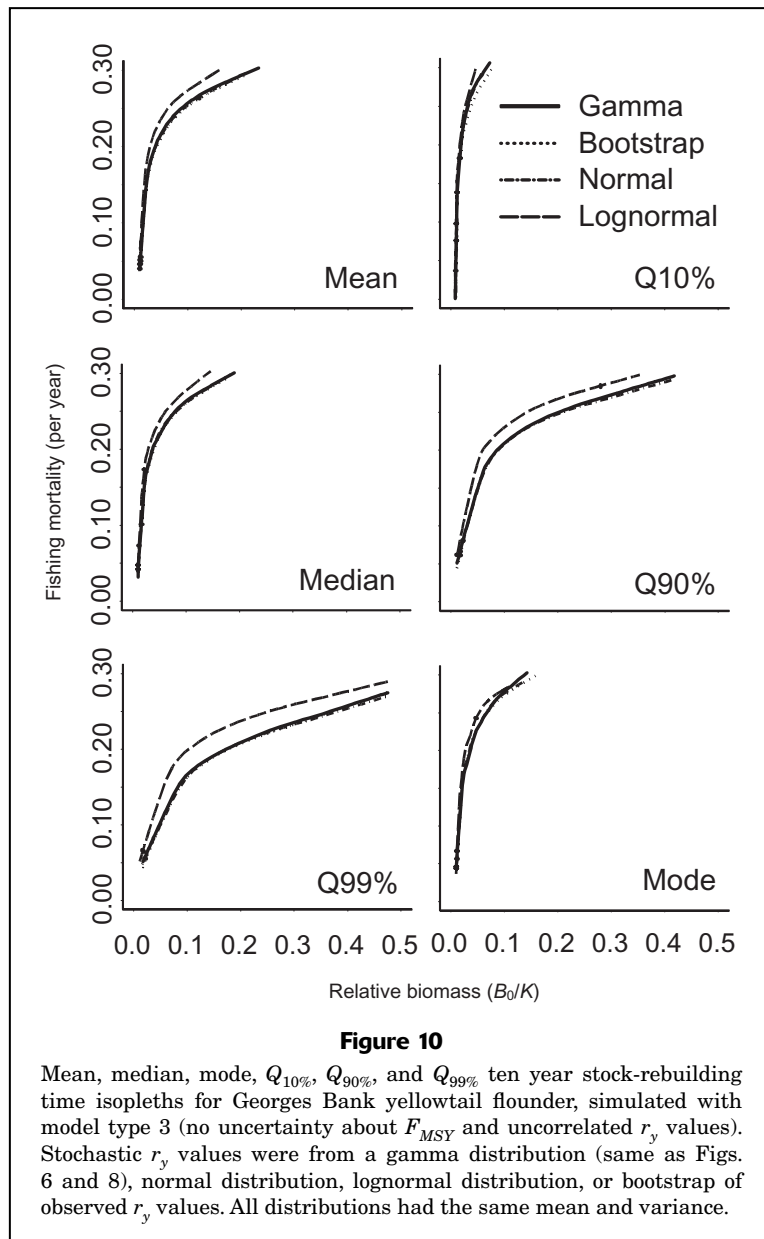
Distributional assumptions

Simulation analyses (Figs. 10–11) indicate that the choice of statistical distribution for simulating process errors in model parameters (e.g. r_y) may be important,

particularly when rebuilding times are long (e.g. those for cowcod rockfish) due to low stock productivity, low stock biomass, unproductive stock dynamics, or autocorrelation in process errors. The choice of statistical distributions for simulating r_y involves choosing between theoretical distributions supported by theory (e.g. autocorrelated gamma distribution with negative values bounded below at $-M$) or bootstrap distributions of observed values. The programming and work required to experiment with alternative distributions is not overwhelming and we recommend sensitivity analyses in cases where distributional assumptions may be important.

Theoretical distributions for stochastic parameters are flexible because many types of distributions are available, most can be modified to include autocorrelation, most can

⁹ Bell, E. D., and G. Stefansson. 1998. Performance of some harvest control rules. NAFO (North Atlantic Fisheries Organization) SCR Doc. 98/7, 1–19. Northwest Atlantic Fisheries Organization, 2 Morris Drive, P. O. Box 638, Dartmouth, Nova Scotia, B2Y 3Y9, Canada.



be modified to include negative or extreme values not evident in short observed time series, and most can give the same mean, variance and autocorrelation levels as estimated from available data. However, as in the case of Georges Bank yellowtail flounder and cowcod, the shape of theoretical and observed distributions may not match closely (Figs. 2–3).

In comparing theoretical and observed distributions for model parameters (e.g. Figs. 2–3), it is important to remember that most observed distributions are based on relatively few observations (Table 2). Furthermore, observed values may be autocorrelated ($\rho=0.33$ for Georges Bank yellowtail flounder and $\rho=0.94$ for cowcod rockfish, Tables 2–3). High levels of autocorrelation reduce the

“effective” number of observations dramatically so that observed values may provide a poor estimate of the shape of their distribution (Bartlett, 1946; Bayley and Hammersley, 1946). For example, Equation 16 in Bayley and Hammersley, with estimated autocorrelations (lags of 1–13 years) for Georges Bank yellowtail flounder, gives an effective sample size $n^*=17$ (compared to $n=25$ r_y values). For cowcod rockfish (with autocorrelations for lags 1–17), $n^*=11$ (compared to $n=47$ r_y values). Thus, autocorrelation in r_y values may reduce the effective sample size and information used to estimate the shape, mean and variance of statistical distributions for r_y values by about 32% for Georges Bank yellowtail flounder and 77% for cowcod rockfish.

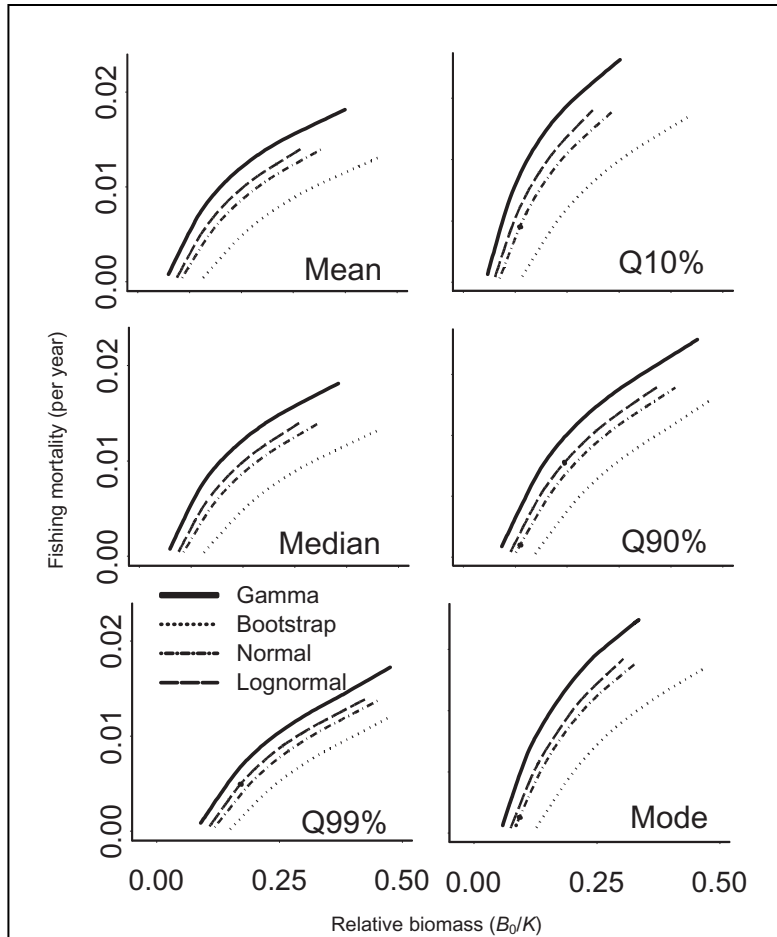


Figure 11

Mean, median, mode, $Q_{10\%}$, $Q_{90\%}$, and $Q_{99\%}$ 75-year stock-rebuilding time isopleths for cowcod rockfish, simulated with model type 3 (no uncertainty about F_{MSY} and uncorrelated r_y values). Stochastic r_y values were from a gamma distribution (same as Figs. 7 and 9), normal distribution, lognormal distribution or bootstrap of observed r_y values. All distributions had the same mean and variance.

Developing, monitoring, and evaluating stock-rebuilding programs

Once the management goal, desired probability of achieving the stock-rebuilding goal, and the time frame for rebuilding are identified (e.g. 10-yr median rebuilding time to a B_{MSY} target), the simplest way to use stock-rebuilding time isopleths in designing a rebuilding plan is to choose a constant- \tilde{F}_{B_0} level from the appropriate rebuilding time isopleth, based on a current estimate of B_0 . Cadrin (1999) has provided an example of this approach.

Stock-rebuilding time isopleths can be used to monitor the progress of any rebuilding plan although interpretation is clearest with constant- F values (Cadrin, 1999). For example, the point defined by current biomass and F for Georges Bank yellowtail flounder in the second year

(1997) of a hypothetical five-year rebuilding plan beginning in 1996 should lie near or within the 3-year rebuilding time isopleth (Fig. 1). If the point lies far outside the 3-year isopleth, then managers could be sure that the rebuilding plan was behind schedule.

Evaluating harvest control rules as stock-rebuilding programs

It may be necessary to evaluate harvest control rules that allow F to vary with biomass (e.g. the common harvest control rule in Fig. 1) as a rebuilding plan. Rebuilding isopleths provide guidance in this situation because they can be used to reject some harvest control rules based on a single necessary criterion. However, the test is weak because a harvest control rule that passes the test may or

may not be sufficient as a rebuilding plan. In this context, it is important to remember that rebuilding time isopleths are constructed based on the assumption of constant fishing mortality rates during the rebuilding program. More complicated rebuilding plans, that allow F to vary according to changes in biomass or other factors, are best evaluated by stock-specific simulations.

The test is based on the notion that control rules that allow F levels above the stock-rebuilding time isopleth for biomass levels above B_0 are unlikely to rebuild the stock with desired probability in the desired time frame. Therefore, as a minimum requirement for meeting rebuilding time goals, harvest control rules used as rebuilding plans should lie on or under the corresponding rebuilding time isopleth for all biomass levels above B_0 . Consider a hypothetical overfished stock for which there is a ten-year median rebuilding time goal. Assume that a harvest control rule proposed as a rebuilding plan has the typical shape (i.e. F increases or stays the same as biomass increases, as in Fig. 1). If the control rule lies above the isopleth for some critical biomass level between B_0 and B_{MSY} , then the rule will allow fishing mortality rates that are generally too high to meet management goals once biomass reaches the critical level.

The example of Georges Bank yellowtail flounder

Georges Bank yellowtail flounder (Fig. 1) can be used to illustrate how rebuilding time isopleths might have been used to evaluate stock-rebuilding plans for hypothetical implementation during 1996 and how stock-rebuilding time isopleths can be used to monitor progress in rebuilding overfished stocks. The discussion is hypothetical, however, because the examples evaluate management approaches that have not been used in rebuilding the stock. In reality, managers kept F for Georges Bank yellowtail flounder during 1996–99 nearly constant at a level well below the 10-year isopleth, and the Georges Bank yellowtail flounder stock was almost rebuilt to the B_{MSY} target level in 1999 after only four years (Fig. 1).

Based on this example, the harvest control rule in Figure 1 would have been marginal for use in a hypothetical median ten-year stock-rebuilding plan for Georges Bank yellowtail flounder starting in 1996, because the rule lies slightly above the 10-year isopleth for biomass levels of 47% B_{MSY} . The harvest rule might have been rejected outright as a five-year rebuilding plan because the rule lies well above the 5-year rebuilding time isopleth. The rebuilding trajectory for Georges Bank yellowtail flounder (Fig. 1) shows that the five-year rebuilding plan, which began in 1996, was on schedule during 1996–98 because fishing mortality and estimated biomass were within the 5-year, 4-year, and 3-year isopleths during successive years.

The Georges Bank yellowtail flounder example provides an important final lesson about uncertainty in actual rebuilding times, even if statistical distributions of potential rebuilding times are characterized accurately. During 1996–99, the stock was managed at a relatively constant F level that was well above the entire 4-year median rebuilding time isopleth (Fig. 1). Despite the relatively high F level, Georges Bank yellowtail flounder reached a

biomass level near B_{MSY} in 1999, after four years. We attribute this fortunate chain of events to stochastic variation in process errors stemming primarily from recruitment and growth of the strong 1997 year class (Cadrin⁵).

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Appendix 1—Independent process errors

We simulated independent production process errors using a three-parameter gamma probability distribution (Johnson et al., 1994). The gamma probability density function for independent (no autocorrelation) r_y values in one simulation (denoted as s) was

$$P(r_{s,y} | \alpha_s, \beta_s, \gamma) = \frac{(r_{s,y} - \gamma)^{\alpha_s - 1} e^{-(r_{s,y} - \gamma)/\beta_s}}{\beta_s^{\alpha_s} \Gamma(\alpha_s)}, \quad (1)$$

where $\Gamma(\alpha_s)$ is the gamma function and the parameters of the gamma distribution are $\alpha_s > 0$, $\beta_s > 0$ and γ .

The expected (mean) value for r_y values from the three-parameter gamma distribution is $\beta_s \alpha_s + \gamma$ and the variance is $\beta_s^2 \alpha_s$. The parameter γ defines a minimum value for the distribution of r_y values; therefore we set $\gamma = -M$ (see text). There were too little data to directly estimate the lower-bound γ for either of the species in our analysis because years with negative production occurred infrequently for stocks in our analysis. The three-parameter gamma distribution has a single mode at $\gamma + \beta_s(\alpha_s - 1)$ if $\alpha_s > 1$. When $\alpha_s \leq 1$, the probability distribution function declines monotonically as r_y increases from the minimum value at $r_y = \gamma$.

The mode for empirical r_y values in our study was always larger than $-M$ so that α_s was larger than one.

Maximum likelihood estimates of the parameters α and β for each stock (conditional on the assumption $\gamma = -M$) were obtained iteratively in a spreadsheet by maximum likelihood with observed r_y values as data. In simulations, we used the simpler method of moments to calculate gamma distribution parameters. Given $\gamma = -M$ and estimates of the mean (μ) and variance (σ^2) for r_y values from real data, the method of moments solves the two equations $\mu = (B\alpha + \gamma)$ and $\sigma^2 = \alpha\beta^2$ for the two unknowns α and β . In particular $\beta = \sigma^2 / (\mu - \gamma)$ and $\alpha = \sigma^2 / \beta^2$. The maximum likelihood approach and method of moments gave similar parameter estimates for real data sets (Figs. 2–3) suggesting that the method of moments was acceptable for simulations. In what comes later, maximum likelihood parameter estimates from data ($\hat{\alpha}, \hat{\beta}, \gamma = -M$) are distinguished from simulated values calculated by the method of moments ($\tilde{\alpha}_s, \tilde{\beta}_s, \gamma = -M$).

In each simulation run, the assumed “true” value of $F_{MSY,s}$ was used to calculate $\bar{r}_s = 2F_{MSY,s}$. In simulation runs with no process error the simple calculation $r_{s,y} = \bar{r}_s$ was used. In runs with process error, \bar{r}_s was the mean of the distribution of stochastic $r_{s,y}$ values (see below).

For runs with independent process errors (no autocorrelation), annual logistic parameter values $r_{s,y}$ were drawn from a gamma distribution with parameters $(\tilde{\alpha}, \tilde{\beta}, \gamma)$ calculated by the method of moments from the mean \bar{r}_s and variance $\sigma_{r_y}^2$ estimated from empirical data.

Appendix 2—Autocorrelated process errors

This appendix describes two algorithms for generating autocorrelated production process errors from gamma distributions. The algorithms are based on first-order autoregressive and moving average error structures used in time-series analysis (Nelson, 1973). The shapes of original uncorrelated gamma distributions and new, correlated probability distributions for both algorithms appeared identical in plots.

Algorithm 1 was for autocorrelations (ρ) in the range zero to 0.5:

$$r_{s,y} = d_{s,y} + \theta d_{s,y-1}, \tag{1}$$

where $r_{s,y}$ was the logistic population growth parameter for year y in simulation run s , and the moving

average parameter (θ) was

$$\theta = -1 + \frac{\sqrt{1 - 4\rho^2}}{2\rho}, \tag{2}$$

with θ in the range (0,1). The random numbers $d_{s,y}$ in algorithm 1 were drawn from a gamma distribution with parameters:

$$\ddot{\alpha}_s = \tilde{\alpha}_s \frac{(1 + \theta^2)}{(1 + \theta)^2} \tag{3}$$

$$\ddot{\beta}_s = \tilde{\beta}_s \frac{(1 + \theta)}{(1 + \theta^2)} \tag{4}$$

and $\gamma = -M$. The adjusted parameter values ($\ddot{\alpha}_s, \ddot{\beta}_s, \gamma$) make the mean and variance of autocorrelated $r_{s,y}$ values from algorithm 1 the same as for an independent series drawn from a gamma distribution with parameters $(\tilde{\alpha}_s, \tilde{\beta}_s, \gamma)$ (see Appendix 1).

Algorithm 2 was for autocorrelations (ρ) in the range (0.5, 1):

$$r_{s,y} = \frac{\sum_{j=1}^L d_{s,y-1-j}}{L}, \tag{5}$$

where the random numbers $d_{s,y}$ were drawn from gamma distributions with parameters $\ddot{\alpha}_{s,r} = \tilde{\alpha}_{s,r} / L, L$ an integer $> 3, \ddot{\beta}_{s,r} = \tilde{\beta}_{s,r} L,$ and γ . The adjusted parameter values make the mean and variance of the autocorrelated and independent $r_{s,y}$ values the same.

The autocorrelation in algorithm 2 is $\rho = (L-1) / L$. For simulations, we chose the smallest value of L that gave an autocorrelation that was at least as large as the value desired. Extremely high autocorrelations (e.g. $\rho > 0.9$) can give $\ddot{\alpha}_{s,y} = \tilde{\alpha}_{s,y} / L \leq 1$ when $\tilde{\alpha}_{s,r}$ is near L . This was a minor problem in some cases because gamma distributions for $d_{s,y}$, and autocorrelated values of $r_{s,y}$, have no mode when $\ddot{\alpha}_{s,y} \leq 1$. We avoided this, where necessary, by setting the maximum value of L to 10 ($\rho = 0.90$) and constraining $\ddot{\alpha}_{s,y} \leq 1$ in simulations.