SPECIFYING A FUNCTIONAL FORM FOR THE INFLUENCE OF HATCHERY SMOLT RELEASE ON ADULT SALMON PRODUCTION

BIING-HWAN LIN[1] AND NANCY A. WILLIAMS[2]

ABSTRACT

The hypothesis of density independent marine survival of salmon has been tested extensively with conflicting results. Unduly restrictive functional form and data deficiency have been suggested as the major contributing factors to the mixed results. Focusing on the issue of functional form selection, this paper utilizes the extended Box-Cox flexible functional form which allows the data to determine the statistical relationship between smolts and adult production without a priori restrictions. The model is applied to Hokkaido chum salmon, *Oncorhynchus keta*, and Oregon coho salmon, *O. kisutch*. Empirical results suggest the existence of density dependence for both Hokkaido chum and Oregon coho salmon. Further, an increasing variability of adult production with respect to an increase in smolts is found for Hokkaido chum salmon but not for Oregon coho salmon.

Two issues pertaining to the relationship between the number of hatchery smolts released and the number of adult salmon returned have been investigated recently in the literature. First, the hypothesis of density independence in the relationship between salmon adults and smolts has been tested. The null hypothesis is a linear relationship between adults returned and smolts released, such that the additional adult salmon produced from an increase in smolts released is constant. The second issue is the relationship between the variability of adult production and the number of smolts released. If, in fact, increases in smolts increase the variability of adults produced, fishery management strategies can be improved by considering the trade-off between the mean and variance of adult salmon returned (Walters 1975; McCarl and Rettig 1983).

The empirical results of the test of density independence have been mixed. Nickelson (1986) provided an excellent discussion of previous results pertaining to the test of density independence for Oregon coho salmon. In short, this hypothesis for marine survival of Oregon coho salmon is rejected by McCarl and Rettig (1983) and Peterman and Routledge (1983), but accepted by Peterman (1981), Clark and McCarl (1983), and Nickelson (1986). In addition, biologists in the Oregon Department of Fish and Wildlife have examined several model specifications and manipulations in data sets and have drawn conflicting conclusions about density independence. This led McCarl and Rettig to suggest that conflicting conclusions are caused by the use of different functional form specifications and to suggest further that resolution of the issue of density independence in Oregon coho salmon requires more refined data. In the case of Hokkaido chum salmon, the null hypothesis of density independence fails to be rejected by McCarl and Rettig.

Regarding the estimation of the variability of adult salmon production, Peterman (1981) pointed out the importance of functional form specification and argued for the use of the multiplicative-error model rather than an additive-error model. McCarl and Rettig (1983) demonstrated that the specification of a multiplicative-error model imposes unwarranted restrictions on the estimation of the variability in adult production. McCarl and Rettig utilized the specification developed by Just and Pope (1978, 1979). As a result, the variability in adult salmon production is estimated and conflicting conclusions of the test of density independence emerged.

It is apparent that functional form specification is critical in the test of density independence and the estimation of variability in adult production. The purpose of this paper is to reexamine these two issues by using the extended Box-Cox flexible functional form. Both Hokkaido chum salmon, *Oncorhynchus keta*, and Oregon coho salmon, *O. kisutch*, data are used in this study.

The next section of the paper discusses the importance of the functional form specification and demonstrates the superior flexibility of the Box-Cox

[1]Department of Agricultural Economics and Rural Sociology, University of Idaho, Moscow, ID 83843.
[2]Department of Economics, Loyola College, 4501 North Charles St., Baltimore, MD 21210-2699.

Manuscript accepted July 1988.
FISHERY BULLETIN: VOL. 86, NO. 4, 1988.

functional form compared with the Just-Pope specification used by McCarl and Rettig (1983). The empirical results of the Box-Cox functional form are then discussed and compared with those of McCarl and Rettig.

METHODS

Previous findings of Peterman (1978, 1981) indicate the importance of the assumption made regarding error term in testing the hypothesis of density independence. In the process of examining the effect of the number of released smolts on the production of adults and its variability, Peterman (1981) employed two alternative (additive-error and multiplicative-error) model specifications of the error term:

$$A_1 = C_1 S^{k_1} + V_1 \quad \ldots\text{Additive-error model} \quad (1)$$
$$A_2 = C_2 S^{k_2} \exp(V_2) \quad \ldots\text{Multiplicative-error model} \quad (2)$$

where A_i = adult production of salmon using specification i,
S = number of smolts,
C_i = survival rate parameter in model i,
k_i = density dependence parameter in model i,
V_i = error term for model i.

By applying these two models to several sets of salmon data, Peterman (1981) concluded that the multiplicative-error model appears to generate better statistical results than its counter model. In addition, the results of the multiplicative-error model suggest that an increase in the number of smolts will increase the variation in total adult returns. Because the variability in adult production is not only influenced by the number of smolts but also other factors affecting the survival of smolts such as the body size of released smolts, Peterman (1981) suggested that model (2) should be modified by including more explanatory variables. By following Peterman's suggestion a third model can be specified with the additional variable body size, B:

$$A_3 = C_3 S^{k_3} B^{d_3} \exp(V_3). \quad (3)$$

The mean and variance of adult production for this model can be expressed as

$$E(A_3) = C_3 S^{k_3} B^{d_3} E(\exp(V_3))$$

$$\text{Var}(A_3) = (C_3 S^{k_3} B^{d_3})^2 \,\text{Var}(\exp(V_3)).$$

The instantaneous rates of change in mean and variance of adult production with respect to smolt body size can be expressed as

$$\partial E(A_3)/\partial B = d_3 E(A_3)/B$$

$$\partial\,\text{Var}(A_3)/\partial B = 2(\text{Var}(A_3)/E(A_3))\,\partial E(A_3)/\partial B.$$

If body size of smolts is enlarged, one would expect higher yields, $\partial E(A_3)/\partial B > 0$. Since both mean and variance are positive, the above model imposes a restriction that the smolt body size has a positive effect on variability, $\partial\,\text{Var}(A_3)/\partial B > 0$. This restriction is unwarranted because of lacking theoretical support; rather the effect (positive, negative, or zero) of body size on variability of adult return should be tested empirically. For this reason, McCarl and Rettig (1983) adopt a model developed by Just and Pope (1978, 1979) which can be expressed as

$$A_4 = C_4 S^{k_4} B^{d_4} + C_5 S^{k_5} B^{d_5} \exp(V_4)$$
$$= f(S,B) + h^{1/2}(S,B)\exp(V_4) \quad (4)$$

where $h^{1/2}(S,B)$ is a component of the standard deviation of adult production as shown below.

The mean and variance of adult production for this model can be expressed as

$$E(A_4) = C_4 S^{k_4} B^{d_4} + C_5 S^{k_5} B^{d_5} E(\exp(V_4))$$

$$\text{Var}(A_4) = (C_5 S^{k_5} B^{d_5})^2 \text{Var}[\exp(V_4)].$$

Because the signs of d_4 and d_5 are to be determined in the estimation, the advantage of model (4) over model (3) is that it allows for body size to have a positive effect on mean return and unknown (positive, negative, or zero) effect on the variability of adult production.

There are problems inherent in model (4), however, the first being that this specification produces a constant percentage change (k_4) in adult production when the number of smolts released changes by 1%, a constant output elasticity, ε_{as}. Output elasticity is an economic term which is widely used in measuring the relationship between input (smolt release) and output (adult production) and has the advantage of being unit free. An output elasticity of 1.0 means that an increase in smolt release by 1% will result in the same percentage increase in adult production, implying density independence. When the hypothesis of density independence is rejected, output elasticity is less than 1.0. Therefore, there is an one-to-one correspondence between the hypothesis of density independence and the value of output elasticity. The purpose of using the concept of output elasticity here is to facilitate the discussion of the restriction inherent in model (4).

There is no theoretical support for imposing the restriction of constant output elasticity, rather it should be treated as a hypothesis to be tested. More important, body size (B) is likely to have a positive effect on the output elasticity, i.e., $\partial \varepsilon_{as}/\partial B > 0$. In other words, when body size of smolts is enlarged, the improved ability of enduring unfavorable environmental conditions should increase the incremental return rate of adult salmon. But, a constant output elasticity implies that body size and the output elasticity are independent.

The comparison between model (3) and model (4) centers around the role of body size in the variability of adult production. However, data on body size is unavailable so that the comparison becomes empirically irrelevant. Consequently, the difference between these two models, in essence, rests on model specification. It should also be pointed out that the estimate of $h^{1/2}(S,B)$ is influenced by the functional form of $f(S,B)$ and vice versa, because $h^{1/2}(S,B)$ is the heteroscedastic error term to be handled by the weighted least squares method. It is, therefore, important to select a more general functional form for the mean and variance of adult production in testing the hypothesis of density independence and in estimating the variability in adult production.

The Box-Cox flexible functional form developed by Box and Cox (1964) and extended by Zarembka (1974) has been a popular tool for both discriminating among alternative functional forms and providing added flexible form in model specification (Moschini and Meilke 1984). The extended Box-Cox functional form for relating adult salmon production to smolts and other explanatory variables X (such as upwelling) can be expressed as

$$A^{(\lambda)} = \alpha_0 + \alpha_1 S^{(\mu)} + \alpha_2 X^{(\theta)} + e \tag{5}$$

$$\text{where } A^{(\lambda)} = \begin{cases} (A^{\lambda} - 1)/\lambda & \text{for } \lambda \neq 0 \\ \ln A & \text{for } \lambda = 0 \end{cases}$$

$$S^{(\mu)} = \begin{cases} (S^{(\mu)} - 1)/\mu & \text{for } \mu \neq 0 \\ \ln S & \text{for } \mu = 0 \end{cases}$$

$$X^{(\theta)} = \begin{cases} (X\theta - 1)/\theta & \text{for } \theta \neq 0 \\ \ln X & \text{for } \theta = 0 \end{cases}$$

$$e \sim \text{NID}(0, \sigma_1^2).$$

Model (5) includes the linear ($\lambda = \mu = \theta = 1$), multiplicative-error or log-log ($\lambda = \mu = \theta = 0$), and log-linear ($\lambda = 0$, $\mu = \theta = 1$) functional forms as special cases. Therefore, models (3) and (4) are special cases of model (5), which allows both nonconstant output elasticity and nonzero effect of X on the output elasticity.[3]

When the variability of adult salmon production is affected by the values of its explanatory variables, the error term has nonconstant variance, i.e., heteroscedasticity. Zarembka (1974) demonstrated that while the Box-Cox model is fairly robust to departures from normality, it is sensitive to heteroscedasticity. Failure to correct for this problem can generate misleading results (Lahiri and Egy 1981). When heteroscedasticity is present, we can also specify a Box-Cox functional form for the variance of the error term in model (5) as the following:

$$e \sim NID(0, h(S,X)\, \sigma_2^2) \tag{6}$$

where $h(S,X) = \beta_1 S^{(\tau)} + \beta_3 X^{(\zeta)}$.

The parameters, α_i, β_i, λ, μ, θ, τ, and ζ can be estimated by maximum-likelihood algorithms (see Appendix for a discussion of the log-likelihood function and estimation methods). The hypothesis of density independence can be tested by estimating the model with the restriction that $\lambda = \mu = 1$ against the unrestricted model.

[3]The superior flexibility of model (5) compared with models (3) and (4) is an important consideration in testing the hypothesis of density independence in light of the following remarks on the comparison of models (1) and (2) in Peterman (1981, p. 1117):

> "This is not to say that model 2 is the 'true' form of natural variability, because there are numerous other models that were not tested here (many of these alternatives cannot be tested in practice). . . ."

We also cannot claim that the extended Box-Cox functional form can produce the "best" or "true" functional form. There exist other flexible functional forms, such as Fourier (Gallant 1984), and the literature is silent in the comparison of these flexible functional forms.

Even though the Box-Cox functional form was first proposed in 1964, its application and investigation of its statistical properties have not received much attention until recently. Therefore, there are numerous aspects of transformations that merit further study (Box and Cox 1982). Lacking software support also makes its application difficult. Nevertheless, the superior flexibility of the Box-Cox functional form compared with other functional forms used traditionally is evident and its application should be encouraged.

RESULTS

In order to test the hypothesis of density independence for salmon utilizing the extended Box-Cox flexible functional form, the two data sets analyzed by McCarl and Rettig (1983) were also used here. The first data set contains total Hokkaido hatchery chum salmon fry releases and brood year adult returns for the years 1950 through 1969 (Moberly and Lium 1977). The second data set pertains to Oregon coho salmon for the years 1960 through 1980 (Oregon Department of Fish and Wildlife 1982). This latter data set was also analyzed by Clark and McCarl.

Hokkaido Chum Salmon Results

Due to the lack of data on body size and other factors affecting the survival rate of fry, adult production (in thousands) is estimated with the single explanatory variable, number of fry released (in millions). As explained in the Appendix, the dependent variable is divided by its geometric mean of 3,332,440. The iterated weighted least squares method produces the following maximum log-likelihood results with the t-statistics given in parentheses below the coefficients:

$$A^{(-0.4)} = \underset{(-4.07)}{-1{,}254} \; \underset{(4.07)}{-1{,}756.3}S^{(-1.4)} \tag{7}$$

$R^2 = 0.48$, Durbin-Watson = 1.5, Log-likelihood = -7.89.

The weight used to remove heteroscedasticity is $S^{(1.06)} = (S^{1.06} - 1)/1.06$. This implies that as the number of fry is increased by 1%, the standard deviation of the adult production increases by 1.06%, which is much smaller than the 2.5% reported by McCarl and Rettig (1983).

The above results suggest that the output elasticity[4] of fry is $1{,}756.3A^{0.4}S^{-1.4}$. When evaluated at the mean values of A and S (which are 1.1278 and 288.36, respectively), a percentage increase in the number of fry increases the adult production by 0.66%, implying density dependence. In addition, this output elasticity is a decreasing function of fry releases. In contrast to these results, McCarl and Rettig (1983) reported a constant output elasticity of 1.09 which was not found to be statistically different from 1.0, supporting density independence.

The hypothesis of density independence was formally tested by estimating the linear relationship between A and S (i.e., the power transformations for A and S are restricted to be one) by using the weighted least squares method (S was treated as the weight) with the following results:

$$A = \underset{(0.6)}{0.18} + \underset{(2.84)}{0.0034}S \tag{8}$$

$R^2 = 0.31$, Durbin-Watson = 1.04, Log-likelihood = -12.56.

The weighted least squares method produces a Durbin-Watson value of 1.04 which is below the lower limit of its critical value, suggesting the possible existence of autocorrelation. However, the Durbin-Watson value is well known to be below the lower limit (or above the upper limit) which could be the cause by model misspecification or autocorrelated error terms. Because the use of improper functional form is a model misspecification, the extended Box-Cox functional form needs to be explored before assuming the existence of an autocorrelation problem in light of low (or high) Durbin-Watson statistics. The extended Box-Cox results have a Durbin-Watson statistic of 1.50 (implying no autocorrelation), and hence, it is concluded that the low Durbin-Watson value in Equation (8) is a result of incorrect functional form. Since the Durbin-Watson statistic is for detecting first-order autocorrelation, the least squares procedure described in Pagan (1974) was applied to test higher-order autocorrelation. It is concluded that the Box-Cox results are free from autocorrelation problems, first or higher orders.

The hypothesis of density independence can be tested by comparing the log-likelihood values of Equations (7) and (8). The test statistic of twice the difference between the log-likelihood functions under the two specifications follows a chi-square distribution with the number of degrees of freedom equal to the number of restrictions (Theil 1971). This test procedure is similar to the Akaike Information Criterion (Akaike 1974) and has the advantage of testing the significance of the difference between the log-likelihood functions of different model specifications. It is concluded that the density-indepen-

[4]Equation (7) can be written as

$(A^{-0.4} - 1)/(-0.4) = -1254 + 1756.3(S^{-1.4} - 1)/(-1.4)$, or $A^{-0.4} = 0.8 + 501.8S^{-1.4}$.

Output elasticity $\varepsilon_{as} = (\%\Delta A)/(\%\Delta S) = dA/dS)\ (S/A) = 1{,}756.3S^{-1.4}A^{0.4}$ $d\varepsilon_{as}/dS = -2{,}458.8S^{-2.4}A^{0.4} < 0$.

dence hypothesis can be rejected at a 1% level with a critical value of 9.21 at 2 degrees of freedom.

For comparison purposes, the Hokkaido chum salmon data was used to fit the multiplicative-error model (model (2)) by applying the weighted least squares method with the following results:

$$\ln(A) = \underset{(2.06)}{-3.27} + \underset{(2.06)}{0.583} \ln(S) \quad (9)$$

$R^2 = 0.19$, Durbin-Watson = 1.15, Log-likelihood = −11.22.

By comparing the values of the log-likelihood function of Equations (7) and (9), it can be concluded that the multiplicative-error model can be rejected a 5% significance level. Even though the multiplicative-error model produces a bigger log-likelihood value than the linear model, the difference between these two log-likelihood values is not statistically signifi-

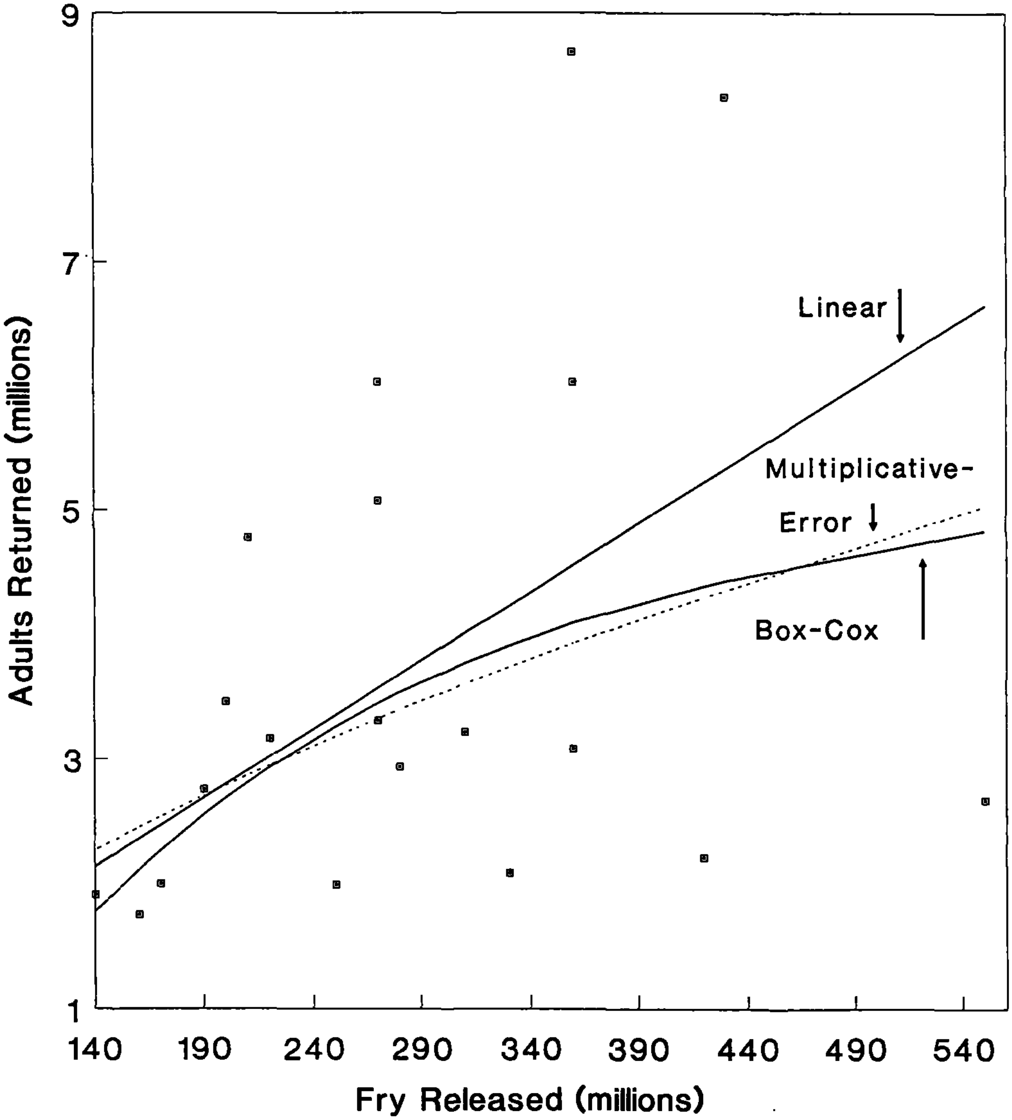

FIGURE 1.—Plot of Hokkaido chum salmon data and estimated relationships: 1950–1969. Dots are actual observations, linear model is Equation (8), multiplicative-error model is Equation (9), and Box-Cox model is Equation (7).

cant. Figure 1 shows the data set with estimated relationships, Equations (7)–(9), superimposed. It is evident that the Box-Cox specification produces a relationship of much bigger curvature and better fit than the multiplicative-error specification.

Oregon Coho Salmon Results

McCarl and Rettig (1983) suggested that aggregated (wild and hatchery) adult coho salmon production (in thousands) is affected by smolt releases (in millions), S, and upwelling index, U. The flexible functional form for Oregon coho salmon can be expressed as Equation (5). The model was estimated by iterated ordinary least squares with the following maximum log-likelihood results:

$$A^{(-0.6)} = -50.08 + 93.71S^{(-2.0)} + 0.59U^{(0.0)} \quad (10)$$
$$\qquad (-1.17) \quad (1.08) \quad (3.37)$$

$R^2 = 0.51$, Durbin-Watson = 2.07, Log-likelihood = -1.66.

To detect any violations of the assumption regarding the homoscedastic error term, a series of tests were conducted by running regressions of squared residuals ($\hat{e}^2$) or logs of ($\hat{e}^2$) on the predicted values of A or the explanatory variables S and U. The regression of $\hat{e}^2$ on all explanatory variables is known as the Breusch-Pagan-Godfrey test and the regression of $\log(\hat{e}^2)$ on all explanatory variables are known as the Harvey test (White 1987). Five tests were conducted using the chi-square distribution, and the assumption of homoscedastic error fails to be rejected at a 5% significance level. The same conclusion is reached when model (2) was fitted by Peterman (1981). The Box-Cox results are also found to be free from autocorrelation problems, first or higher orders.

Empirical results as summarized in Equation (10) indicate that the number of smolt released contributes positively to adult production at a 15% significance level. Upwelling also positively affects adult production at a 1% significance level. The Box-Cox results produce a nonlinear relationship between adult production and smolt release and an output elasticity of less than one, suggesting that the null hypothesis should be rejected. To formally test the hypothesis of density independence, the power transformations for A and S are restricted to be 1.0 and the Box-Cox functional form is reestimated with the following results:

$$A = -0.58 + 0.00065S + 0.084U^{(0.39)} \quad (11)$$
$$\qquad (-1.2) \quad (0.1) \quad (3.6)$$

$R^2 = 0.41$, Durbin-Watson = 2.09, Log-likelihood = -6.54.

By comparing the values of the log-likelihood functions for Equations (10) and (11) and following a chi-square test with 2 degrees of freedom, it is concluded that the hypothesis of density independence for Oregon coho salmon can be rejected. The same conclusion was reached by Peterman and Routledge (1983) and McCarl and Rettig (1983).

CONCLUSION

The findings of testing the hypothesis of density-independent marine survival for salmon and of the effect of the number of smolts released on the variability of adult production have important implications for fishery managers as noted in the literature. If the hypothesis of density independence fails to be rejected, there is no technical maximum[5] for the adult production from releasing smolts. A technical maximum of adult production exists when the number of smolts has a positive and decreasing effect on adult production. If the variability of adult production is positively affected by the number of smolts, it will be useful for fishery managers and the fishing industry to know the form of variability to evaluate the effectiveness of salmon hatchery operations. Further, fishery managers can improve management strategies by considering the trade-off between the mean and variance of adult production. The hypothesis of density independence has been tested extensively for different sets of data with conflicting results. Functional form selection and data deficiency have been suggested as the causes of conflicting findings.

Results of this study confirm that functional form selection is critical in testing the hypothesis of density independence and estimating the form of the variability of adult production. By using the extended Box-Cox functional form, it is concluded that there exists a density-dependent relationship be-

[5]A "technical maximum" refers to the maximum adult production in physical terms. This may not be an appropriate objective for fishery managers to achieve, because the release of smolts at technical maxima may not generate maximum benefits to the fishing industry. Maximization of the return to hatchery operations appears to be a more suitable objective of a single-attribute model to be accomplished without considering the risk factor.

tween the adult production and the number of chum salmon fry released in Hokkaido. Also, as the number of fry increases, the variability in adult production increases as well. The results reported by McCarl and Rettig (1983), using the Just-Pope specification (a special case of extended Box-Cox), conclude that the hypothesis of density independence fails to be rejected and the effect of the number of fry on the variability of adult production is more than twice that of this study. The Box-Cox results of aggregated Oregon coho salmon also indicate density dependence, and the same conclusion is also reached by McCarl and Rettig (1983) and Peterman and Routledge (1983). Because Nickelson (1986) reached a different conclusion using disaggregated data, the use of the extended Box-Cox specification to analyze disaggregated data for Oregon coho salmon is, therefore, recommended by the authors as a possible research need. However, partitioning the data set according to high and low upwelling, for example, will lead to the problem of insufficient degrees of freedom, as pointed out by an anonymous reviewer. This can be overcome only after a sufficient number of years of data collection have transpired. Finally, data reliability needs to be secured before the selection of functional form can improve our understanding of this issue.

ACKNOWLEDGMENTS

This work is the result of research sponsored by the Alaska Sea Grant College Program, cooperatively supported by the U.S. Department of Commerce, NOAA Office of Sea Grant and Extra-Mural Program, under grant number NA86AA-D-SG041, project number R/14-09; and by the University of Idaho with funds appropriated by the state.

LITERATURE CITED

AKAIKE, H.
1974. A new look at the statistical model identification. IEEE Trans. Autom. Control AC-19:716-723.
BOX, G. E. P., AND D. R. COX.
1964. An analysis of transformations. J. Royal Stat. Soc. 26(Ser. B):211-243.
1982. An analysis of transformations revisited, rebutted. J. Am. Stat. Assoc. 77:209-210.
CLARK, J., AND B. A. MCCARL.
1983. An investigation of the relationship between Oregon coho salmon (*O. kisutch*) hatchery releases and adult production utilizing the law of the minimum regression. Can. J. Fish. Aquat. Sci. 40:516-523.
GALLANT, R.
1984. The fourier flexible form. Am. J. Agric. Econ. 66: 204-208.
JUST, R. E., AND R. D. POPE.
1978. Stochastic specification of production functions and economic implications. J. Econ. 7:67-86.
1979. Production function estimation and risk related considerations. Am. J. Agric. Econ. 61:276-284.
KOUTSOYIANNIS, A.
1977. Theory of econometrics. 2d ed. The McMillan Press, Lond.
LAHIRI, K., AND D. EGY.
1981. Joint estimation and testing for functional form and heteroscedasticity. J. Econ. 15:299-307.
MCCARL, B. A., AND R. B. RETTIG.
1983. Influence of hatchery smolt releases on adult salmon production and its variability. Can. J. Fish. Aquat. Sci. 40:1880-1886.
MOBERLY, S. A., AND R. LIUM.
1977. Japan salmon hatchery review. Alaska Dep. Fish Game, Div. Fish. Rehabil. Enchance. Dev., 124 p.
MOSCHINI, G., AND K. D. MEILKE.
1984. Parameter stability and the U.S. demand for beef. West. J. Agric. Econ. 9:271-282.
NICKELSON, T. E.
1986. Influences of upwelling, ocean temperature, and smolt abundance on marine survival of coho salmon (*Oncorhynchus kisutch*) in the Oregon production area. Can. J. Fish. Aquat. Sci. 43:527-535.
OREGON DEPARTMENT OF FISH AND WILDLIFE.
1982. Comprehensive plan for production and management of Oregon's anadromous salmon and trout. Part II - coho salmon plan. Oreg. Dep. Fish. Wildl., Portland, OR, 188 p.
PAGAN, A. R.
1974. A generalized approach to the treatment of autocorrelation. Aust. Econ. Pap. 13:267-280.
PETERMAN, R. M.
1975. "Ocean effects" in salmon. PR-3 Inst. Resour. Ecol., 37 p. Univ. Br. Columb., Vancouver, B.C.
1978. Testing for density dependent marine survival in Pacific salmonids. J. Fish. Res. Board Can. 35:1434-1450.
1981. Form of random variation in salmon smolt-to-adult relations and its influence on production estimates. Can. J. Fish. Aquat. Sci. 38:1113-1119.
PETERMAN, R. M., AND R. D. ROUTLEDGE.
1983. Experimental management of Oregon coho salmon (*Oncorhynchus kisutch*): designing for yield of information. Can. J. Fish. Aquat. 40:1212-1223.
SEAKS, T. G., AND S. K. LAYSON.
1983. Box-Cox estimation with standard econometric problems. Rev. Econ. Stat. 65:160-164.
SPITZER, J. J.
1982. A primer on Box-Cox estimation. Rev. Econ. Stat. 64:307-313.
1984. Variance estimates in models with the Box-Cox transformation: implications for estimation and hypothesis testing. Rev. Econ. Stat. 66:605-653.
THEIL, H.
1971. Principles of econometrics. J. Wiley and Sons, N.Y.
WALTERS, C. J.
1975. Optimal harvest strategies for salmon in relation to environmental variability and uncertain production param-

eters. J. Fish. Res. Board Can. 32:1777–1784.
WHITE, K. J.
1987. Shazam: A general computer program for econometric methods (Version 5). Am. Stat. 41:80.
ZAREMBKA, P.
1974. Transformation of variables in econometrics. *In* P. Zarembka (editor), Frontiers in econometrics, p. 81–104. Academic Press, N.Y.

APPENDIX

Because the estimation of the extended Box-Cox functional form is carried out by maximum likelihood procedures, the log-likelihood function for the extended Box-Cox functional form and estimation methods are briefly presented in this appendix.

Under the assumptions that S and B are nonstochastic and the error term is normally and independently distributed with zero mean and constant variance, σ_1^2, the log-likelihood function of model (5) can be expressed following Spitzer (1982):

$$L(\alpha, \lambda, \mu, \theta, \sigma_1^2) = -T/2(\ln 2\pi + \ln \sigma_1^2) - (2\sigma_1^2)^{-1}(e'e) + (\lambda - 1)\Sigma \ln A \tag{12}$$

where T is the number of observations. To reduce the dimension of the estimation problem, the parameter σ_1^2 can be eliminated from Equation (12) to derive the concentrated log-likelihood function as follows:

$$L(\alpha, \lambda, \mu, \theta) = -T/2(\ln 2\pi + \ln \hat{\sigma}_1^2) + (\lambda - 1)\Sigma \ln A$$

where $\hat{\sigma}_1^2 = (1/T)\hat{e}'\hat{e}$.

When heteroscedasticity is present, the concentrated log-likelihood function for model (5) and the error term expressed in model (6) can be expressed as

$$L(\alpha, \lambda, \mu, \theta) = -T/2(\ln 2\pi + \ln \hat{\sigma}_2^2 + 1)$$
$$- \Sigma \ln (\beta_1 + \beta_2 S^{(\tau)} + \beta_3 B^{(\zeta)}) + (\lambda - 1)\Sigma \ln A \tag{14}$$

where $\hat{\sigma}'^2_2 = (1/T)\hat{e}'V^{-1}\hat{e}$ and V is a nxn matrix (n is the number of observations) in which off-diagonal elements are zeros and diagonal elements are $\beta_1 + \beta_2 S^{(\tau)} + \beta_3 B^{(\zeta)}$.

The maximum log-likelihood parameter estimates for (α, λ, μ, θ, and β) can be obtained by nonlinear least squares methods or iterated ordinary (weighted) least squares procedures. Seaks and Layson (1983) provide an example of the iterated ordinary (weighted) least squares method using the Time Series Processor (TSP) computer package for estimating Box-Cox flexible functional form with standard econometric problems; i.e., heteroscedasticity and autocorrelation.

As Spitzer (1984) pointed out, the ordinary least squares method underestimates the variance of the error term while the first derivative only gradient estimation methods (e.g., Marquardt) overestimate the variance. In order to compress the range of under- and overestimation of the error variance, Spitzer suggested that the dependent variable be divided by its geometric mean. This scaling process will then eliminate the last term in the concentrated log likelihood function in Equations (12)–(14).