Estimation of weight-length relationships from group measurements

William H. Lenarz
Tiburon Fisheries Laboratory
National Marine Fisheries Service, NOAA
3150 Paradise Drive, Tiburon, CA 94920

Catch sampling provides data that are basic to fisheries research and is often an important component of research budgets. Samplers typically select fish randomly, measure length, remove ageing structures, and determine sex for each individual. In many schemes for sampling commercial (e.g., Sen, 1986; Tomlinson, 1971) and survey catches (e.g., Gunderson and Sample, 1980), sample weight is needed to expand the sample results to the total catch. Individual weights are usually not needed to satisfy the main objectives. Often only the aggregate weight of the sample is taken to save time, and if at sea, to avoid difficult logistics. While sampling costs are easily justified by program objectives, scientists frequently use the data for additional research.

Investigators often use weight-length relations to study possible correlations between condition of fish and environmental factors or population density (e.g., Patterson, 1992). A literature search revealed only two previous developments of methods of estimating weight-length relations from samples of individual lengths and aggregate weights (WLRAW). Cammen (1980) used a general nonlinear regression program from the BMDP package (Dixon, 1983) as a WLRAW method. He tested the method with simulated data and compared the results of regression using unweighted observations to using observations weighted by the inverse of sample weights, and with various estimates made when individual weights were known. Since the data were simulated, assuming a multiplicative error term, it would have been more appropriate to use the inverse of sample weight squared for weighting. The nonlinear method produced good fits to the simulated data, and weighted parameter estimates were closer to the true values than unweighted estimates. Damm (1987) developed two nonlinear WLRAW methods. One method is a biased approximation, and his report indicated that the other method did not always produce estimates of the parameters.

In this note I describe a new WLRAW method, compare it with Cammen's method, explore error term characteristics, and describe bootstrap estimates of confidence limits of estimates. The methods of Damm (1987) were not studied because his biased approximation method requires as much calculation as my new method and his other method does not always work.

Methods

The relation between expected weight and length of an individual fish is usually assumed to be the power equation,

$$E(W_i) = \alpha L_i^{\beta} \tag{1}$$

Where W_i = weight of fish i,
α = parameter,
L_i = length of fish i,
β = parameter.

For the new WLRAW method I modeled the weight-length relationship as

$$\overline{W}_j = \frac{\alpha}{n_j}\sum_{i=1}^{n_j}\left(L_{ij}^{\beta}\right) + \varepsilon_j, \tag{2}$$

where $\overline{W}_j$ = average weight of fish in sample j,
n_j = number of fish in sample j,
L_{ij} = length of fish i in sample j,
ε_j = error term for sample j,
j = 1, . . . , T,
T = number of samples.

I assumed that error was additive because under field conditions much of the error was due to limits to the accuracy in measurement of sample weights. Because the dependent variable in Equation 2 was a sample average, its variance should contain a component which is proportional to the inverse of n_j. Thus in the new estimation procedure, I weight each observation by n_j to stabilize the variance. I made the assumption that, after weighting by sample size, error was random and independent of j.

The new method treated estimation of parameters of (Eq. 2) as a separable least-squares problem (Seber and Wild, 1989). For a trial value of β (β'), γ_j was calculated for each sample,

$$\gamma_j = \left(\sum_{1}^{n_j} L_{ij}^{\beta'}\right)/n_j. \tag{3}$$

With the new notation, Equation 2 becomes

$$\overline{W}_j = \alpha'\gamma_j + \varepsilon_j. \tag{4}$$

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I then obtained an estimate of α (α') corresponding to β' by using the standard least squares linear regression with zero intercept method. I used a nonlinear least squares procedure to obtain the estimate of β ($\hat{\beta}$). This procedure was analogous to finding the transformation, $L_{ij}^{\hat{\beta}}$, that minimized the sum of squares about the linear regression (Eq. 4). Using this procedure, I estimated brackets for ensuring that the searching range included $\hat{\beta}$ with the procedure MNBRAK (Press et al., 1989). Then I used the iterative procedure BRENT (Press et al., 1989) to obtain the final estimate. BRENT uses parabolic interpolation to minimize the sum of squares as a function of β'. Convergence is assumed when the procedure does not change the value of β' more than a tolerance specified by the user. As previously stated, observations were weighted by n_j to stabilize the variance. I implemented the WLRAW method in double precision using Sun FORTRAN for a Sun SPARC2 work station.

Bootstrap approximations of confidence intervals about the line were calculated for the new method. The literature contains a variety of bootstrap methods proposed to approximate confidence intervals (e.g., DiCiccio et al., 1992). I used the nonparametric BC_a method of Efron (1987) because it often produces good results and is relatively easy to use.

BC_a stands for accelerated bias corrected bootstrap confidence intervals. Efron (1987) showed that, in the parametric case, the method is approximately correct if a transformation to a normally distributed variable exists. The transformation does not need to be known and the variance does not need to be constant. While the correctness of the BC_a has not been mathematically proven for nonparametric cases, such as the WLRAW, Efron (1987) stated, "...empirical results look promising." The BC_a confidence limits of an estimate of parameter θ, $\hat{\theta}$, are

$$IBS(N(z[\alpha])) \le \theta \le IBS(N(z[1-\alpha])). \qquad (5)$$

$IBS(P)$ is the value of θ that corresponds to the percentile P of the cumulative bootstrap frequency distribution. $N(Z)$ is the percentile of the cumulative normal probability distribution that corresponds to the standard deviate Z. $z[\alpha]$ is given by Efron (1987) as

$$z[\alpha] = z_0 + \frac{z_0 + z^{(\alpha)}}{1 - a\left(z_0 + z^{(\alpha)}\right)}. \qquad (6)$$

$z^{(\alpha)}$ is the standard deviate that corresponds to the α percentile of the normal cumulative distribution. z_0 is the standard deviate of the normal cumulative distribution that corresponds to the percentile that corresponds to $\hat{\theta}$ in the cumulative bootstrap frequency distribution. Efron (1987) called z_0 the *bias constant*. Efron called a the *acceleration constant*. It is related to the skewness of the bootstrap frequency distribution. Efron gave the following approximation for a:

$$a \approx \frac{1}{6}\left[\frac{(\sum_{j=1}^{T} U_j^3)}{(\sum_{j=1}^{T} U_j^2)^{\frac{3}{2}}}\right], \qquad (7)$$

where $U_j = \dfrac{\hat{\theta}_j^{(\Delta)} - \hat{\theta}}{\Delta}$

$\hat{\theta}^{(\Delta)}$ = estimate of θ when jth sample has a very small amount of extra weighting (Δ).

If a and z_0 are zero, then Equation 7 becomes the percentile method that is the most frequently used bootstrap method in the fisheries literature (e.g., Sigler and Fujioka, 1988).

I chose to approximate 90% confidence bands rather than 95% or 99% bands because 90% nonparametric bootstrap intervals tend to perform better than intervals that cover a wider portion of the distribution (Efron, 1988). Following the advice of Efron, I used 1,000 bootstrap replicates.

Cammen (1980) used the general nonlinear regression program of BMDP to estimate the parameters of Equation 2, except that he assumed that the error term is multiplicative and used total sample weight instead of average weight as the dependent variable. The BMDP program uses the Gauss-Newton algorithm. I used the same algorithm in the nonlinear regression program of the SAS package (SAS Institute Inc., 1989) on a Sun SPARC2 to compare parameter estimates and execution times with the new method. Since the correct error model is not known, I also estimated the parameters using no weighting and weight set to $1/\overline{W}_j$, $1/\overline{W}_j^2$, $n_j/\overline{W}_j$, and $n_j/\overline{W}_j^2$, and compared asymptotic standard errors of the parameter estimates. The new estimation procedure is simpler than the Gauss-Newton approach because it searches for the least squares by iteratively changing the value of one parameter instead of two.

I used data collected on chilipepper rockfish (*Sebastes goodei*) by a cooperative landing sampling program of the California Department of Fish and

Game and National Marine Fisheries Service to examine utility of the WLRAW method. Samplers collected two groups of fish from each sampled landing. For each group a container that holds 22.7 kg of fish was filled with fish regardless of species. Then the sampler obtained total group weights to the nearest lb (0.45 kg) for each species and the total length of each fish was measured to the nearest mm. I converted weights to kg. I changed lengths to decimeters to minimize potential scaling problems in the computations. Before using the WLRAW method, I combined groups within a landing because they may not be independent.

I first used data for all months during 1991 from all ports between Morro Bay and Crescent City, California, to develop, test, and time the software. Results of the test runs are described briefly in the Results and Discussion section. More detailed results are presented for a more typical application of the method. Investigators are more likely interested in results from a smaller number of samples taken from more restrictive scales of time and area than from data sets like the one used in the preceding example. I used data for chilipepper rockfish taken during July and August 1991 from Morro Bay to illustrate use of the method.

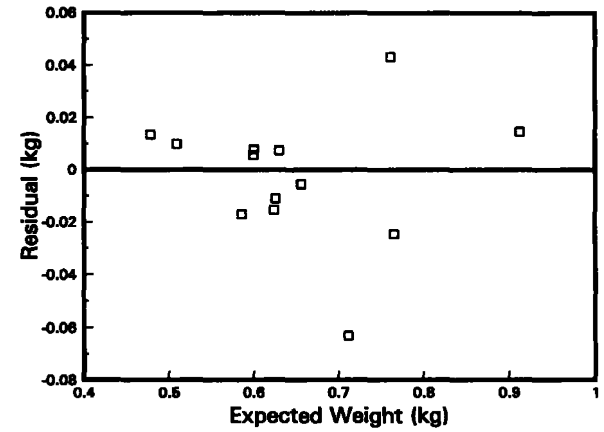

Figure 1

Residual of average weight (kg) as a function of expected weight (kg) for chilipepper rockfish (*Sebastes goodei*) collected in samples taken from Morro Bay during July and August 1991.

Results and discussion

The data from all ports consisted of measurements from 7,687 fish taken in 186 samples. The procedure required 1.6 seconds, compared with 18.8 seconds for the Gauss-Newton method. The Gauss-Newton and new methods produced parameter estimates that were identical to six decimal places. Predicted weights were very close to the results of Phillips (1964), who used data from individually measured fish. Sums of squares plotted against β' indicated that there were no local minima. Residuals were not related to weight, indicating that the additive error assumption is correct. Sometimes transformation of β' to $\ln(\beta')$ when estimating parameters of power equations avoids problems due to curvature (Ratkowsky, 1983). Transformation was tried and parameter estimates were identical to the results when β' was not transformed. When β' was transformed, the procedure required more time to complete, so the transformation was not used.

Data were available for 583 fish taken from 13 samples taken in Morro Bay, during July and August 1991. There were no strong trends between the residual and expected weight (Fig. 1). There was a tendency for absolute values of residuals to be negatively correlated with the number of fish in a sample (Fig. 2A). The tendency was reduced when residuals were multiplied by $\sqrt{n_j}$, as expected under the assumption that variance is proportional to the inverse of sample size (Fig. 2B). Also, n_j produced the lowest asymptotic standard errors of the parameter estimates of the six weighting factors explored (Table 1). The results shown in Table 1 and Figures 1 and 2 indicated that the additive error model with weighting by n_j was appropriate for these data. Bootstrap estimates of standard error using the new method were higher than asymptotic estimates using the Gauss-Newton method. The bootstrap and asymptotic normal confidence intervals were narrow and similar within the range of most observed average weights but diverged when expected weight was greater than 0.75 kg even though individual fish of larger size occurred in many of the samples (Table 2). The bootstrap confidence intervals were skewed at the larger sizes. However, the bootstrap estimates of absolute bias were less than 0.01 kg except they were –0.01 kg for 450-mm fish and –0.02 kg for 500-mm fish. All estimates of the absolute value of a were about 0.015, which indicated that a could have been ignored for this set of data.

The new WLRAW method performed well. Good fits to the data were obtained and the residuals agreed with the assumptions. Approximate confidence limits indicated that precise estimates of expected weight are obtained with a small number of samples under field conditions for sizes of fish within the range of most observed average weights. The method is fast when used on a work station or on a modern personal computer. The new method is 10 times faster than using the Gauss-Newton ap-

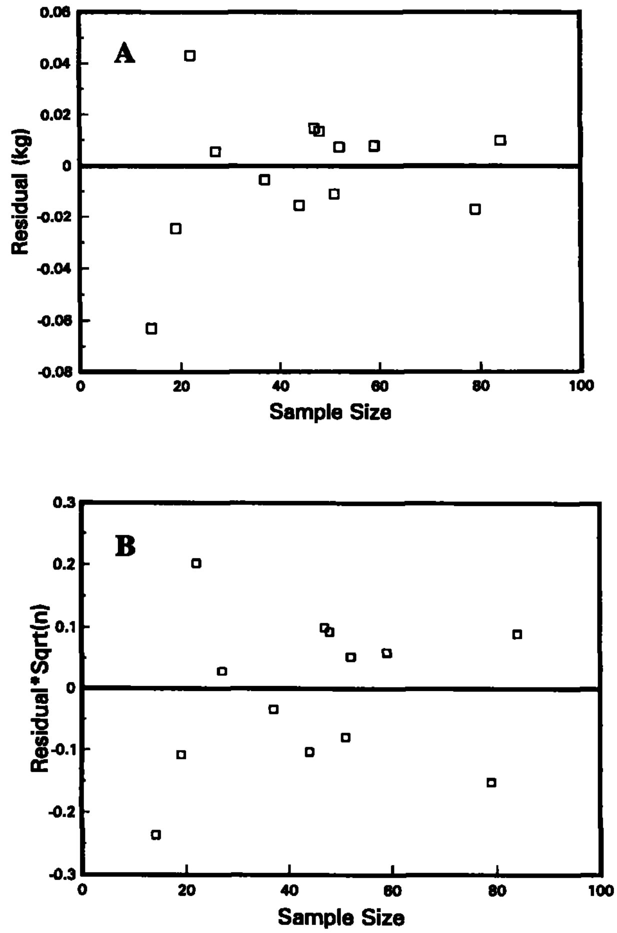

Figure 2

(**A**) Residual of average weight (kg) as a function of sample size for chilipepper rockfish (*Sebastes goodei*) collected in samples taken from Morro Bay during July and August 1991. (**B**) Residual multiplied by $\sqrt{n_j}$ as a function of sample size.

proach with a standard statistical package. Some of the difference is probably due to the overhead involved with using the statistical package. When computationally intensive methods such as bootstrapping are used, time saved by using the new method is significant.

The widening confidence limits for expected weights beyond the range of most observed average weights indicated use of expected weights beyond the observed range is extrapolation and should not be done. This also applies to comparison of parameter estimates from different sets of data. If the range of observed average weights differ much among the data sets, comparison of parameter estimates is not meaningful. Estimates of the two parameters of the weight-length relation are highly correlated even when individuals are weighed and standard linear regression is used (Lenarz, 1974). Thus, regardless of the type of data or statistical

Table 1

Estimates of standard errors of parameter estimates of weight-length model for chilipepper rockfish (*Sebasted goodei*) collected from Morro Bay during July and August 1991. The Gauss-Newton method was used with observations weighted by six factors to estimate the parameters, and the new method with n_j as the weighting factor. Asymptotic standard errors are shown for the Gauss-Newton method and bootstrap standard errors for the new method. Coefficients of variation of the parameter estimates are shown in parentheses.

Weighting factor	Standard error α	Standard error β
Gauss-Newton method		
none	0.0028 (0.30)	0.2159 (0.07)
n_j	0.0019 (0.21)	0.1489 (0.05)
$n_j/\overline{W}_j$	0.0020 (0.20)	0.1528 (0.05)
$n_j/\overline{W}_j^2$	0.0022 (0.20)	0.1547 (0.05)
$1/\overline{W}_j$	0.0030 (0.29)	0.2129 (0.07)
$1/\overline{W}_j^2$	0.0032 (0.28)	0.2069 (0.07)
New method		
n_j	0.0046 (0.50)	0.2211 (0.07)

Table 2

Expected weights for chilipepper rockfish (*Sebastes goodei*) collected from Morro Bay during July and August 1991, and 90% confidence about the line. Confidence limits were approximated using the bootstrap BC_a (bootstrap) and the asymptotic normal methods (normal). Expected weights were calculated from the estimated weight-length relation (0.0091819 Length$^{3.1758673}$).

Total length (dm)	Expected weight (kg)	Confidence limits: Normal Lower (kg)	Normal Upper (kg)	Bootstrap Lower (kg)	Bootstrap Upper (kg)
3.00	0.30	0.28	0.32	0.28	0.33
3.50	0.49	0.47	0.51	0.47	0.50
3.75	0.61	0.60	0.62	0.60	0.62
4.00	0.75	0.74	0.76	0.73	0.76
4.50	1.09	1.04	1.14	0.99	1.12
5.00	1.52	1.42	1.63	1.31	1.61

procedure, I recommend comparison of weight-length relations among data sets by comparison of expected weights of fish at sizes within the range of observed average weights common to all data sets of interest.

The results of this study suggest that an additive error term is more appropriate than a multiplicative error term for modeling weight-length relations. Most previous studies have assumed multiplicative error, which is implied when the log-log transformation is used to estimate parameters of the model from individually measured fish by linear regression. The multiplicative error assumption has not been demonstrated correct even when data are available from fish weighed individually. While good fits to data are usually obtained under the multiplicative assumption, if the assumption is not valid, statistical inferences may be erroneous. Pienaar and Thomson (1969) assumed that the error term was additive for their data and discussed statistical aspects of the assumption. Further examination of the error term form would be interesting.

Copies of the FORTRAN code used in this study are available from the author.

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