THE ESTIMATION OF A CATCH LEVEL WHICH STABILIZES THE PARENTAL BIOMASS OF AN EXPLOITED FISH STOCK

J. MAJKOWSKI AND J. HAMPTON[1]

ABSTRACT

This paper addresses the problem of determining a catch level which stabilizes the parental biomass of a fish population at its present level. Initially, two methods are presented, both based on a simple catch equation and requiring estimates of recruitment, natural mortality, weights at age, proportions of sexually mature fish in each age class, and present parental biomass. Method I deals with a fishery assumed to have complete control over age composition of catches and requires age composition to be specified. Estimation of the catch age composition which results in the absolute maximum catch weight while holding parental biomass constant is also demonstrated. In method II no control over catch age composition is assumed; this composition is determined by catchability coefficients and the age structure of the population. Then, a simple modification of method II, method III, is presented for a fishery which has limited control over catch age composition through selective allocation of relative fishing effort among components of the fishery, the age composition of each component being different but not controllable by the fishery. This allows the determination of catches stabilizing the parental biomass for different allocations of relative fishing effort. Maximization of catch weight using methods I and III can be regarded as an improved yield per recruit analysis having the explicitly incorporated conditions of constant parental biomass and, as a consequence of other assumptions inherent in the methods, of constant recruitment. Consequences of incomplete compliance with assumptions inherent in these methods and their management implications are discussed. These methods are applied to southern bluefin tuna, *Thunnus maccoyii*, population and fishery data collected prior to 1981, and indicate that a total stabilizing catch of about 30,000 t per year is possible under the existing pattern of fishing.

Management of many commercial fisheries involves the determination of the maximum sustainable yield (MSY), (see reviews in Ricker 1975; Gulland 1977) and the imposition of restrictions which ensure that catches do not exceed this MSY. The estimation of MSY is usually made with the aid of production models which typically require historical catch per unit fishing effort data (Ricker 1975; Gulland 1977). Even if such data are available, assumptions underlying the use of these models are often violated or at least poorly complied with in real fisheries situations. Models taking account of the population age structure such as those reviewed by Getz (1979) are, in many cases, more appropriate for determining the MSY, but, in addition to estimates of natural mortality and growth rate, they also require information on the stock-recruitment relationship.

It is often the case that the stock-recruitment relationship for an exploited population is poorly known and the production models cannot be used for the reasons outlined. In this situation, a sensible management strategy is to stabilize the parental biomass at its present level which, through experience, is known to provide an adequate recruitment. As well as the assurance of reproductive success, the stabilization of parental biomass (and, as a consequence of the approach to be applied, of the entire age structure of the population) may be desired for completely different reasons, e.g., the preservation of a certain ecological equilibrium in a community of interacting species in which the exploited species is a member.

The objective of this paper is to demonstrate how a level of yearly catch that will stabilize the parental biomass of a population at its present level can be determined. This catch is hereafter referred to as the stabilizing catch. Initially, two methods developed for this purpose are presented. Method I deals with a fishery which has complete control over age composition of catches, while method II is relevant to a fishery having no such control. Method III, a simple modification of method II, is then presented for a fishery having limited control over catch age composition. It is assumed that this fishery can be divided into components, the catch age composition of each being different but not controllable by the fishery.

An estimation of stabilizing catch based on method I can be made for nearly any age composition of catches. This method allows also the determination of an age composition which results in the absolute maximum stabilizing-catch weight. If a fishery has

---

[1] Division of Fisheries Research, CSIRO Marine Laboratories, P.O. Box 21, Cronulla, NSW 2230, Australia.

Manuscript accepted May 1983.

limited control over age composition of catches, this maximum catch is usually not attainable. Method III allows the determination of stabilizing catches when relative fishing effort in one or more of the components of the fishery is changed. This enables the selection of a fishing strategy that is most suitable for achieving a management objective (e.g., the attainable maximum catch weight).

Maximization of catch weight using methods I and III can be regarded as an improved yield-per-recruit analysis. The advantage of this approach in comparison with the classical one (Beverton and Holt 1957) is the explicit incorporation of the condition of constant parental biomass and, as a consequence of other assumptions inherent in the methods, of constant recruitment. The condition of constant recruitment must be assumed in the classical analysis to make the interpretation of results practically useful. However, the difficulty in the classical approach is that the effect of changing the fishing strategy upon the recruitment level is not considered. As pointed out by Dunning et al. (1982), this may result in long-term yield losses if the reproductive potential of the population is reduced.

The three methods differ in their data requirements. Method I requires the relative catch age composition to be specified, while methods II and III can be used if estimates of age class-specific catchability coefficients (defined as the fraction of all fish in the age class caught using one unit of fishing effort) are available. These catchability coefficients in the case of method III have to be known for all components of the fishery being investigated. Also, the three methods require estimates of recruitment, natural mortality, weights at age, proportions of sexually mature fish in each age class, and present parental biomass.

Although few fisheries will exactly comply with all the assumptions underlying the use of these methods, fisheries scientists and managers, knowing the characteristics of their fisheries and the data available, should be able to decide which method best suits their particular cases. The consequences of incomplete compliance with the assumptions inherent in the methods and their management implications are discussed. The methods are illustrated by their application to southern bluefin tuna, *Thunnus maccoyii* (Castlenau), population and fishery data collected prior to 1981.

METHODS

Theoretical Background

A population satisfying the following assumptions is considered:

(a) Both recruitment to the fishable portion of the population and spawning are discrete events with respect to time and take place once per year at the same time each year.

(b) The magnitude of recruitment is dependent only upon the magnitude of parental biomass.

(c) Instantaneous rate of natural mortality may be dependent on age class only.

(d) Average weight of fish in the population at the time of spawning is a function of age only.

(e) Average weight of caught fish from any age class does not change from year to year.

If 1) these assumptions are satisfied, 2) both the magnitudes of yearly catches decomposed into age classes and their variability within a year do not change from year to year, and 3) a catch level is being maintained which ensures that the magnitude of parental biomass at spawning is constant, the population is in a regime referred to in this paper as a steady-state.

The question posed is what level of yearly catch would lead to the maintenance of parental biomass, P, at a specific level PS at the time of spawning over an infinite number of years. If a steady-state exists, only a single cohort need be considered to address this question.

The catch determination methods to be presented are based on the Pope (1972) catch equation:

$$\mathbf{C}_i = \mathbf{No}_i \exp(-0.5\mathbf{M}_i) - \mathbf{Ne}_i \exp(0.5\mathbf{M}_i) \qquad (1)$$

where C_i is the yearly catch (in number) of fish from age class i (age class i is defined as a group of fish at age $i - 1$ to i years), No_i and Ne_i are, respectively, the initial and final abundances of fish in age class i, i.e., at age $i - 1$ and i years of age (in steady-state $Ne_i = No_{i+1}$), and M_i is the instantaneous rate of natural mortality for age class i. This equation was derived assuming that the entire yearly catch is taken in the middle of the year and M_i is constant during the year. It is a modification of the equation (Ricker 1975)

$$C_i = \frac{F_i}{Z_i} No_i(1 - \exp(-Z_i)) \qquad (2)$$

where Z_i and F_i are the yearly average rates of total and fishing mortalities, respectively, for age class i. Both equations are equally effective in the majority of cases, but the use of Equation (2) is complicated from the computational point of view (see discussions in Pope 1972; Ricker 1975). Therefore, Equation (1) is used as the basis of the methods presented in this paper.

The assumptions associated with Equation (1) can be relaxed by using a time interval smaller than 1 yr. For example, if monthly periods were used, 12 equations could be formulated, each expressing the relationship between the monthly catch and the numbers of fish at the beginning and end of a month. In such a case, M_i and the fishing intensity could vary from month to month.

Method I (Complete Control Over Age Composition)

Here we consider a fishery which has complete control over the age composition of catches. The dynamics of a single cohort are described in such a case by the system of equations:

$$Cf_i = No_i \exp(-0.5M_i) - No_{i+1} \exp(0.5M_i) \quad i = r, \ldots n \tag{3}$$

where C is the total yearly catch in number, f_i is the fraction of the total catch belonging to age class i ($f_i = C_i/C$), and r and n, respectively, denote the youngest and oldest age classes numerously represented in the fishable portion of the population. The abundances of age classes should satisfy the condition:

$$\sum_{i=r}^{n+1} \alpha_i No_i Ws_i = PS \tag{4}$$

where α_i is the fraction of sexually mature fish in the ith age class and Ws_i is the average weight of a sexually mature fish belonging to age class i at the time of spawning.

The system of $n - r + 2$ algebraic Equations (3) and (4) is solvable for C and No_i's if the values of PS, No_r, f_i, M_i, α_i, and Ws_i ($i = r, \ldots, n$) are known. Meaningful solutions are restricted by the conditions

$$C \geq 0$$

and

$$No_i \geq 0 \quad i = r + 1, \ldots, n + 1. \tag{5}$$

Because of these conditions a meaningful solution may not exist for a given set of input values. In such a case, a change in fishing strategy (and also, therefore, in the f_i values) may resolve the problem. The stabilizing total catch, CB, can be found from the formula:

$$CB = \sum_{i=r}^{n} Cf_i \overline{W}_i \tag{6}$$

where $\overline{W}_i$ is the average weight of a fish from the ith age class.

If the basic management objective is to maintain the parental biomass at its present level, only the coefficients f_i may be subject to manipulation. According to their definition, they have to satisfy the following conditions:

$$f_i \geq 0 \quad i = r, \ldots, n$$

and

$$\sum_{i=r}^{n} f_i = 1 \tag{7}$$

but their individual values can be selected freely to the extent determined by the nature of Equations (3), (4), and (5). If alternative fishing strategies defined by different values of f_i are feasible, it is of interest to know 1) which of these are possible under the basic management objective of stabilizing the parental biomass and 2) what catches and age structures of the population are associated with these strategies. These questions can be easily addressed by solving the system of Equations (3) and (4).

It may be desirable to select such a fishing strategy which, in addition to maintaining the parental biomass at its present level, would yield the absolute maximum weight of yearly catch. This strategy could be determined by finding the set of f_i coefficients which maximizes CB. The problem is readily solvable with the aid of linear programming methods using Equation (6) as an objective function (treating Cf_i as a single variable C_i, thus making the problem linear) constrained by Equations (3), (4), and (5).

Method II (No Control Over Age Composition)

Here we consider a fishery which may be age selective, but that selectivity is beyond the fishermen's control. This being the case, changes in the age composition of catches can only be caused by alterations in the age composition of the population.

In this case, C_i can be expressed as

$$C_i = q_i E No_i \exp(-0.5M_i) \quad i = r, \ldots, n \tag{8}$$

where q_i is the catchability coefficient for age class i and E is an index of effective fishing effort. Substituting for C_i in Equation (1) we obtain

$$q_i E No_i \exp(-0.5M_i) = No_i \exp(-0.5M_i) - No_{i+1} \exp(0.5M_i) \quad i = r, \ldots, n.. \tag{9}$$

If the values of No_r, PS, q_i, α_i, and Ws_i ($i = r, \ldots, n$) are known, the system of $n - r + 2$ algebraic Equations (9) and (4) can be solved with respect to E and No_i ($i = r + 1, \ldots, n + 1$). Note that this equation system can be reduced to an $n - r + 1$ th order polynomial equation and solved for E using a standard computer routine which finds the zeroes of a function. The values of No_i can then be found by substitution. Meaningful solutions for E are constrained by the conditions

$$E \geq 0$$

and

$$No_i \geq 0 \quad i = r + 1, \ldots, n + 1. \qquad (10)$$

Knowing E and No_i's, the C_i values can be determined on the basis of Equation (8).

Method III (Limited Control Over Age Composition)

If a fishery can be divided into components, each of which is characterized by a unique set of q_i values, some control over the age composition of the total catch can be exercised by varying the relative amount of fishing effort expended in these components. In such a case, it is appropriate to replace q_iE in Equation (9) with $\sum_{j=i}^{K} q_{ij} E_j$ where j denotes one of k components of the fishery. The system of Equations (4) and (9) (modified) may then have a number of solutions (i.e., sets of E_j's and No_i's), but only the management strategies defined by nonnegative solutions will be possible for implementation. Determination of the meaningful solutions and the associated catches will be helpful for fisheries managers in selecting a fishing strategy that is most suitable for achieving their objectives (e.g., the attainable maximum yearly catch weight).

VALIDITY OF ASSUMPTIONS AND MANAGEMENT IMPLICATIONS

Assumption (a) limits the number of species for which the methods can be applied. It is not satisfied for most tropical species (see review in Saila and Roedel 1980), but does hold well for many temperate species (see reviews in Gulland 1969, 1977; Ricker 1975).

The magnitude of recruitment to most fish stocks is affected to some degree by environmental variation; therefore, assumption (b) will rarely be strictly satisfied. However, as long as the environmentally induced variation in recruitment is random and not large, results derived on the basis of the methods should provide a good indication of the stabilizing catch level.

Assumptions (c) to (e) are standard for most fisheries analyses (see reviews in Gulland 1969; Ricker 1975) although their validity is not always obvious. If both the age structure of the population and the environment are stable, assumptions (c) to (e) will likely be satisfied. The assessment of compliance with assumption (c) is extremely difficult. Simple methods used for estimating M_i (Gulland 1969, 1977; Ricker 1975) are usually unsuitable for testing this assumption. More complex methods are available (e.g., Majkowski 1981), but these have considerable data requirements and are frequently impractical. Assumptions (d) and (e) can usually be tested, especially if a technique of direct age determination is available for the species under consideration.

A management policy defined by the values of C and f_i's (satisfying Equations (3) and (4)) or E and q_i's or E_j's and q_{ij}'s (satisfying Equations (4) and (9)) can be effective immediately if the age structure of the population at the beginning of the first year of policy implementation is identical to that defined by the calculated values of No_i's (corresponding to the values of C and f_i's, E and q_i's or E_j's and q_{ij}'s). This will rarely be the case because of historical variation in catches. As a consequence, the parental biomass during an initial period of harvesting CB may fall below or increase above the specified level. As long as this has no effect upon recruitment, the population age structure will approach that defined by the calculated No_i's over the life span of the species.

The accuracy of the input parameters is implicitly assumed. Uncertainty in the management recommendations (i.e., in the value of CB) due to inaccuracies (caused by estimation errors and/or natural variability) in estimates of the input parameters for the procedures is generally difficult to predict, but can be examined for each specific application of the procedures using a sensitivity analysis technique (see reviews in Majkowski et al. 1981a; Majkowski 1982, in press; Majkowski and Hampton 1983).

EXAMPLE

Application of the methods described is demonstrated by using southern bluefin tuna population and fishery data collected prior to 1981.[2] This

---

[2] This analysis has since been updated (Hampton et al. in press).

species is highly migratory and spawns in waters off the south coast of Java. Juveniles migrate to waters off the coast of Australia, passing Western and South Australia and New South Wales. The general direction of their movement within the 200 mi Australian Fishing Zone (AFZ) is from west to east; however, some fish also move in the reverse direction. Schools of juveniles within the AFZ support the most important and valuable Australian finfish fishery. The fishing methods used are pole and line, purse seining, and, to a small extent, trolling. Southern bluefin tuna passing Australia gradually leave the nearshore fishing areas and become available to the Japanese longline fishery.

The parental biomass of this population, presently (i.e., in 1980) equal to about one-third of the preexploitation level, has been continuously and significantly reduced over the period of exploitation (Murphy and Majkowski 1981). Recruitment to the fishable portion of the stock has been quite stable over the same period, although reliable recruitment estimates are available only to 1976. Due to the absence of accurate information on the southern bluefin tuna stock-recruitment relationship and the lag in evaluation of the recruitment level, a conservative approach to fisheries management is most appropriate at this stage. Therefore, it is recognized by scientists of Australia, Japan, and New Zealand, the countries involved in the southern bluefin tuna fishery, that the present level of parental biomass should not be reduced further. This scenario provided the impetus for this paper.

Determination of Input Parameters

The input values required for the application of method I, their symbols, descriptions, and reference sources are presented in Table 1. The values of No_r and PS were estimated by cohort analysis while the f_i values were derived from the 1980 catch-at-age data. The catchability coefficients (calculated by using No_i's from cohort analysis), required for the application of methods II and III, are presented in Table 2.

Results

Results from several applications of methods I and II are presented in Table 3. Values of CB calculated on the basis of method I using the catch age composition specified in Table 1 and calculated on the basis of method II using the global catchability coefficients (see Table 2) are almost identical (30,012 and 29,013 t, respectively). The associated age structures of both population and catch produced by the two methods are also very similar (Table 4).

Estimates of CB derived using method I are dependent on the specified values of f_i. CB is maximized at 52,690 t/yr under the condition that only age classes 10 and 11 are fished. This fishing strategy requires catches of 123,100 fish from age class 10 and 688,100 fish from age class 11. The result is obtained using a linear programming computer program from the Numerical Algorithm Group Library, utilizing the contracted simplex method (McMillan 1970).

Within method II it is possible to examine the effect of various fishing regimes on CB simply by varying the values of q_i. Examples are presented in Table 3. The catchability coefficients used in these examples reflect the operation of 1) single components of the southern bluefin tuna fishery (i.e., the Australian fisheries off the coasts of Western Australia, South Australia, or New South Wales, or the Japanese fishery), or 2) the entire fishery with the exclusion of a selected component. The range of CB's estimated in this way was 12,523-44,695 t. These extreme values of CB corresponded to the lone operation of the Western Australian and Japanese fisheries, respectively.

Method III allows calculations of various combinations of stabilizing catches by the components of the global fishery. Examples of possible catch combinations are given in Table 5. These catch values are generated by specifying combinations of E's associated with the Western Australian, South Australian, and New South Wales fisheries, then calculating the Japanese fishing effort index and all related catches which would enable the stabilization of parental biomass. These results show that, as the fishing efforts of the Western Australian, South Australian, and New South Wales components increase, the Japanese and global stabilizing catches decrease. The minimum and maximum CB values generated by using method III are equivalent to the values in Table 3, relating to the lone operation of the Western Australian and Japanese fisheries, respectively.

Sensitivity Analysis

The sensitivity analysis technique used is referred to as ordinary sensitivity analysis (Majkowski and Bramall 1980; Majkowski and Waiwood 1981; Majkowski 1982, in press). The procedure consists in individually perturbing input parameters by various relative amounts and observing the resultant changes in CB.

The results of ordinary sensitivity analyses of the CB estimates of 30,012 and 29,013 t derived by using methods I and II, are presented in Tables 6 and 7, re-

TABLE 1.—Input parameter values for southern bluefin tuna necessary for the evaluation of CB using method I.

Parameter	Symbol	Value	Source of Information
Youngest exploited age class	r	2	Majkowski et al. (1981b)
Oldest exploited age class	n	20	Shingu (1978)
Proportion of fish belonging to age class which is sexually mature	α_i		
	i = 2-8	0	Shingu (1978)
	i = 9-20	1	
Abundance of fish about to enter age class r	No_r	5,258,422	J. Hampton (unpubl. data)
Yearly average rate of natural mortality	M	0.2	Hayashi et al. (1972)
Existing parental biomass (kg)	PS	167,371,601	J. Hampton (unpubl. data)
Fraction of the total 1980 catch (in number) belonging to age-class i (1980 calendar year)	f_i		J. Hampton (unpubl. data)
	i = 2	0.085321	
	3	0.411359	
	4	0.221376	
	5	0.053785	
	6	0.041523	
	7	0.022783	
	8	0.027565	
	9	0.031522	
	10	0.043063	
	11	0.034070	
	12	0.015556	
	13	0.007112	
	14	0.003083	
	15	0.001021	
	16	0.000505	
	17	0.000175	
	18	0.000096	
	19	0.000046	
	20	0.000037	
Weight (kg) at the beginning of the year for a fish belonging to age class i	Ws_i		Robins (1963) Kirkwood (1983)
	i = 9	47.11	
	10	55.07	
	11	62.53	
	12	69.42	
	13	75.68	
	14	81.31	
	15	86.34	
	16	90.80	
	17	94.72	
	18	98.17	
	18	101.17	
	20	103.79	
Weight (kg) at the middle of the year for a fish belonging to age class i	$\overline{W}_i$		Robins (1963) Kirkwood (1983)
	i = 2	1.81	
	3	5.55	
	4	11.19	
	5	18.26	
	6	26.22	
	7	34.58	
	8	42.99	
	9	51.14	
	10	58.87	
	11	66.05	
	12	72.62	
	13	78.57	
	14	83.90	
	15	88.64	
	16	92.82	
	17	96.50	
	18	99.72	
	19	102.53	
	20	104.97	

TABLE 2.—The catchability coefficients characterizing the global southern bluefin tuna fishery and its major components, the Australian fisheries off the coasts of Western Australia (WA), South Australia (SA), and New South Wales (NSW), and the Japanese fishery.

Age	Fishery				
class	Global	WA	SA	NSW	Japanese
2	0.0404	0.0259	0.0145	—	—
3	0.2608	0.1139	0.1430	0.0036	0.0003
4	0.1906	0.0148	0.1509	0.0170	0.0078
5	0.0609	0.0002	0.0167	0.0235	0.0206
6	0.0649	—	0.0066	0.0231	0.0352
7	0.0490	—	0.0005	0.0063	0.0422
8	0.0896	—	—	0.0074	0.0822
9	0.1057	—	—	0.0015	0.1042
10	0.1506	—	—	0.0002	0.1504
11	0.1827	—	—	—	0.1827
12	0.1413	—	—	—	0.1413
13	0.0964	—	—	—	0.0964
14	0.0654	—	—	—	0.0654
15	0.0421	—	—	—	0.0421
16	0.0224	—	—	—	0.0223
17	0.0063	—	—	—	0.0063
18	0.0050	—	—	—	0.0050
19	0.0024	—	—	—	0.0024
20	0.0023	—	—	—	0.0023

TABLE 3.—Estimates of CB and fishing effort index associated with various fishing strategies. WA = Western Australia, SA = South Australia, NSW = New South Wales, n.a. = not available.

Method	Specification of f_i's or q_i's	CB (t)	Effort index
I	f_i's from Table 1	30,012	n.a.
	f_i's corresponding to the "absolute" maximum catch weight	52,690	n.a.
II	Global q_i's	29,013	0.798
	WA q_i's	12,523	4.842
	SA q_i's	17,212	2.495
	NSW q_i's	29,314	10.968
	Japanese q_i's	44,695	1.606
	WA q_i's subtracted from global q_i's	32,479	0.928
	SA q_i's subtracted from global q_i's	35,256	1.136
	NSW q_i's subtracted from global q_i's	29,015	0.856
	Japanese q_i's subtracted from global q_i's	16,773	1.501

TABLE 4.—The estimated age composition of the southern bluefin tuna population and its catches (C_i) associated with the fishing strategies determined by the f_i values from Table 1 and the q_i values for the global fishery from Table 2.

Age	Fishing strategy defined by			
	f_i values		q_i values	
class	No_i	C_i	No_i	C_i
2	5,258,422	146,536	5,258,422	153,436
3	4,172,641	706,495	4,166,397	784,576
4	2,777,006	380,206	2,701,244	371,679
5	1,929,596	92,374	1,875,282	82,522
6	1,496,236	71,314	1,460,682	68,482
7	1,160,487	39,129	1,133,941	40,158
8	914,721	47,342	892,056	57,695
9	706,073	54,138	678,149	51,767
10	529,098	73,959	508,381	55,268
11	366,268	58,514	366,218	48,305
12	246,929	26,717	256,126	26,123
13	177,994	12,215	186,061	12,948
14	134,677	5,295	140,618	6,637
15	105,473	1,754	109,123	3,319
16	84,767	867	86,339	1,393
17	68,617	301	69,428	317
18	55,907	165	56,555	205
19	45,623	79	46,118	81
20	37,282	64	37,685	63

spectively. Several facts are evident from these results:

1) Both estimates of CB are most sensitive to perturbations of M_i and N_r.

2) Approximately linear relationships exist between CB and No_r, M_i, PS, and W_i (but not Ws_i) in the case of both CB estimates.

3) Perturbing all q_i values (method II) by the same percentage has no effect on CB, but does produce changed values of E.

4) Method II appears slightly more robust than method I in that CB (Method II) is less sensitive to changes in No_r, M_i, PS, and Ws_i than CB (Method I).

The results of sensitivity analysis presented in Tables 6 and 7 reflect the sensitivity of the CB estimates to changes in the individual input parameters. In the case of southern bluefin tuna, the input parameter estimates are related and this complicated the interpretation of the results. The effect of such interrelationships upon the CB estimates is illustrated by examining the dependence of No_r and PS upon M_i. These three parameters are probably subject to the greatest estimation error.

In the presented example, No_r and PS are estimated on the basis of cohort analysis for which M_i is an input parameter. Therefore, both No_r and PS values are dependent on the M_i estimate in which a relative error of up to ±50% may have existed. Table 8 shows the effects of perturbations in M_i, and the consequent changes in No_r and PS, upon the CB estimates of 30,012 and 29,013 t. Here, the percentage changes in CB in both cases are much smaller than the corresponding changes brought about by perturbations in M_i only (Tables 6, 7). This mostly results from the fact that No_r estimated on the basis of cohort analysis is a strongly increasing function of M_i and the effects of No_r and M_i on the CB estimates are antagonistic. Therefore, if No_r is estimated from cohort analysis, the results of both methods are considerably less sensitive to perturbations in M_i than in the case when No_r and M_i are independently estimated. If, however, M_i and No_r for southern bluefin tuna are independently estimated, a high degree of accuracy is necessary to confidently evaluate CB. Note that the degree of sensitivity of a CB estimate to changes in M_i is dependent also on the age composition of CB. For example, the sensitivity of the CB estimate of 52,690 t (age classes 10 and 11 only are fished), derived by using method I, to changes in M_i is much higher than that of 30,012 t (age classes 2-20 are fished). More extensive sensitivity examinations are beyond the scope of this

TABLE 5.—Possible combinations of stabilizing catches (in tonnes) in the four component fisheries when fishing effort index is specified for Western Australia (WA), South Australia (SA), and New South Wales (NSW) and calculated for Japan.

WA		SA		NSW		Japan		Total catch
Effort	Catch	Effort	Catch	Effort	Catch	Effort	Catch	
0.5	1,546	0.5	4,321	0.5	1,504	1.33	30,307	37,678
0.5	1,546	0.5	4,310	1.0	2,958	1.26	28,253	37,067
0.5	1,545	0.5	4,298	1.5	4,363	1.19	26,250	36,456
0.5	1,517	1.0	8,062	0.5	1,282	1.16	23,115	33,976
0.5	1,516	1.0	8,041	1.0	2,519	1.09	21,346	33,422
0.5	1,515	1.0	8,020	1.5	3,714	1.02	19,620	32,869
0.5	1,487	1.5	11,255	0.5	1,079	0.97	16,845	30,666
0.5	1,487	1.5	11,225	1.0	2,121	0.90	15,337	30,170
0.5	1,486	1.5	11,195	1.5	3,125	0.83	13,868	29,674
1.0	3,025	0.5	4,084	0.5	1,382	1.30	27,317	35,808
1.0	3,024	0.5	4,073	1.0	2,717	1.23	25,425	35,239
1.0	3,023	0.5	4,063	1.5	4,007	1.16	23,580	34,673
1.0	2,966	1.0	7,604	0.5	1,170	1.12	20,597	32,337
1.0	2,965	1.0	7,584	1.0	2,300	1.05	18,977	31,826
1.0	2,964	1.0	7,563	1.5	3,390	0.98	17,399	31,316
1.0	2,909	1.5	10,589	0.5	978	0.93	14,782	29,258
1.0	2,907	1.5	10,560	1.0	1,921	0.86	13,413	28,801
1.0	2,906	1.5	10,532	1.5	2,830	0.79	12,079	28,347
1.5	4,437	0.5	3,853	0.5	1,263	1.26	24,475	34,028
1.5	4,435	0.5	3,842	1.0	2,484	1.19	22,740	33,501
1.5	4,434	0.5	3,832	1.5	3,663	1.12	21,049	32,978
1.5	4,350	1.0	7,156	0.5	1,062	1.09	18,218	30,786
1.5	4,349	1.0	7,136	1.0	2,087	1.01	16,745	30,317
1.5	4,347	1.0	7,117	1.5	3,076	0.94	15,309	29,849
1.5	4,265	1.5	9,938	0.5	880	0.90	12,851	27,934
1.5	4,263	1.5	9,911	1.0	1,729	0.82	11,617	27,520
1.5	4,262	1.5	9,885	1.5	2,546	0.75	10,414	27,107

TABLE 6.—Results of ordinary sensitivity analysis of the CB estimate of 30,012 t derived by using method I.

Parameter perturbed	Perturbation magnitude			
	−25%	−1%	+1%	+25%
[2]M_i	[1]+52.3	+2.1	−2.1	−53.5
No_r	−39.9	−1.6	+1.6	+39.9
[3]f_i	+24.8	+1.0	−1.4	−36.7
[2]W_i	−25.0	−1.0	+1.0	+25.0
[2]Ws_i	−29.9	−0.6	−0.6	+12.0
PS	+14.9	+0.6	−0.6	−14.9

[1] All values in this table represent relative changes (expressed in percentages) in CB.

[2] All values (i.e., for i = 2-20) are simultaneously perturbed by the same percentage indiated in the first row.

[3] The f_i values for i = 2-6 are simultaneously perturbed by the percentage indicated in the first row, and all reamining f_i values (i.e., for i = 7-10) are changed by a percentage such that $\sum_{i=2}^{20} f_i$ is equal to one.

TABLE 7.—Results of ordinary sensitivity analysis of the CB estimate of 29,013 t derived by using method II.

Parameter perturbed	Perturbation magnitude			
	−25%	−1%	+1%	+25%
[2]M_i	[1]+45.0	+1.9	−1.9	−50.8
No_r	−35.9	−1.4	+1.4	+33.4
[2]$\bar{W}_i$	+25.0	+1.0	+1.0	+25.0
[2]Ws_i	−15.9	−1.2	−0.4	+ 6.3
PS	+ 8.1	+0.4	−0.4	−10.5
[2]q_i	0.0	0.0	0.0	0.0

[1] All values in this table represent relative changes (expressed in percentages) in CB.

[2] All values (i.e., for i = 2-20) are simultaneously perturbed by the same percentage indiated in the first row.

TABLE 8.—Effects of perturbations in M_i's(i = 2,...,20) and consequent perturbations in No_r and PS upon the CB estimates of 30,012 and 29,013 t derived by using methods I and II, respectively.

Method	Perturbation magnitude					
	−50%	−25%	−1%	+1%	+25%	+50%
I	[1]+1.5	+1.9	+0.1	−0.1	−5.1	−14.5
II	+14.8	+7.3	+0.3	−0.3	−8.0	−17.8

[1] All values in this table represent relative changes (expressed in percentages) in CB.

paper. These examples are presented to illustrate the use of the sensitivity analysis technique in the case of the CB estimation rather than to make a final judgment of the validity of the CB estimates presented. The knowledge, lacking in the case of southern bluefin tuna, of probability distributions of all input parameters would allow the application of stochastic sensitivity analysis (Majkowski and Waiwood 1981; Majkowski et al. 1981a; Majkowski 1982, 1983; Majkowski and Hampton 1983, in press) and assist in making such a judgment.

CONCLUDING REMARKS

The methods presented are mathematically very simple. The system of equations associated with method I is linear with respect to the CB and No_i variables. Therefore, the estimate of CB related to a specified set of f_i coefficients can be obtained analytically. The determination of the age composition which results in the absolute maximum stabilizing catch weight requires a standard computer program from a linear programming package. The system of equations associated with methods II and III cannot be solved analytically; a computer program which finds the zeros of a function has to be used.

Data requirements for the methods are not extensive. They are usually available from an established and well-documented fishery. Data required for method I may be more easily obtained than those for methods II or III. This is the major advantage of method I.

Assumptions associated with methods II and III are more realistic for most fisheries than those related to method I. Therefore, if the data for methods II and III are available, these methods will be superior in the majority of cases. However, the example presented clearly indicates that methods I and II, at least for southern bluefin tuna, provide nearly identical results if the present age specific pattern of fishing is considered.

The example also suggests that the methods, at least in the case examined, are relatively robust with respect to uncertainties in the input parameters. This is a very important consideration because many population and fisheries parameters are frequently poorly known.

Because of their deterministic nature and the assumptions involved, none of the methods will be completely realistic in their description of the fishery and population. However, they should provide managers with a valuable tool for gaining an impression of the level of sustainable catch and for assessing the relative merits of harvesting alternatives.

ACKNOWLEDGMENTS

We are grateful to Geoffrey P. Kirkwood (CSIRO Division of Fisheries Research, Cronulla, NSW, Australia), Ron Sandland (CSIRO Division of Mathematics and Statistics, Cronulla, NSW, Australia), Robin L. Allen (Ministry of Agriculture and Fisheries, Fisheries Research Division, Wellington, New Zealand), and Patrick K. Tomlinson (Inter-American Tropical Tuna Commission, La Jolla, Calif., USA) for suggestions for improvements to an earlier version of this paper.

LITERATURE CITED

BEVERTON, R. J. H., AND S. J. HOLT.
1957. On the dynamics of exploited fish populations. Fish. Invest. Minist. Agric, Fish Food (G.B.), Ser. II, 533 p.

DUNNING, D. J., Q. ROSS, AND J. GLADDEN.
1982. Evaluation of minimum size limits for St. Lawrence River northern pike. North Am. J. Fish. Manage. 2:171-175.

GETZ, W. M.
1979. The ultimate-sustainable-yield problem in nonlinear age-structured populations. Math. Biosci. 48:279-292.

GULLAND, J. A.
1969. Manual of methods for fish stock assessment Part 1. Fish population analysis. FAO Man. Fish Sci. 4, 154 p.

GULLAND, J. A. (editor).
1977. Fish population dynamics. John Wiley and Sons, New York and Toronto, 372 p.

HAMPTON, J., J. MAJKOWSKI, AND G. I. MURPHY.
In press. The 1982 assessment of the southern bluefin tuna (*Thunnus maccoyii*) population and the determination of catch levels stabilizing the parental biomass. CSIRO Mar. Lab. Rep.

HAYASHI, S., C. SHINGU, AND K. HISADA.
1972. An inference on rational utilization of southern bluefin tuna based on data obtained from 1957 to 1969. Far Seas Fish. Res. Lab. S. Series 6:63-88.

KIRKWOOD, G. P.
1983. Estimation of von Bertalanffy growth curve parameters using both length increment and age-length data. Can. J. Fish. Aquat. Sci. 40:1405-1411.

MAJKOWSKI, J.
1981. Application of a multispecies approach for assessing the population abundance and the age-structure of fish stocks. Can. J. Fish. Aquat. Sci. 38:424-431.
1982. Usefulness and applicability of sensitivity analysis in a multispecies approach to fisheries management. *In* D. Pauly and G. I. Murphy (editors), Theory and management of tropical fisheries, p. 149-165. ICLARM Conference Proceedings, 9. ICLARM, Manila, Phil., and Div. Fish. Res., CSIRO, Cronulla, Aust.
In press. Usefulness and applicability of sensitivity analysis in resource management. *In* Proceedings of the Resource Management Workshop, James Cook University of North Queensland, 13-16 July 1981.

MAJKOWSKI, J., AND L. BRAMALL.
1980. Sensitivity of bioenergetic growth models of animals to changes in the energy balance parameters. J. Theor. Biol. 85:643-655.

MAJKOWSKI, J., AND J. HAMPTON.
1983. The effect of parameter uncertainties in an age length relationship upon estimating the age composition of catches. Can. J. Fish. Aquat. Sci. 40:272-280.
In press. Deterministic partitioning of the catch of southern bluefin tuna, *Thunnus maccoyii*, into age classes using an age-length relationship. *In* E. D. Prince and L. M. Pulos (editors), Age determination of oceanic pelagic fishes: Tunas, billfishes, and sharks. Proceedings of the International Workshop.

MAJKOWSKI, J., J. M. RIDGEWAY, AND D. R. MILLER.
1981a. Multiplicative sensitivity analysis and its role in development of simulation models. Ecol. Model. 12:191-208.

MAJKOWSKI, J., AND K. G. WAIWOOD.
1981. A procedure for evaluating the food biomass consumed by a fish population. Can. J. Fish. Aquat. Sci. 38:1199-1208.

MAJKOWSKI, J., K. WILLIAMS, AND G. I. MURPHY.
1981b. Research identifies changing patterns in Australian

tuna fishery. Aust. Fish. 40(2):5-10.

MCMILLAN, C.
1970. Mathematical programming. John Wiley and Sons, N.Y., 496 p.

MURPHY, G. I., AND J. MAJKOWSKI.
1981. State of the southern bluefin tuna population: fully exploited. Aust. Fish. 40(11):20-29.

POPE, J. G.
1972. An investigation of the accuracy of virtual population analysis using cohort analysis. Int. Comm. Northwest Atl. Fish. Res. Bull. 9:65-74.

ROBINS, J. P.
1963. Synopsis of biological data on bluefin tuna *Thunnus thynnus maccoyii* (Castlenau) 1872. FAO Fish. Rep. 6:562-587.

RICKER, W. E.
1975. Computation and interpretation of biological statistics of fish populations. Fish. Res. Board Can. Bull. 191, 382 p.

SAILA, S. B., AND P. M. ROEDEL (editors).
1980. Stock assessment of tropical small-scale fisheries. Proceedings of an International Workshop held September 19-21, 1979, at the University of Rhode Island, Kingston, R.I., 198 p.

SHINGU, C.
1978. Ecology and stock of southern bluefin tuna. [In Jpn.] Jpn. Assoc. Fish. Res. Prot. Fish. Study 31, 88 p. [Engl. transl. Aust. CSIRO Div. Fish. Oceanogr. Rep. 131, 1980, 79 p.]