A comparison of alternative methods for the estimation of age from length data for Atlantic coast bluefish (*Pomatomus saltatrix*)

Mark Terceiro
National Marine Fisheries Service
Northeast Fisheries Science Center
Woods Hole, MA 02543

Jeffrey L. Ross
North Carolina Department of Environment,
Health, and Natural Resources
Division of Marine Fisheries
Morehead City, NC 28557

Abstract.–Bluefish (*Pomatomus saltatrix*) are found along the US Atlantic coast from Maine to Florida, and are the object of a major recreational fishery as they migrate northward from the South Atlantic Bight in the spring and return southward in the late fall. Acceptance of analytic assessment results for bluefish has been hindered by the lack of a consistent time series of geographically comprehensive age-length keys for converting bluefish lengths to age. We used bluefish length-age data from North Carolina commercial fisheries during 1986 to 1989 to compare the utility of two simple methods for estimating age from length-frequency data with two more rigorous statistical methods. The simple methods were cohort slicing using von Bertalanffy growth parameters for bluefish and the application of a pooled age-length key compiled with data from different times and fishing areas. The two statistical approaches were the iterated age-length key method and MULTIFAN. Combined 1986–1989 proportions at age estimated by cohort slicing, pooled age-length key, and iterated age-length key methods were significantly different from those in the test data, based on a quantitative comparison using the Kolmogorov-Smirnov cumulative distribution test. Proportions at age estimated by MULTIFAN were not significantly different from the test data. Our work suggests that MULTIFAN is the best alternative to a time series of fishery-specific age-length keys for the estimation of Atlantic coast bluefish ages from length data.

Manuscript accepted 16 April 1993.
Fishery Bulletin 91:534–549 (1993).

Bluefish (*Pomatomus saltatrix*) are found along the U.S. Atlantic coast from Maine to Florida, migrating northward from the South Atlantic Bight in the spring and returning southward in the late fall. They are the target of a major recreational fishery along the Atlantic coast, with catches averaging 47,000 metric tons (t) per year during 1980 to 1990. For the same period, the commercial catch of bluefish, mainly by otter trawls, averaged 6,400 t per year.

Atlantic coast bluefish exhibit fast growth during the first two years of life, attaining fork lengths of over 40 cm by age 2 (Hamer, 1959; Lassiter, 1962; Richards, 1976; Wilk, 1977). They may reach ages of up to 12 years and sizes in excess of 100 cm fork length and 14 kg in weight. About fifty percent of bluefish reach sexual maturity by the second year of life (Wilk, 1977). Spawning occurs during two major periods: March and April in the South Atlantic Bight near the inner edge of the Gulf Stream, with a peak about 1 April; and June through September in the Mid-Atlantic Bight, with a peak about 1 August (Wilk, 1977; Kendall and Walford, 1979; Nyman and Conover, 1988). There is evidence that spring and summer spawning fish mix extensively during their lifespan, as summer spawning fish have been observed to originate from both cohorts (Chiarella and Conover, 1990). Year classes of bluefish therefore consist of varying proportions of spring- and summer-spawned cohorts.

In anticipation of a fisheries management plan to regulate commercial and recreational catches of Atlantic coast bluefish, the Atlantic States Marine Fisheries Commission (ASMFC) and the Mid-Atlantic Fisheries Management Council (MAFMC), in cooperation with the National Marine Fisheries Service (NMFS), began work on stock assessment in 1986. Length-frequency data collected by the Northeast Fisheries Science Center (NEFSC) (bottom trawls, 1976–1986) and by NMFS Marine Recreational Fishery Statistics Survey (MRFSS) (1979–1985) were available for analytical assessment (i.e., virtual population analysis). However, no consistent time series of geographically comprehensive age-length keys were available for estimating age-frequency from length-frequency data. Therefore, an age-length key (hereafter referred to as the ASMFC pooled key) was developed by pooling data collected by the fisheries agencies of several Atlantic coast states during 1982 to 1986 (Crecco et al.[1]; Terceiro[2]).

There were concerns about the potential for biased results in using the ASMFC pooled key, because of the influence of interannual variation in growth, recruitment, or mortality rate, given the broad temporal and geographic scale over which the age-length data were collected (Westrheim and Ricker, 1978). As a result, no consensus was reached on the utility of the ASMFC pooled key, catch-at-age data, or subsequent analyses presented in Crecco et al.[1] and Terceiro[2].

Following implementation of the bluefish Fisheries Management Plan in 1990, stock assessment work for bluefish was renewed by ASMFC. A yield-per-recruit analysis was developed to provide biological reference points for the new assessment. Parameters of the von Bertalanffy growth function $L_t = L_{\text{inf}}[1 - e^{-K(t-t_0)}]$), derived from weighted mean calculated lengths at annulus formation presented in NOAA (1989), were used as an expedient alternative to the ASMFC pooled key to age MRFSS-sample bluefish length-frequency data by cohort slicing (i.e., solving for t in the von Bertalanffy growth equation, given L_{inf}, L_t, K, and t_0). Length at age, weight at age, and partial recruitment vectors for the yield-per-recruit analysis were developed from this version of the MRFSS length-age data.

Published estimates of mean calculated bluefish length at annulus formation exhibit considerable variation (Barger, 1990; Chiarella and Conover, 1990; Hamer, 1959; Lassiter, 1962; NOAA, 1989; Richards, 1976; Wilk, 1977) (Table 1). The variation is likely due to 1) variation in growth over time, 2) sampling and availability bias, 3) varying influence of Lee's phenomenon (Jones, 1958), 4) use of different conventions in fixing the assumed birthdate of bluefish, and 5) differential proportions of spring and spawned fish in the length frequency samples collected by different investigators (Wilk, 1977; Kendall and Walford, 1979; Chiarella and Conover, 1990). Variation in mean lengths at age and subsequently derived growth parameters, likely inherent when data from several disparate sources are considered, could lead to biased results from cohort slicing. The Eleventh NEFSC SAW[3] recommended testing alternative methods, such as mixture of distributions methods, and data sources in lieu of a time series of age-length keys for the estimation of bluefish ages from length-frequency data.

This work compares results from the application of 1) the cohort slicing method using growth parameters, 2) a pooled age-length key applied in the standard manner, 3) the iterated age-length key method (IALK) as presented by Kimura and Chikuni (1987) and Hoenig and Heisey (1987), and 4) MULTIFAN, a mixture of distributions method (Fournier et al., 1990), to bluefish length-age data collected by the North Carolina Division of Marine Fisheries (NCDMF) from 1986 to 1989. Our objective was to determine how these methods interpret the known-age test data, and suggest whether any are an acceptable alternative to the resource-intensive method of developing fisheries specific age-length keys.

Methods

Data sources

North Carolina bluefish length-age test data Bluefish are landed in the commercial long haul seine (April–October), pound net (May–October), and winter otter trawl (October–May) fisheries in the estuarine and coastal waters of North Carolina. The NCDMF has collected biological sample data from these commercial fisheries for bluefish since 1982. This time series is the largest and most consistent body of bluefish length-age data available. NCDMF bluefish length-age data collected from spring 1986 to winter 1989 (n = 1469, 13–84 cm, ages 0 to 9, ages determined from scale annuli), aggregated by calendar year, were used as a test data set for comparing the performance of alternative methods for estimating bluefish ages from length data (Table 2, Figure 1).

Investigators have not agreed on a birthdate convention for Atlantic coast bluefish. NCDMF fisheries biologists employ an unweighted average date of 1 June, between peak spawning of the spring (1 April) and summer (1 August) aggregations. This date roughly corresponds with the time of annulus formation during late May to mid-June.

Bluefish recruit to North Carolina fisheries during the summer fishing season. Bluefish with fork lengths ranging from 15 to 32 cm that are captured before 1 June and possess scales that do not yet exhibit the first annulus are typically classified as age 0. Inspection of modes in length-frequency data and the timing of spawning suggests that the largest of these bluefish originate from the spring cohort of the previous calendar year, while the smallest are fish from the summer

---

[1] Crecco, V., M. Terceiro, and C. Moore. 1987. A stock assessment of Atlantic coast bluefish, *Pomatomus saltatrix*. Special report prepared for the Atlantic States Marine Fisheries Commission, Washington, D.C.

[2] Terceiro, M. 1987. Status of Atlantic coast bluefish—1987. U.S. Dep. Commer. NOAA, Natl. Mar. Fish. Serv., Northeast Fish. Cent., Woods Hole lab. Ref. Doc. 87-10, 54 p.

[3] NEFSC. 1990. Report of the Eleventh NEFSC Stock Assessment Workshop, Fall 1990; Woods Hole, MA. U.S. Dep. Commer., NOAA, Natl. Mar. Fish. Serv., Northeast Fish. Sci. Cent. Ref. Doc. 90-09, 121 p.

Table 1

Weighted mean calculated fork lengths (cm) at annulus formation of bluefish (*Pomatomus saltatrix*) from various studies for fish from the U.S. Atlantic coast.

		Age (yr)											
Source	*N*	1	2	3	4	5	6	7	8	9	10	11	12
Hamer 1959	?	31.8	38.1	41.9	50.6	55.0	63.2	70.0		72.6			
Lassiter 1962													
spring spawned	290	26.6	37.6	48.4	53.5	61.2	68.2	74.1	79.2				
summer spawned	154	14.8	30.3	41.9	48.4								
Richards 1976	64	23.0	40.0	49.0	58.0	64.0	69.0	71.0					
Wilk 1977	7,425	20.8	33.6	44.4	55.1	63.1	67.3	71.9					
NOAA 1989	429	22.6	38.1	50.2	59.8	67.3	73.1	77.8	81.4	84.2	86.5		
Barger 1990	588	29.0	36.1	41.5	47.3								
Chiarella and Conover 1990	147	26.5	43.5	53.2	60.7	65.9	69.3	72.0	75.4	79.3	83.8	85.8	87.0

Table 2

North Carolina Division of Marine Fisheries 1986–1989 bluefish (*Pomatomus saltatrix*) length-age test data, n=1469.

	Numbers at age									
Length (cm)	0	1	2	3	4	5	6	7	8	9
14	1									
15	2									
16	11									
17	4	6								
18	15	5								
19	18	9								
20	10	8								
21	9	11								
22	11	5	1							
23	10	6								
24	18	6								
25	28	10								
26	38	14								
27	39	21								
28	54	10	1							
29	29	21	1							
30	21	21	1							
31	13	17	1							
32	17	28	2							
33	4	37	4							
34	3	32	3							
35	1	52	3							
36		43	4							

Table 2 (Continued)

	Numbers at age									
Length (cm)	0	1	2	3	4	5	6	7	8	9
37	1	35	2							
38		36	8							
39		45	1							
40		29	7							
41		24	8							
42		13	9							
43		8	3							
44		1	8							
45		6	4							
46		4	6							
47		1	7							
48		2	12							
49		1	7							
50			10							
51			11							
52			5							
53			2	2						
54			5	2						
55			2	3						
56				5						
57			2	9						
58			1	5						
59			1	6	1					
60				7	3	1				
61				2	5	1				
62			2	7	11					
63				5	9	2				
64				2	5	1				
65				3	12	4				
66				1	11	11				
67				1	9	16	2			
68					5	9	2			
69					6	11	10			
70					3	21	8			
71						16	14	1		
72						10	13	4		1
73						5	8	6		
74							6	8	1	
75						1	7	6	1	
76							3	5	3	
77						1	2	4	2	
78							3	3		1
79							1	1	2	3
80								1	4	1
81							1	1	2	
82									5	1
83									1	1
84								1		1
Age totals	357	567	144	60	80	110	80	41	21	9
Mean length (cm)	25.7	33.7	44.7	59.4	64.8	68.9	72.2	75.2	79.2	79.6
SD	3.6	6.5	3.7	1.1	1.0	1.2	1.1	0.7	0.5	0.4
CV (%)	14.0	19.3	8.3	1.9	1.5	1.7	1.5	0.9	0.6	0.5

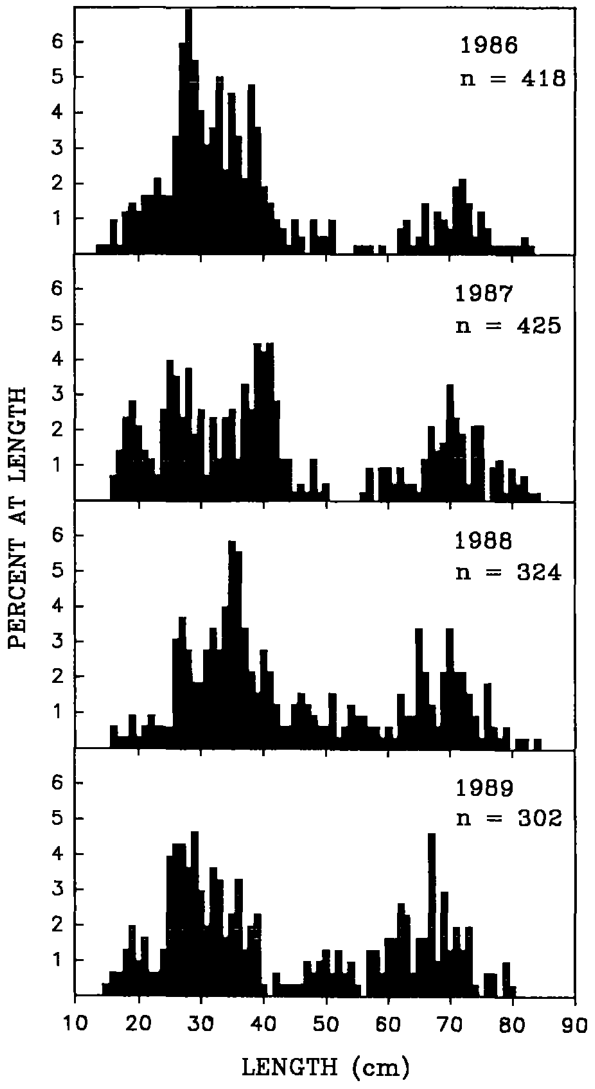

Figure 1
North Carolina Division of Marine Fisheries (NCDMF) 1986–1989 bluefish (*Pomatomus saltatrix*) length-age test data: percent at length distributions by year.

cohort of the previous calendar year. Depending on the exact time of spawning and sampling, the largest age-0 fish may be 12–15 months old, while the smaller age-0 fish are about 10 months old. Under the birthdate convention of 1 January used by NEFSC fisheries biologists, all of these fish would be classified as age class 1. This difference in classification schemes has caused confusion in previous stock assessments and other research for bluefish. The NCDMF length-age test data in this paper are based on the 1 June birthdate convention.

Connecticut, New Hampshire, North Carolina (CNN) length-age data Length-age sample data (n = 3883, 12–87 cm, ages 0 to 11, ages determined from scale annuli) were available for bluefish collected by state agencies in Connecticut (1984–1985, n = 1452), New Hampshire (1986, n = 76), and North Carolina (1982–1985, n = 2355) (Fig. 2). The 1 June birthdate convention was employed in aging. Connecticut and New Hampshire fish were collected in research surveys, mainly during the summer and fall when most recreational fishing occurs (June–October). North Carolina fish were taken throughout the year in proportion to the number of fish landed by commercial fisheries. These data were combined to form a matrix of the distribution of age at length for bluefish. This matrix was applied as a standard age-length key (CNN ALK) to the NCMDF 1986–1989 annual length distributions

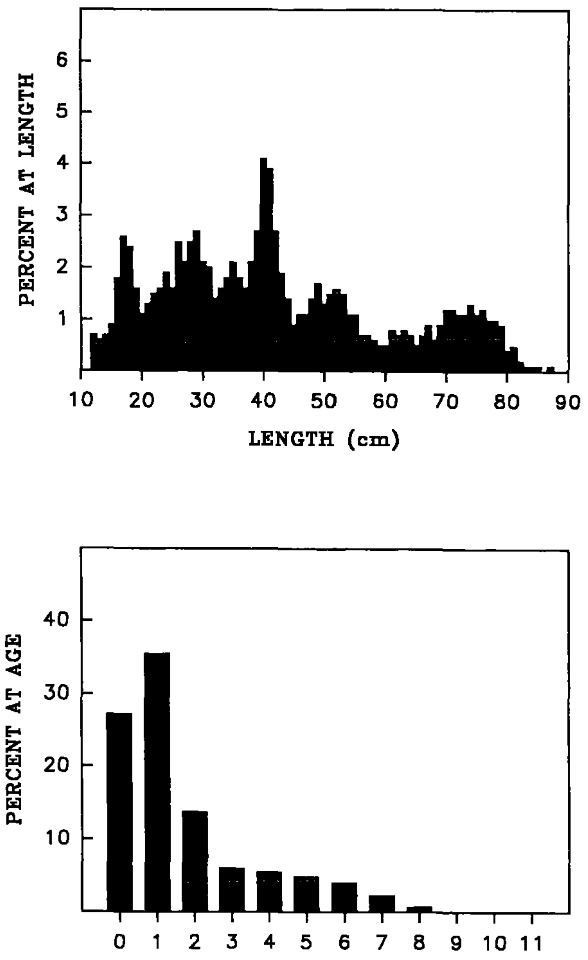

Figure 2
Connecticut, New Hampshire, and North Carolina length-age data (n=3,883) for bluefish (*Pomotomus saltatrix*) used as the basis for the Connecticut, New Hampshire, and North Carolina age-length key (CNN ALK) and the interated age-length key (IALK) method.

to estimate proportions at age for comparison with the NCDMF ages. It is important to note that the North Carolina part of the CNN length-age data and the NCDMF 1986–1989 length-age data were compiled by the same age-reader and therefore were not completely independent.

NOAA (1989) length-age data Bluefish length-age data collected during NEFSC autumn bottom trawl research surveys, 1985–1987, formed the basis for weighted, mean-calculated lengths at annulus formation presented in NOAA (1989). Samples were aged according to the 1 January birthdate convention. These data provided the von Bertalanffy growth parameters of L_{inf} = 94.6 cm, K = 0.242, and t_0 = –0.128, which were used in cohort slicing for estimation of bluefish ages from length data in the revised yield-per-recruit analysis for bluefish[3].

The NOAA (1989) value of t_0 was adjusted to reflect the difference between the 1 January (used by NEFSC) and 1 June (used by NCDMF) birthdate conventions. The difference of 151 days, or 0.414 years, was added to the ages of fish in the raw data used to estimate the parameters. Refitting the data provided a new value of t_0 = –0.542 (e.g., a 25-cm bluefish sampled on 1 April would typically be classified as age 0 by NCDMF staff; a 25-cm bluefish aged by cohort slicing with NOAA 1989 parameters and t_0 = –0.128 would be 1.140 years old, and classified as age 1; while a 25-cm bluefish aged by cohort slicing with NOAA 1989 parameters and t_0 = –0.542 would be 0.726 years old and classified as age 0). The NOAA 1989 growth parameters with the adjusted value of t_0 were used in cohort slicing of the NCDMF 1986–1989 annual length-frequency data for comparison with the NCDMF 1986–1989 annual and combined age distributions.

Statistical methods

Iterated age-length key The iterated age-length key (IALK) method was introduced by Kimura and Chikuni (1987). The method combines the standard age-length key (Kimura, 1977; Westrheim and Ricker, 1978) and mixture of distributions approaches (Hasselblad, 1966; Macdonald and Pitcher, 1979; Schnute and Fournier, 1980) for the resolution of lengths to ages. Application of the standard age-length key method without bias requires that the length-age data and length data have the same underlying age composition (Kimura, 1977). The IALK method has been presented as a potential solution to the problem of applying age-length keys to length-frequency distributions with different time and space characteristics, and therefore potentially different underlying age distributions (Kimura and Chikuni, 1987; Hoenig and Heisey, 1987).

Assumptions for the IALK method are somewhat less stringent than for standard application of the age-length key. The method requires only that the length-age data be adequately sampled and that the associated probabilities of length at age are applicable to the observed length distribution to be aged, although the potential for bias increases if the two data sets have very different temporal and geographic characteristics. The goal of the iterative procedure is to modify the length distribution of the length-age data so that it more closely approximates the length-frequency data to be aged, until the underlying age distributions of the length-age data and length-frequency data also become similar, thus satisfying the assumptions of the standard age-length key concept.

Kimura and Chikuni (1987) presented the IALK method in the form of an algorithm, gave conditions for which the algorithm converged to a unique solution, and showed the algorithm was an application of the expectation-maximization (EM) algorithm (Dempster et al., 1977) to mixtures of distributions. Hoenig and Heisey (1987) subsequently used a similar approach for the IALK as an EM algorithm, while making different assumptions about the sampling errors of the data. The steps of the IALK algorithm as implemented by Kimura and Chikuni (1987) are the following:

1) estimate the proportions at age, with all ages initially having equal probability. Hoenig and Heisey (1987) suggested that the initial estimate of proportions at age might be set the same as those in the length-age data;

2) using observed probabilities of length at age calculated from the length-age sample data and the current estimate of proportions at age, calculate the corresponding age-length key (probabilities of age at length). An intermediate result of this step is the IALK estimate of the observed length distribution;

3) apply the observed length distribution to be aged to the calculated age-length key of step 2;

4) calculate the maximum absolute deviation over ages between the newly estimated proportions at age of the observed length frequency and the proportions at age in the previous iteration. If the deviation is less than some small constant, then stop; otherwise continue with the next iteration of the age-length key.

We used the CNN length-age data (Fig. 2) as the source for the probabilities of length at age used in the calculation of the IALK applied to the observed NCDMF 1986–1989 length distributions (Fig. 1). Initial proportions at age were set to the same values as the CNN length-age data, as suggested by Hoenig and Heisey (1987), for ages 0 to 11:

[0.272 0.355 0.138 0.060 0.055 0.048 0.040
0.023 0.007 0.001 0.0001 0.0009].

The IALK algorithm was judged to have converged when the maximum absolute deviation summed over iterated proportions at age was less than 0.001 (0.1%). The IALK method was applied to the annual NCDMF 1986–1989 length distributions separately.

MULTIFAN MULTIFAN was introduced by Fournier et al. (1990) as an extension of the methods of Schnute and Fournier (1980) and Fournier and Breen (1983). MULTIFAN is a likelihood-based method using the mixture of distributions approach for the classification of lengths to age, with consideration of biological constraints, to simultaneously analyze several length-frequency distributions sampled at different times. The major assumptions of the method include the following: 1) the lengths of the animals in each age class are normally distributed about the mean length at age; 2) growth follows the von Bertalanffy function; and 3) standard deviation of lengths about the mean length at age varies as a simple function of mean length at age.

The log-likelihood function used in MULTIFAN compares the expected probability that a fish chosen randomly will lie in a given length interval with the observed number of fish in that interval for the set of growth parameters being tested. The formal statistical basis of MULTIFAN allows for a structured and relatively objective means of evaluating alternative interpretations of the processes producing the observed length- frequency distributions (e.g., the growth rate and resulting number of significant age classes, gear selectivity on younger age classes, and evidence of a length dependent trend in the standard deviation of length at age). The basic set of parameters of the MULTIFAN model include the following: 1) the proportions at age; 2) the mean length of the first age group; 3) the mean length of the last age group; 4) the von Bertalanffy growth parameter *K*; 5) two parameters which determine the standard deviation of length at age; 6) a parameter determining the extent of selectivity bias on the first age group; and 7) a parameter determining the overall variance of the sampling errors in the length-frequency data sets (see Fournier et al. 1990 for detailed notation of the MULTIFAN model parameterization).

For example, in the case of two suspected age classes underlying a set of five length-frequency distributions, the model would have a total of nine parameters. The addition of one age class to the model (one age, five distributions) would increase the number of parameters by five, to 14. Accounting for a length dependent trend in standard deviation of length at age (LDSD) in the three age-class case would increase the number of parameters by one, to 15; accounting for selectivity bias on the first age group would again increase the number of parameters estimated by one, to 16. A chi-square test is used to determine the best fitting growth structure. The statistical significance of increases in the log-likelihood values obtained by the addition of age classes are determined by the 0.90 point of the chi-square random variable, to reduce the probability of rejecting an additional age class when it is actually present in the data. The significance of the addition of other parameters is determined at the more conventional 0.95 level (Fournier et al., 1990).

After an initial inspection of modes apparent in the length-frequency data, initial constraints for MULTIFAN included reasonable ranges for 1) the number of expected age classes, 2) the corresponding values for the von Bertalanffy parameter *K*, and 3) a range for the mean length at age of an obvious mode. Initial constraints for the NCDMF 1986–1989 bluefish length-frequency data were potential significant age classes from 5 to 12, *K* range from 0.20 to 0.40, and mean length range for presumed age-0 fish in the 1986, 1987, and 1989 samples set at 15–30 cm, and for age 1 in 1988 at 30–40 cm. Preliminary MULTIFAN runs determined the number of significant age classes underlying the NCDMF 1986–1989 sample length frequencies. Additional runs with MULTIFAN were made to test for evidence of selectivity bias (SEL) on the first age class and for a length dependent trend in standard deviation of length at age (LDSD) as means of improving the fits.

Comparison of methods

A qualitative assessment of the utility of each method was made by inspection of the estimated mean lengths at age determined by the different alternatives. The result of ultimate interest from the application of these methods to the NCDMF length frequency distributions, however, is the number of fish per age class. We assessed the performance of the alternatives by using the Kolmogorov-Smirnov (K–S) cumulative distribution test (Sokal and Rohlf, 1981) to compare the calculated annual and combined 1986–1989 proportions at age from each aging approach with the NCDMF annual and combined 1986–1989 proportions at age.

This non-parametric method tests for differences in both the shape and location of two frequency distributions under the null hypothesis that the samples are taken from populations with the same underlying distribution. The proportions at age being compared are considered as two samples from the commercial landings. If the maximum unsigned difference between cumulative frequency distributions (proportions at age) exceeds some critical value (*D*) for a given confidence level and sample size, then the null hypothesis is rejected (Sokal and Rohlf, 1981). The test provides an indication of how closely the alternative methods can match the interpretation of the age structure commercial landings of bluefish in North Carolina that would be provided by the NCDMF 1986–1989 length-age test data. By extension, the utility of these methods as alternatives to a time series of geographically appropriate age-length keys can be evaluated on a quantitative scale.

Results

Application of the alternative methods to the NCDMF 1986–1989 length data provided estimates of mean length at age that were lower than the NCDMF value for age-0 fish, and generally higher for ages 1 and older. Mean lengths at age estimated by MULTIFAN for ages 3 to 5 were the exception, as they were lower than the NCDMF values (Table 3).

For the annual and combined NCDMF 1986–1989 length distributions, cohort slicing using the NOAA (1989) growth parameters consistently provided the poorest match with the NCDMF proportions at age (Tables 4–8, pages 543–547); there were large differences relative to the NCMDF data for ages 0, 4, and 5. Differences were significant ($P > 0.05$) in 1988 and for the combined distribution.

Standard application of the CNN ALK provided improved results over simple cohort slicing, and estimated age proportions were not significantly different from the true NCDMF proportions, except when data were combined. Estimated proportions at age for fish of less than 40 cm were very close to the NCDMF proportions. The largest value of D always occurred at age 4 or 5, reflecting problems in resolving fish in the 60–75 cm length interval to the correct age when using the standard age-length key application of the CNN length-age data (Tables 4–8).

The IALK method provided initial results that were surprisingly poor. The convergence criteria proposed by Kimura and Chikuni (1987) for IALK estimated proportions at age were intended to stop iteration of the key when the underlying age distributions of the length-age data and the length frequency to be aged closely matched, thus better satisfying the conditions necessary for unbiased application of the key.

Two characteristics of bluefish biology, multiple spawned cohorts for the same year class and differential availability to fisheries by age class, appear to have combined to result in enough difference in the length distribution patterns of the CNN and NCDMF data to significantly hinder the effectiveness of the IALK model when the final iteration of the key was used. Iteration of the annual CNN length-age distributions reduced the number of bluefish in the 50–60 cm interval, and increased the number of bluefish in the 20–40 cm (ages 0 and 1) and 65–75 cm (ages 4 and 5) length intervals. The resulting length and age distributions of the input CNN length-age data matched the NCDMF lengths more closely than the initial distributions. However, when the final iterated key was applied to NCDMF lengths, the cumulative proportions at age generally provided large values of D for ages 0 and 1, or ages 4 and 5 (Table 9, page 548).

Intermediate iterations of the CNN length-age data (generally the first or second iteration) provided a better match to the NCDMF proportions at age than the final iteration (generally about the 15th iteration). In effect, we limited the degree to which the CNN length-age distribution could be changed by evaluating the application of each iteration of the CNN ALK, and selecting as the best solution the estimated proportions at age providing the smallest cumulative difference from the NCDMF cumulative proportions. Under this procedure of evaluation, the IALK provided the same or slightly improved results relative to standard application of the CNN length-age data. Only for the combined 1986–1989 length distributions were the IALK estimated proportions at age significantly different from the NCDMF proportions at the 5% level (Tables 4–8).

MULTIFAN results were inconsistent when considered on an annual basis. The growth parameters selected by MULTIFAN as best describing the growth pattern of bluefish during 1986–1989 (Table 10, page 549) performed relatively poorly in matching the NCDMF 1986 and 1987 proportions at age, but provided a close match to the NCDMF 1988 and 1989 proportions (Tables 4–8). Variation in growth and recruitment to the sampled fishery among cohorts contributed to varying success in matching the annual NCDMF proportions with the MULTIFAN method.

The best fitting MULTIFAN run provided the only non-significant value of D (at age 6, the "plus group" of the MULTIFAN model interpretation) for the combined 1986–1989 length distributions obtained among the compared methods. The largest values of D provided by the other methods occurred at ages 4 or 5, indicating problems in resolving the ages of fish 60 cm and larger in accordance with the NCDMF interpretation. Only MULTIFAN provided proportions at ages 4 and 5 that accurately reflected the NCMDF proportions in the combined distributions (Table 8). The improved performance of MULTIFAN when the NCDMF data are evaluated in combined fashion reflects better adherence to the underlying concept of the MULTIFAN model (Fournier and Breen, 1983).

Table 3

Combined mean fork lengths (cm) at age from application of alternative methods for conversion of North Carolina Division of Marine Fisheries (NCDMF) 1986–1989 bluefish (*Pomatomus saltatrix*) length-frequency data to ages, compared with NCDMF values.

	Mean fork length at age (cm)											
Method	0	1	2	3	4	5	6	7	8	9	10	11
CNN ALK	25.0	34.6	47.9	60.7	67.3	71.3	74.9	77.8	81.1	82.2	—	—
NOAA 1989 $t_0 = -0.542$	24.6	35.9	48.6	59.4	66.5	72.1	77.4	81.1	83.6	—	—	—
IALK	25.1	34.7	47.8	60.7	67.3	71.3	74.9	77.7	80.2	82.4	83.1	82.0
MULTIFAN	24.9	34.5	45.5	54.5	62.0	68.2	73.3	—	—	—	—	—
NCDMF 1986–1989	25.7	33.7	44.7	59.4	64.8	68.9	72.2	75.2	79.2	79.6	—	—

Discussion

Cohort slicing, standard application of the CNN ALK, the IALK method, and MULTIFAN are increasingly complex solutions to the problem of estimating ages from length data when appropriate age-length keys are not available. All four methods provided reasonable interpretations of the NCDMF bluefish data by accurately reflecting the dominance of age-0 and age-1 fish in the length distributions. Cohort slicing using the NOAA (1989) parameters generally provided the poorest match with the NCDMF proportions at age, because the method fails to account for the variability in length at age owing to multiple spawning events and the variation in growth among cohorts.

The CNN ALK and IALK provided satisfactory resolution of bluefish lengths to age, nearly matching those estimated by the best MULTIFAN fit for the combined NCDMF 1986–1989 data. Performance of the CNN ALK and IALK was poorest for those bluefish ages with a high degree of overlap in the NCDMF length distributions (usually ages 4 and 5) because of the inability of these methods to accurately portray the variation in growth among cohorts. However, the CNN ALK and IALK provided the most accurate estimates of mean length at age for very large bluefish (age 7 and older) which often may not be sufficiently abundant in length-frequency sample distributions to form significant modes.

We note that with the IALK method, the length-age data would usually be applied to a length distribution of unknown age composition, in which case the alternative method we used for selecting the best IALK solution would not be an option. Kimura and Chikuni (1987) suggested the IALK method should reduce bias in the resulting age distribution if an age-length key is applied to length-frequency data with potentially different underlying growth or selectivity patterns. Our work confirmed that the IALK algorithm achieves the goal of reducing the differences in underlying age distribution of the length-age data used as a key and the length frequency to be aged. However, when these differences are so large that even the less restrictive assumptions of the IALK method are violated, the IALK method provides little improvement over standard application of the age-length key.

MULTIFAN performed well in resolving the underlying age structure of bluefish length frequency samples, given the reasonable number (e.g., through age 5 or 6) of age groups that were apparent in the NCDMF 1986–1989 length samples. MULTIFAN was the only method to correctly estimate the relative proportions at ages 4 and 5, by accurately reflecting the variation in length at age of fish from the length frequency mode at 65–75 cm.

Key attributes of the MULTIFAN model which contributed to good performance in this exercise with bluefish data include 1) the large number of parameters estimated by the model, and consideration of variation of mean length at age and selectivity bias on the first age class, 2) use of the entire NCDMF length data series to estimate the growth pattern, and 3) the hypothesis testing approach to model evaluation. Based on qualitative evaluation of the estimates of mean lengths at age and the quantitative comparison of estimated proportions at age with the Kolmogorov-Smirnov cumulative distribution test, we believe that MULTIFAN was the best current alternative to a time series of fishery-specific age-length keys for the estimation of bluefish ages from lengths data.

The MULTIFAN model is a good starting point for future method development. MULTIFAN assumes

Table 4

North Carolina Division of Marine Fisheries (NCDMF) 1986 bluefish (*Pomatomus saltatrix*) proportions at age (*P*) and cumulative proportions at age (*C*) compared with estimates derived by cohort slicing using NOAA (1989) parameters, by standard application of the Connecticut/New Hampshire/North Carolina age-length key (CNN ALK), the CNN age-length key applied using the iterated age-length key method (IALK), and by MULTIFAN. Maximum difference in cumulative proportions at age (*D*) in underlined bold print. $D_{0.05,n=418}$=0.094.

Age	NCDMF *P*	NCDMF *C*	NOAA 1989 *P*	NOAA 1989 *C*	NOAA 1989 *D*	CNN ALK *P*	CNN ALK *C*	CNN ALK *D*	IALK *P*	IALK *C*	IALK *D*	MULTIFAN *P*	MULTIFAN *C*	MULTIFAN *D*
0	0.323	0.323	0.361	0.361	**0.038**	0.313	0.313	0.010	0.318	0.318	0.005	0.258	0.258	**0.065**
1	0.428	0.751	0.416	0.778	0.026	0.459	0.773	0.022	0.455	0.773	0.022	0.490	0.749	0.002
2	0.072	0.823	0.045	0.823	0.000	0.053	0.825	0.002	0.053	0.825	0.002	0.065	0.813	0.010
3	0.012	0.835	0.017	0.840	0.005	0.026	0.852	0.017	0.026	0.852	0.017	0.007	0.821	0.014
4	0.036	0.871	0.055	0.895	0.024	0.053	0.904	**0.033**	0.053	0.904	**0.033**	0.005	0.825	0.045
5	0.065	0.935	0.079	0.974	0.038	0.053	0.957	0.022	0.053	0.957	0.022	0.067	0.892	0.043
6	0.038	0.974	0.014	0.988	0.014	0.024	0.981	0.007	0.024	0.981	0.007	0.108	1.000	0.026
7	0.007	0.981	0.010	0.998	0.017	0.012	0.993	0.012	0.012	0.993	0.012	0.000	1.000	0.019
8	0.014	0.995	0.002	1.000	0.005	0.005	0.998	0.002	0.005	0.998	0.002	0.000	1.000	0.005
9	0.000	1.000	0.000	1.000	0.000	0.002	1.000	0.000	0.000	0.998	0.002	0.000	1.000	0.000
10	0.000	1.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	0.998	0.002	0.000	1.000	0.000
11	0.000	1.000	0.000	1.000	0.000	0.000	1.000	0.000	0.002	1.000	0.000	0.000	1.000	0.000

Table 5

North Carolina Division of Marine Fisheries (NCDMF) 1987 bluefish (*Pomatomus saltatrix*) proportions at age (P) and cumulative proportions at age (C) compared with estimates derived by cohort slicing using NOAA (1989) parameters, by standard application of the Connecticut/New Hampshire/North Carolina age-length key (CNN ALK), the CNN age-length key applied using the iterated age-length key method (IALK), and by MULTIFAN. Maximum difference in cumulative proportions at age (D) in underlined bold print. $D_{0.05,n=425}$=0.093.

	NCDMF	NCDMF	NOAA	NOAA	NOAA	CNN	CNN	CNN	IALK	IALK	IALK	MULTIFAN	MULTIFAN	MULTIFAN
			1989	1989	1989	ALK	ALK	ALK						
Age	P	C	P	C	D	P	C	D	P	C	D	P	C	D
0	0.209	0.209	0.308	0.308	**0.099**	0.261	0.261	0.052	0.268	0.268	0.059	0.388	0.388	**0.179**
1	0.419	0.628	0.360	0.668	0.040	0.402	0.664	0.035	0.395	0.664	0.035	0.214	0.602	0.026
2	0.082	0.711	0.040	0.708	0.002	0.052	0.715	0.005	0.052	0.715	0.005	0.056	0.659	0.052
3	0.031	0.741	0.045	0.753	0.012	0.040	0.755	0.014	0.040	0.755	0.014	0.007	0.666	0.075
4	0.042	0.784	0.075	0.828	0.045	0.092	0.847	0.064	0.092	0.847	0.064	0.052	0.718	0.066
5	0.073	0.856	0.122	0.951	0.094	0.075	0.922	**0.066**	0.075	0.922	**0.066**	0.045	0.762	0.094
6	0.078	0.934	0.024	0.974	0.040	0.042	0.965	0.031	0.042	0.965	0.031	0.238	1.000	0.066
7	0.038	0.972	0.021	0.995	0.024	0.024	0.988	0.016	0.024	0.988	0.016	0.000	1.000	0.028
8	0.019	0.991	0.005	1.000	0.009	0.007	0.995	0.005	0.007	0.995	0.005	0.000	1.000	0.009
9	0.009	1.000	0.000	1.000	0.000	0.005	1.000	0.000	0.002	0.998	0.002	0.000	1.000	0.000
10	0.000	1.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	0.998	0.002	0.000	1.000	0.000
11	0.000	1.000	0.000	1.000	0.000	0.000	1.000	0.000	0.002	1.000	0.000	0.000	1.000	0.000

Table 6

North Carolina Division of Marine Fisheries (NCDMF) 1988 bluefish (*Pomatomus saltatrix*) proportions at age (*P*) and cumulative proportions at age (*C*) compared with estimates derived by cohort slicing using NOAA (1989) parameters, by standard application of the Connecticut/New Hampshire/North Carolina age-length key (CNN ALK), the CNN age-length key applied using the iterated age-length key method (IALK), and by MULTIFAN. Maximum difference in cumulative proportions at age (*D*) in underlined bold print. $D_{0.05, n=324}$=0.107.

	NCDMF	NCDMF	NOAA 1989	NOAA 1989	NOAA 1989	CNN ALK	CNN ALK	CNN ALK	IALK	IALK	IALK	MULTIFAN	MULTIFAN	MULTIFAN
Age	*P*	*C*	*P*	*C*	*D*	*P*	*C*	*D*	*P*	*C*	*D*	*P*	*C*	*D*
0	0.182	0.182	0.173	0.173	0.009	0.160	0.160	0.022	0.170	0.170	0.012	0.117	0.117	**0.065**
1	0.346	0.528	0.401	0.574	0.046	0.420	0.580	0.052	0.414	0.583	0.056	0.451	0.568	0.040
2	0.151	0.679	0.105	0.679	0.000	0.102	0.682	0.003	0.102	0.685	0.006	0.096	0.664	0.015
3	0.062	0.741	0.059	0.738	0.003	0.074	0.756	0.015	0.074	0.759	0.019	0.065	0.728	0.012
4	0.071	0.812	0.114	0.852	0.040	0.105	0.861	0.049	0.105	0.864	0.052	0.022	0.750	0.062
5	0.059	0.870	0.105	0.957	**0.086**	0.077	0.938	**0.068**	0.074	0.938	**0.068**	0.123	0.873	0.003
6	0.065	0.935	0.034	0.991	0.056	0.034	0.972	0.037	0.034	0.972	0.037	0.127	1.000	0.065
7	0.049	0.985	0.006	0.997	0.012	0.015	0.988	0.003	0.015	0.988	0.003	0.000	1.000	0.015
8	0.009	0.994	0.003	1.000	0.006	0.003	0.991	0.003	0.006	0.994	0.000	0.000	1.000	0.006
9	0.006	1.000	0.000	1.000	0.000	0.009	1.000	0.000	0.003	0.997	0.003	0.000	1.000	0.000
10	0.000	1.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	0.997	0.003	0.000	1.000	0.000
11	0.000	1.000	0.000	1.000	0.000	0.000	1.000	0.000	0.003	1.000	0.000	0.000	1.000	0.000

Table 7

North Carolina Division of Marine Fisheries (NCDMF) 1989 bluefish (*Pomatomus saltatrix*) proportions at age (*P*) and cumulative proportions at age (*C*) compared with estimates derived by cohort slicing using NOAA (1989) parameters, by standard application of the Connecticut/New Hampshire/North Carolina age-length key (CNN ALK), the CNN age-length key applied using the iterated age-length key method (IALK), and by MULTIFAN. Maximum difference in cumulative proportions at age (*D*) in underlined bold print. $D_{0.05, n=302}$=0.111.

	NCDMF	NCDMF	NOAA	NOAA	NOAA	CNN	CNN	CNN	IALK	IALK	IALK	MULTIFAN	MULTIFAN	MULTIFAN
			1989	1989	1989	ALK	ALK	ALK						
Age	*P*	*C*	*P*	*C*	*D*	*P*	*C*	*D*	*P*	*C*	*D*	*P*	*C*	*D*
0	0.245	0.245	0.311	0.311	0.066	0.265	0.265	0.020	0.268	0.268	0.023	0.325	0.325	**0.079**
1	0.325	0.570	0.262	0.573	0.003	0.311	0.576	0.007	0.305	0.573	0.003	0.272	0.596	0.026
2	0.099	0.669	0.086	0.659	0.010	0.086	0.662	0.007	0.086	0.659	0.010	0.036	0.632	0.036
3	0.073	0.742	0.096	0.755	0.013	0.109	0.772	0.030	0.109	0.768	0.026	0.076	0.709	0.033
4	0.079	0.821	0.149	0.904	**0.083**	0.126	0.897	**0.076**	0.126	0.894	**0.073**	0.103	0.811	0.010
5	0.109	0.930	0.070	0.974	0.043	0.066	0.964	0.033	0.066	0.960	0.030	0.116	0.927	0.003
6	0.033	0.964	0.023	0.997	0.033	0.023	0.987	0.023	0.023	0.983	0.020	0.073	1.000	0.036
7	0.020	0.983	0.003	1.000	0.017	0.010	0.997	0.013	0.010	0.993	0.010	0.000	1.000	0.017
8	0.013	0.997	0.000	1.000	0.003	0.003	1.000	0.003	0.007	1.000	0.003	0.000	1.000	0.003
9	0.003	1.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000
10	0.000	1.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000
11	0.000	1.000	0.000	1.000	0.000	0.000	1.000	0.000	0.002	1.000	0.000	0.000	1.000	0.000

Table 8

North Carolina Division of Marine Fisheries (NCDMF) 1986–1989 bluefish (*Pomatomus saltatrix*) proportions at age (*P*) and cumulative proportions at age (*C*) compared with estimates derived by cohort slicing using NOAA (1989) parameters, by standard application of the Connecticut/New Hampshire/North Carolina age-length key (CNN ALK), the CNN age-length key applied using the iterated age-length key method (IALK), and by MULTIFAN. Maximum difference in cumulative proportions at age (*D*) in underlined bold print. $D_{0.05, n=1469}$=0.050.

Age	NCDMF *P*	NCDMF *C*	NOAA 1989 *P*	NOAA 1989 *C*	NOAA 1989 *D*	CNN ALK *P*	CNN ALK *C*	CNN ALK *D*	IALK *P*	IALK *C*	IALK *D*	MULTIFAN *P*	MULTIFAN *C*	MULTIFAN *D*
0	0.243	0.243	0.294	0.294	0.051	0.255	0.255	0.012	0.261	0.261	0.018	0.278	0.278	0.035
1	0.386	0.629	0.365	0.659	0.030	0.404	0.658	0.029	0.398	0.658	0.029	0.357	0.635	0.006
2	0.098	0.727	0.065	0.724	0.003	0.070	0.728	0.001	0.070	0.728	0.001	0.063	0.698	0.029
3	0.041	0.768	0.050	0.775	0.007	0.058	0.786	0.018	0.058	0.786	0.018	0.034	0.732	0.035
4	0.054	0.822	0.093	0.868	0.046	0.091	0.877	**0.054**	0.091	0.877	**0.054**	0.042	0.775	**0.048**
5	0.075	0.897	0.095	0.963	**0.066**	0.067	0.944	0.047	0.067	0.943	0.046	0.083	0.858	0.039
6	0.054	0.952	0.023	0.986	0.035	0.031	0.975	0.024	0.031	0.975	0.023	0.142	1.000	0.048
7	0.028	0.980	0.011	0.997	0.018	0.016	0.991	0.012	0.016	0.990	0.011	0.000	1.000	0.020
8	0.014	0.994	0.003	1.000	0.006	0.005	0.996	0.002	0.006	0.997	0.003	0.000	1.000	0.006
9	0.006	1.000	0.000	1.000	0.000	0.004	1.000	0.000	0.001	0.998	0.002	0.000	1.000	0.000
10	0.000	1.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	0.998	0.002	0.000	1.000	0.000
11	0.000	1.000	0.000	1.000	0.000	0.000	1.000	0.000	0.002	1.000	0.000	0.000	1.000	0.000

Table 9

North Carolina Division of Marine Fisheries (NCDMF) 1986–1989 bluefish (*Pomatomus saltatrix*) annual proportions at age (*P*) compared with estimates derived by initial application of CNN age-length key using the iterated age-length key method (IALK). Cumulative proportions are omitted for brevity. Maximum difference in cumulative proportions at age (*D*) in underlined bold print. 1986 $D_{0.05,n=418}$=0.094, 1987 $D_{0.05,n=425}$=0.093, 1988 $D_{0.05,n=324}$=0.107, 1989 $D_{0.05,n=302}$=0.111.

	1986			1987			1988			1989		
Age	NCDMF *P*	IALK *P*	IALK *D*	NCDMF *P*	IALK *P*	IALK *D*	NCMDF *P*	IALK *P*	IALK *D*	NCMDF *P*	IALK *P*	IALK *D*
0	0.323	0.289	0.033	0.209	0.245	0.035	0.182	0.082	**0.099**	0.245	0.291	0.046
1	0.428	0.507	**0.045**	0.419	0.447	0.064	0.346	0.524	0.080	0.325	0.295	0.017
2	0.072	0.029	0.002	0.082	0.014	0.005	0.151	0.061	0.009	0.099	0.040	0.043
3	0.012	0.012	0.002	0.031	0.021	0.014	0.062	0.077	0.006	0.073	0.132	0.017
4	0.036	0.067	0.033	0.042	0.134	**0.078**	0.071	0.136	0.071	0.079	0.172	**0.109**
5	0.065	0.074	0.043	0.073	0.073	0.078	0.059	0.086	0.099	0.109	0.046	0.046
6	0.038	0.007	0.012	0.078	0.031	0.031	0.065	0.021	0.056	0.033	0.003	0.017
7	0.007	0.002	0.007	0.038	0.016	0.009	0.049	0.006	0.012	0.020	0.000	0.003
8	0.014	0.010	0.002	0.019	0.009	0.000	0.009	0.002	0.003	0.013	0.020	0.003
9	0.000	0.001	0.002	0.009	0.005	0.005	0.006	0.001	0.003	0.003	0.000	0.000
10	0.000	0.000	0.002	0.000	0.000	0.005	0.000	0.004	0.000	0.000	0.000	0.000
11	0.000	0.002	0.000	0.000	0.005	0.000	0.000	0.002	0.000	0.000	0.000	0.000

Table 10

Matrix of MULTIFAN inverse log-likelihood function values for alternative interpretations of the number of significant age classes underlying the North Carolina Division of Marine Fisheries 1986–1989 bluefish (*Pomatomus saltatrix*) length-frequency test data. LISD = length independent standard deviation in length at age, LDSD = length dependent standard deviation in length at age, NOSEL = no selectivity bias on the first age group, SEL = selectivity bias on the first age group. Values in parentheses are the number of parameters in model being tested (e.g., proportions at age, mean lengths at age, *K*, LDSD, or SEL). Bold underlined value is combination of characteristics selected for final evaluation as the best fitting model, based on the significance of increases in value with added parameters.

Ages	LISD+NOSEL	LISD+SEL	LDSD+NOSEL	LDSD+SEL
6	4171.7 (24)	4171.9 (25)	4172.4 (25)	4174.2 (26)
7	4169.9 (28)	**4180.7** (29)	4175.6 (29)	4178.4 (30)
8	4160.4 (32)	4170.0 (33)	4167.1 (33)	4168.9 (34)
9	4158.9 (36)	4169.7 (37)	4172.7 (37)	4185.8 (38)
10	4161.9 (40)	4175.7 (41)	4168.5 (41)	4187.0 (42)
11	4160.5 (44)	4171.7 (45)	4169.9 (45)	4188.1 (46)
12	4160.7 (48)	4170.3 (49)	4169.6 (49)	4187.9 (50)
13	4161.1 (52)	4170.0 (53)	4169.1 (53)	4187.8 (54)

growth follows the von Bertalanffy function. Other growth functions such as the logistic or Gomphertz may be better models of growth for the youngest age classes of fish, or for species that experience periods of rapid growth at intermediate ages followed by slow growth as they approach asymptotic sizes (Moreau, 1987; Terceiro[4]).

The normal distribution of length at age assumed in MULTIFAN is appropriate for many fish species (Fournier and Breen, 1983; Fournier et al., 1990; Terceiro et al., 1992). Other probability distributions might better describe the variation in length at age for species with special life-history characteristics. Species characterized by multiple spawning events or extreme variation in growth among cohorts due to density dependence often exhibit log-normal (long-tailed) distributions of length at age. Consideration of alternative probability distributions would be a useful addition to the MULTIFAN model. For species subject to a high degree of size selectivity within age, or those experiencing very slow growth at the oldest ages, the inclusion of parameters that account for departures from normality (skewness and kurtosis) could provide a MULTIFAN model that better estimates age from asymmetric distributions of length at age.

---

[4] Terceiro, M. 1991. Length composition analysis of Atlantic sea scallop using the MULTIFAN method. *In* Report of the Twelfth Northeast Regional Stock Assessment Workshop (12th SAW), Spring 1991, Appendix 1, p. 1–29. U.S. Dep. Commer., NOAA, Natl. Mar. Fish. Serv., Northeast Fish. Sci. Cent., Woods Hole Lab. Ref. Doc. 91-03, 187 p.

Acknowledgments

We appreciate the efforts of an anonymous reviewer, whose critical comments greatly improved the manuscript.

Literature cited

Barger, L. E.
1990. Age and growth of bluefish *Pomatomus saltatrix* from the northern Gulf of Mexico and U.S. South Atlantic coast. Fish. Bull. 88:805–809.

Chiarella, L. A., and D. O. Conover.
1990. A Spawning season and first-year growth of adult bluefish from the New York Bight. Trans. Am. Fish. Soc. 119:455–462.

Dempster, A. P., N. M. Laird, and D. B. Rubin.
1977. Maximum likelihood via the EM algorithm. J. Royal Stat. Soc. Ser. B: Methodological 39:1–22.

Fournier, D. A., and P. A. Breen.
1983. Estimation of abalone mortality rates with growth analysis. Trans. Am. Fish. Soc. 112:403–411.

Fournier, D. A., J. R. Sibert, J. Majkowski, and J. Hampton.
1990. MULTIFAN a likelihood-based method for estimating growth parameters and age composition from multiple length frequency data sets illustrated using data for southern bluefin tuna (*Thunnus maccoyii*). Can. J. Fish. Aquat. Sci. 47:301–317.

Hamer, P. E.
1959. Age and growth studies of bluefish (*Pomatomus saltatrix*) of the New York Bight. M.S. thesis, Rutgers Univ., New Brunswick, NJ.

Hasselblad, V.
1966. Estimation of parameters for a mixture of normal distributions. Technometrics 8:431–444.

Hoenig, J. M., and D. M. Heisey.
1987. Use of a Log-linear model with the EM algorithm to correct estimates of stock composition and to convert length to age. Trans. Am. Fish. Soc. 116:232–243.

Jones, R.
1958. Lee's phenomenon of "apparent changes in growth-rate" with particular reference to haddock and plaice. *In* International Commission for the Northwest Atlantic Fisheries, Special Publ. 1. Halifax, Canada.

Kendall, A. W. Jr., and L. A. Walford.
1979. Sources and distribution of bluefish, *Pomatomus saltatrix*, larvae and juveniles off the east coast of the United States. Fish. Bull. 77:213–277.

Kimura, D. K.
1977. Statistical assessment of the age-length key. J. Fish. Res. Board. Can. 34:317–324.

Kimura, D. K., and S. Chikuni.
1987. Mixtures of empirical distributions: an iterative application of the age-length key. Biometrics 43:23–35.

Lassiter, R. R.
1962. Life history aspects of the bluefish, *Pomatomus saltatrix* Linnaeus, from the coast of North Carolina. M.S. thesis, North Carolina State College, Raleigh, NC.

Macdonald, P. D. M., and T. J. Pitcher.
1979. Age groups from size frequency data: a versatile and efficient method of analyzing distribution mixtures. J. Fish. Res. Board. Can. 36:987–1001.

Moreau, J.
1987. Mathematical and biological expression of growth in fishes: recent trends and further developments. *In* R. C. Summerfelt and G. E. Hall (eds.), Age and growth of fish, p. 81–113. Iowa State Univ. Press. Ames, IA.

NOAA.
1989. Guidelines for estimating lengths at age for 18 northwest Atlantic finfish and shellfish species. NOAA Tech. Mem. NMFS-F/NEC-66, 39 p.

Nyman, R. M., and D. O. Conover.
1988. The relation between spawning season and the recruitment of young-of-the-year bluefish, *Pomatomus saltatrix*, to New York. Fish. Bull. 86:237–250.

Richards, S. W.
1976. Age, growth, and food of the bluefish (*Pomatomus saltatrix*) from east-central Long Island Sound from July through November 1975. Trans. Am. Fish. Soc. 105:523–525.

Schnute, J., and D. A. Fournier.
1980. A new approach to length frequency analysis: growth structure. J. Fish. Res. Board. Can.37:1337–1351.

Sokal, R. R., and F. J. Rohlf.
1981. Biometry (2nd edition). W. H. Freeman and Company, NY.

Terceiro, M., D. A. Fournier, and J. R. Sibert.
1992. Comparative performance of MULTIFAN and Shepherd's Length Composition Analysis (SRLCA) on simulated length-frequency distributions. Trans. Am. Fish. Soc. 121:667–677.

Westrheim, S. J., and W. E. Ricker.
1978. Bias in using an age-length key to estimate age-frequency distributions. J. Fish. Res. Board. Can. 35:184–185.

Wilk, S. J.
1977. Biological and fisheries data on bluefish, *Pomatomus saltatrix* (Linnaeus). NMFS, NEFC, Sandy Hook Lab. Tech. Ser. Rep. 11, Highlands, NJ, 56 p.