A LINEAR-PROGRAMMING SOLUTION TO SALMON MANAGEMENT[1]

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ABSTRACT

A linear-programming model was constructed to allocate the catch of salmon among the days of the salmon run. The objective of the model was to derive a management schedule for catching the salmon which would result in maximizing the value of the landings given certain constraints. These constraints ensured that cannery capacity was not exceeded, and that escapement of both male and female fish was "adequate." In addition to considering the allocation of the catch in the primal problem, the dual problem considered the shadow prices or marginal value of the various sizes of fish, eggs, and cannery capacity, thus enabling the manager to view his decisions in light of the marginal values of these entities. As an example, the model was applied to a run of sockeye salmon in the Bristol Bay system. In the particular example, which was chosen to replicate the 1960 run, the additional value of the catch owing to optimality amounted to an ex-vessel value of a few hundred thousand dollars. In addition it appeared that the required processing time could be reduced by several days. The optimum allocation was obtained through conformance to the linear-programming model. The cost of this conformance was not, however, determined.

The Pacific salmon fisheries have been cited as an example of irrational conservation (Crutchfield and Pontecorvo, 1969). Much of this irrationality is reflected in the dissipation of a sizable fraction of the available economic rent, a situation which results from the open-access nature of the fishery and legislated inefficiency. The remedy for this situation is to alleviate the open-access and inefficiency problem. Such alleviation would require the dissolution of rather formidable institutional problems. In the present paper, we examine the salmon problem from a slightly different vantage point than Crutchfield and Pontecorvo. We examine the salmon problem under the status quo; we do not consider the optimal amount of gear or its efficiency (this should not, however, be construed as reflecting any diminution in the importance of these problems); rather we consider, as an interim approach, whether it is possible, under the stringent condition of knowing in advance the structure of the run, to increase the value of the fish on the dock by optimally allocating the catch among the days of the run.

The traditional approach to salmon management might be considered, at the risk of several simplifications, as consisting of (1) forecasting the magnitude of the run; (2) setting an escapement goal and a catch implied by the forecast and the escapement; and (3) daily fishing closures and other devices which allocate the catch in varying quantities to the days of the run. The traditional approach, then, also involves an allocation of the catch to the days of the run. In the traditional approach, the allocations are usually based on the experience of management biologists. Although the objectives of their allocations are not always clearly and explicitly stated, there is a tendency for the primary objective of management to be simply the attainment of the escapement goal. Our approach is to use the theory of linear programming to advise on a non-intuitive optimum allocation of the salmon catch among the days of the run where the objective of management does not explicitly involve escapement. Rather, we develop our allocation strategy to maximize the value of the catch on the dock given a variety of constraints which include the necessity for a given number of fish to escape the fishery. The objective of maximizing the value of the fish on the dock and the constraints explicitly define the objectives of the management scheme.

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We consider these problems in three additional sections. In the first, we describe the linear-programming allocation model, which we believe to be applicable, with simple modifications, to a variety of salmon management situations. In the second, we consider how the model might be applied to a run of salmon in the Naknek-Kvichak system of Bristol Bay, Alaska. As an example, we choose data from the 1960 run to that system and obtain an optimum allocation of large and small, male and female fish, on each day of the run to the daily catch. This optimum allocation served to maximize the value of the fish on the dock subject to constraints which ensured that the catch did not exceed the daily run, that the catch would be less than the cannery capacity, and that an "adequate" escapement, both in terms of the number of eggs and sex ratio, passed the fishery. Thus, in addition to managing the run by a non-intuitive optimum allocation and satisfying an escapement goal, we also considered the quality of the run in terms of its sex and age composition. In order, however, to achieve this optimum allocation we needed certain data on the structure of the run in advance and we also needed a mechanism by which we could select large and small male and female fish. It would most likely be impractical to have either a precise prediction of the daily run or an ability to select, with high precision, large or small, male or female fish. We show that even if we had the necessary data, a technique for precise selection of the various entities of fish, and maintained the 1960 escapement and sex-ratio conditions, optimum allocation would yield us a catch having a value of several hundred thousand dollars more than the actual catch. Thus given the cost of obtaining the necessary information to perform the optimum allocation and the constraints extant in 1960, it is questionable whether biological management could yield a better allocation than that which was obtained. This serves to re-emphasize the approach of Crutchfield and Pontecorvo, indicating that the system is most sensitive to variables which lie outside the objective and constraint equations specified in the present paper. On the other hand, our results show that it is possible, at least in terms of the model, to reduce the number of days during which the cannery operates and yet process the same number of fish. Furthermore as previously indicated, we constrained our example to fit the statistics of the 1960 run and thus we had, in our example, a nearly 1:1 sex ratio; but as we indicate later, we could have caught a considerably larger number of male fish and still would have had sufficient male fish in the escapement to ensure the efficient production of fertilized eggs. And finally the model was quite sensitive to decreasing the escapement but unfortunately there is little guidance in the literature which would indicate the optimum escapement for the Naknek-Kvichak system and furthermore there appears to be little hope of learning the magnitude, in the reasonably near future, of the optimum escapement for the Naknek-Kvichak system. Thus evaluation of the cannery processing time, catch problem, and relaxation of sex ratio and escapement constraints might result in an added value to the catch which would make some attempts at allocation practical. We also, in the second section, place some stress on interpretation of the shadow prices of the various variables in the problem. This is of interest to operations researchers because it provides an example, in addition to those conventionally used, of an application of the interpretation of the linear-programming primal-dual relation. The shadow prices are of interest to the fishery manager because from them it is possible to impute values to the various resources under the manager's control, and, in making a decision, the manager can thus consider these values which, as we show, are not always intuitively obvious. In the third and final section we conclude the paper with a general discussion of salmon management in a linear-programming setting.

MODEL

Most linear-programming models generally involve finding values X_i which maximize (or minimize) an objective function $\Sigma c_i X_i$, subject to a set of constraints each of which has the form $\Sigma \beta_i X_i \leqslant L_j$, where the inequality can be in either direction or can, in fact, be an equality.

The β_i's and the L_j's are constants appropriate to a particular problem. The details of the LP (linear-programming) procedure can be found in the many treatises on the subject (e.g., Gass, 1964) or in most texts on operations research (e.g., Hillier and Lieberman, 1967).

In our application of the LP model, we maximize the following objective function

$$Z = \sum_{i=1}^{M} \sum_{j=1}^{N} c_{ij} X_{ij}, \qquad (1)$$

where M refers to the total number of age-sex categories and N refers to the days of the run. The variable X_{ij} is the number of fish caught in the ith entity on the jth day of the run and c_{ij} corresponds to the value of the fish caught in the ith entity on the jth day (Table 1). The age-sex category classification results from the fact that salmon runs are comprised of a variety of age-groups. Because each age-group is usually of a different average size, the individuals in each age-group also have a different average value which we denote by c_{ij}. It should be mentioned that size is not the only criterion which can be used for classification. For example, in the Naknek-Kvichak run of Bristol Bay, the sex of the fish can also be used because within an age-group the male fish tend to be larger than the female fish and thus more valuable in terms of weight of fish-flesh; but, on the other hand, the eggs of the females are a valuable commodity and thus the per-pound value of females may be greater than the per-pound price of males. If the value of the fish were constant during the course of the run, we could replace the c_{ij} with c_i and the allocation problem would become rather uninteresting. But the value, however, does tend to vary during the course of the run. One reason for this is a deterioration of the quality of fish, as indicated by declining oil content and reduction in color intensity with the progression of the run. Another way in which c_{ij} could vary is that the average value of the fish on a particular day would tend to vary during the course of the run because of a within-entity trend in the average size of the fish during the course of the run; this, however, is not considered in the present paper. It is obvious that, if we had sufficient information, we could establish a large number of different c_{ij}'s.

TABLE 1.—Linear program model notation.

M	–	The total number of age-sex categories.
N	–	The total number of days in the run.
X_{ij}	–	The number of fish in the ith age-sex category which are caught on day j of the run.
c_{ij}	–	The value of a fish caught in the ith age-sex category on day j of the run.
R_{ij}	–	The number of fish in the ith age-sex category which run past the fishery on day j of the run.
K_j	–	The capacity, in numbers of fish, of the canneries on day j of the run.
K'	–	The total seasonal capacity of the canneries in numbers of fish.
W_{ij}	–	The number of fish of the ith age-sex category in the escapement on day j of the run.
a_i	–	The average number of eggs in each fish of the ith age-sex category.
T	–	The total number of eggs contained in the escapement and catch.
E	–	The minimum number of eggs required in the escapement.
$\mathcal{M}$	–	The total number of males in the escapement and catch.
F	–	The average fecundity of the female age-sex categories, expressed in number of eggs.
H	–	The sex ratio desired in the escapement, expressed as the number of females per male.
L_i	–	The number of fish of the ith age-sex category desired in the season's escapement.
S_i	–	The number of fish in the total season run of the ith age-sex category.
$P(j)$	–	The proportion of the run that arrives by day j of the run.
$P'(j)$	–	The proportion of the run that arrives on day j of the run.

As indicated previously, equation (1) is maximized subject to a variety of constraints. For the salmon problem, the first set of constraints is rather obvious and constrains the catch, of any entity, on any day, to be less than, or equal to, the number of fish in that entity in the run. These constraints are of the form

$$X_{ij} \leqslant R_{ij}. \qquad (2)$$

X_{ij} is always $\geqslant 0$ and R_{ij} is the number of fish of the ith entity which run past the fishery on the jth day. There can be as many as $M \times N$ constraints of this form, but in some applications, either the number of entities or the number of days will be collapsed owing to either the nature of the problem or a lack of information. Note also we can easily "close" the fishery for any entity or all entities on any day simply by setting the appropriate $R_{ij} = 0$.

The second set of constraints constrains the catch on any day to be less than the daily capacity of the canneries. Thus, we have the set of constraints,

$$\sum_{i=1}^{M} X_{ij} \leqslant K_j \qquad (3)$$

where K_j is the capacity of the cannery or canneries on the jth day. Another form of this constraint might be incorporated in situations such as in the Bristol Bay fishery, which has a short season, is remote from supply points, and thus has a finite seasonal capacity; these constraints can be expressed by

$$\sum_{i=1}^{M} \sum_{j=1}^{N} X_{ij} \leqslant K'. \qquad (4)$$

The constraint is redundant if $K' \leqslant \sum_{j=1}^{N} K_j$.

and hence is only used when the season's capacity is less than the sum of the daily capacities, i.e.,

$$K' \leqslant \sum_{j=1}^{N} K_j.$$

The next set of constraints results from the need to ensure that an adequate number of fish escape the fishery and are thus permitted to spawn. We formulate this constraint in terms of the egg complement of the number of females escaping the fishery rather than the number of females escaping per se or the total number of fish (males and females) escaping.

In order to formulate this constraint set, we define T as

$$\sum\sum a_i W_{ij} + \sum\sum a_i X_{ij} = T \qquad (5)$$

where a_i is the average number of eggs in each fish in the ith entity, W_{ij} is the escapement of fish in the ith entity on the jth day and thus T is the egg complement of the escapement and the catch. Now if we need a minimum number of eggs to represent the escapement, a quantity which we denote by E, we must have

$$\sum\sum a_i W_{ij} \geqslant E. \qquad (6)$$

So substitution of (6) into (5) yields the constraint set conformable to the X_{ij}'s of our other constraints, viz.

$$\sum\sum a_i X_{ij} \leqslant T\text{-}E. \qquad (7)$$

We can see that by using the same reasoning we could construct a constraint set which would constrain the egg complement to be less than some maximum egg complement.

Our next constraint involves the sex ratio of the spawning fish. The utility of allowing a particular egg complement to escape the fishery could be negated by not allowing a sufficient number of males to escape for the purpose of fertilizing the eggs. In order to ensure that an adequate number of males escape the fishery, we formulate the necessary constraint by noting that

$$\sum W_j + \sum X_j = \mathscr{M} \qquad (8)$$

where in this particular equation the W's refer to the number of males in the escapement and the X's to the number of males in the catch and M to the total number of males in the run. Now to satisfy our constraint, we must have

$$\sum W_{ij} \leqslant \frac{E \times H}{F}. \qquad (9)$$

where the sum extends over all male entities and days of the run; F is the average fecundity of the female entities in the run, and H is the desired male to female sex ratio. Substitution of (9) into (8) then yields the desired constraint

$$\sum X_{ij} \leqslant \mathscr{M} - \frac{E \times H}{F}. \qquad (10)$$

This constraint can also be formulated in a variety of ways.

In addition to escapement goals established in terms of eggs and a sufficient number of males to fertilize the eggs, we desired to establish, for some sample problems, escapement goals in terms of total numbers of fish of each entity in the escapement; this would greatly simplify the simulation of the actual escapements for any historic salmon season. Hence, we developed the constraint

$$\sum_j W_{ij} \geq L_i \qquad (11)$$

where L_i is an escapement goal set for entity i which would ensure escapements identical with any historic year. To make (11) conform to our constraint set, we note that

$$\sum_j W_{ij} + \sum_j X_{ij} = S_i \qquad (12)$$

where W_{ij} and X_{ij} are the escapement and catch (respectively) of entity i on day j and S_i is the total season's run of entity i, we can substitute (11) into (12) and get the constraint in the proper form:

$$\sum_j X_{ij} \leq S_i - L_i. \qquad (13)$$

Thus a seasonal limit can be placed upon the catch of any entity i.

As indicated earlier, once the objective function and the constraints have been formulated, then optimization of the objective function, given the constraints, is a standard procedure outlined in some detail in the literature, and, with a computer facility, is a relatively simple task. There are, however, interpretations which are of further interest than the solution of the objective function. These interpretations rest in the primal-dual relationship of the LP problem.

In order to demonstrate this relation, we will denote a general form of the primal problem as

$$\text{maximize} \quad \sum_{i=l}^{m} c_i X_i$$

$$\text{subject to} \quad \sum_{i=1}^{m} a_{ij} X_i \leq b_j$$

$$j = 1, \ldots, n \qquad (14)$$

where we have used slightly different notation than in the salmon problem, but the analogue between (14) and the salmon problem should, nevertheless, be quite clear. If (14) is the primal, then the dual of this primal is

$$\text{minimize} \quad \sum_{j=1}^{m} b_j Y_j$$

$$\text{subject to} \sum_{j=1}^{m} a_{ij} Y_j \geq c_i$$

$$i = 1, \ldots, m \qquad (15)$$

In the primal we are allocating scarce resources, the b_j's (number of fish in the run, cannery capacity, egg complement, and male fish), among the i activities (which consist in our problem of catching a particular entity of fish on a particular day). The intensity of the activity is X_i, the catch of the ith entity on the jth day and the "profit" per unit of the activity is, of course, c_j. It might be mentioned at this point that most LP computer codes provide as output the value of slack variables which in (14) is the difference between the right-hand side and the left-hand side of the constraint equations. The slack variables have a rather important interpretation for the salmon problem in that the slack variable in the run constraint (2) is the escapement; in the cannery constraint (3), it is the unused capacity of the cannery which we might want, in viewing the problem from a different context, to minimize; in the egg constraint (7), it is the number of eggs which are not caught that could be caught —another quantity which we might wish to minimize—and finally we have the slack variable associated with constraint (10) denoting the number of males which are not caught, but could be caught, and which we might, similarly, wish to minimize. Thus, for example, we simultaneously derive, by virtue of the LP model, as we have formulated it, *both* the escapement and the catch.

Now, in the dual, we can place unit values on the scarce resources rather than on the levels of the activities, as in the primal. It is thus helpful, in making management decisions, to know the imputed unit value of a unit of cannery capacity, a large male fish, an egg, etc. These imputed values are commonly known as shadow prices and correspond to the optimal values of the Y_j's in (15); they will be discussed briefly in our interpretation of the salmon model. It should also be mentioned that we have slack variables in the dual formulation just as we had slack variables in the primal.

The dual slack variables can be viewed as opportunity costs in the sense that if we fail to meet a constraint, this is an opportunity foregone; and the dual slack variable then gives the value foregone by the "bad" management either of nature (that is, the vagaries induced in the system which are uncontrollable by the management agency) or of the management agency.

Finally, it is worth noting a feature of the shadow prices vis-à-vis the relation of the right- and left-hand side of the constraint equations. If in some solution of a particular problem, the right-hand side becomes equal to the left-hand side, then we say the constraint is binding. If the constraint is binding, then the shadow price has some positive value, namely the imputed value of an additional unit of scarce resource; but if, on the other hand, the constraint is not binding, then an additional unit of the resource is "free" within the bounds of the problem formulation—consequently the shadow price of the free resource is zero.

EXAMPLES BASED ON THE NAKNEK-KVICHAK RUN

As an example, we have decided to consider the implications of a LP approach to implementing the management framework of one of the most important salmon runs in North America, the sockeye salmon run to the Naknek-Kvichak system of Bristol Bay, Alaska. Our approach was to use the LP model described in the previous section employing actual data where available for the constraints and the objective function. Although we examined behavior of the model for several of the years for which we had data, we are presenting in this paper our partial analysis for the 1960 run only. Initially, we indicate how we assigned values to the various coefficients in the problem and then we give the actual examples.

First, we assigned values to the objective function (1) which is to be maximized. With respect to the number of entities in the objective function, there is a relatively large number of ocean-age groups represented in the Naknek-Kvichak run, but the very great majority are either the relatively large .3 ocean-age fish or the relatively small .2 ocean-age fish. Because, as we will see in subsequent paragraphs, the male fish are valuated differently than the female fish, we used four entities: male or female, .2 or .3 ocean-age fish.

Next, in order to assign values c_{ij} to each entity in the objective function according to the conventional per-fish management unit, we used the aforementioned observation that the male fish in each age group tend to be larger and hence more valuable than the female fish. On the other hand, the eggs which are contained in the females are processed into a caviar-type product, "sujiko." by Japanese firms in the Bristol Bay canneries. Thus, the females, because of the eggs which they contain, are more valuable than males of the same weight. Taking these factors into consideration and using an average ex-vessel value of $0.25/pound, we have computed the average value for each entity. These calculations are set forth in Table 2 which shows, among other things, that the added value of eggs tends to offset the reduced value of females relative to males of the same age class.

As indicated previously, the $0.25/pound is an *average* value and it should be emphasized at this point that it is not a computed average since generally speaking a fixed price is paid for fish throughout the season. But as we indicated earlier, some fish are certainly more

TABLE 2.—Computations used to determine the value of each entity classification for the model.

Entity	Sex	Age	Average weight[1]	Average no. of eggs[2]	Average value $\bar{c}_i$
			Lb.	*Number*	*$*
1	Male	2-ocean	5.1	--	1.28
2	Male	3-ocean	7.4	--	1.85
3	Female	2-ocean	4.5	3,700	1.38
4	Female	3-ocean	6.2	4,384	1.84

[1] With the price of salmon at $0.25/pound, the value of each entity can be calculated as follows:

entity 1 — 5.1 X $0.25 = $1.28;
entity 2 — 7.4 X $0.25 = $1.85;
entity 3 — 4.5 X $0.25 = $1.13;
entity 4 — 6.2 X $0.25 = $1.55.

[2] With the price of salmon eggs at $0.50/pound, the additional value of each female entity can be calculated as follows (it should be noted that eggs were not processed in 1960):

entity 3 — 3,700 eggs X .061 g/egg ÷ 453.6 g/lb X $0.50 = $0.25

entity 4 — 4,384 eggs X .061 g/egg ÷ 453.6 g/lb X $0.50 = $0.29.

valuable than others. Thus, we deduced that this fixed price must reflect an average value and we note, parenthetically, the important point that the bias and precision (we take the liberty of using these terms in the statistical sense even though the estimation procedure may not be statistical in nature) with which this average is estimated is a subject of significance to the management of the salmon stocks.

In addition to the value of salmon differing among entities, the value of salmon usually deteriorates within an entity during a season. Thus, even though a fixed price is paid for salmon during a season, the value decreases owing to a reduction in quality. For example, the value of pink salmon may be 25 % less near the end of the run than near the beginning of the run. The decline in value of red salmon is not so severe, amounting to a range of about $0.03/pound from the beginning to the end of the season. (While a few cents decline in value during the course of the season may seem to be a negligible quantity, we must remember that this factor must be multiplied by the several pounds in weight of each fish and the several million fish that are involved in the value reduction.) So just as we deduced $0.25/pound to be an average price among entities, we must likewise deduce that the values tabulated in Table 2 are average values for each entity for the season. In the Naknek-Kvichak run, which usually begins around June 27 and ends around July 15, the decline in value appears to be centered on July 4.

In order to arrive at a unique allocation, we must deduce how the c_{ij}'s for each of the j days of the run differ from the average of the c_{ij}'s listed in Table 2. The ideal way of doing this would be to develop a model which is descriptive of the value change during the run. Unfortunately, we have no information upon which to base such a model, so we used three arbitrarily chosen functions to describe the day-to-day value change of the salmon. An example of these is shown in Figure 1.

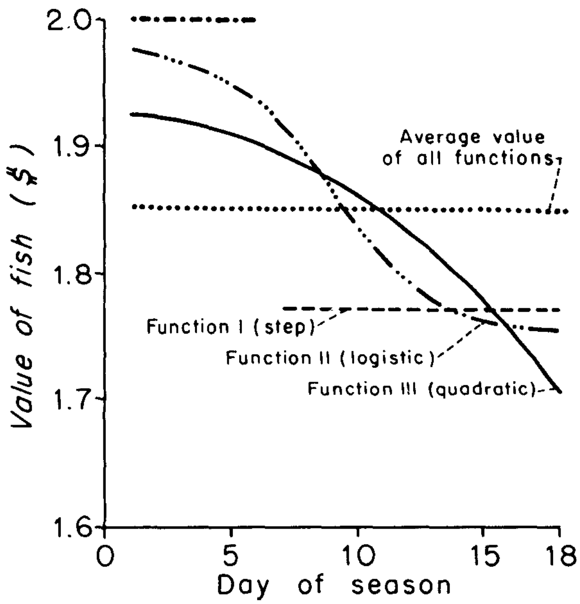

FIGURE 1.—Value functions as objective function coefficients in the linear-programming model for entity 2 (large male fishes).

Next we needed to determine the quantity, in the objective function, of N, the number of days of the run. We utilized an empirical equation presented by Royce (1965) to obtain both N and the daily run for each entity R_{ij}. The function Royce used for each of several years to describe the temporal change in the Naknek-Kvichak catch is

$$P_k(j) = \frac{1}{1 + e^{-(a_k + b_k j)}} \tag{16}$$

where k specifies the year of the run, a_k and b_k are constants, j is time in days, and $P_k(j)$ is the cumulative proportion of the total catch. In order to choose N, we defined the fishing season to be those days for which $.05 \leqslant P(j) \leqslant .95$ and the difference between the initial and closing day was, to the nearest integral value, set equal to N.

We also used expression (16) to obtain R_{ij} from $P'_k(j) = P_k(j) - P_k(j-1)$ and then $R_{ij} = P'_k(j)\ S_i$, where S_i is the number of fish of the ith entity in the run during the year under consideration. The assumptions involved in obtaining the R_{ij}'s are (1) the catch is proportional to the "average" run as defined by the fitted curve (16); and (2) R_{ij} is proportional to $R_{.j}$. It should also be mentioned that our R_{ij}'s are certainly different from the traditionally used R_{ij}'s because the latter are based on counts made several days after the fish enter the fishery area. This, however, is not important in the allocation whereas the relative daily size of the run is important. We feel that these assumptions are reasonable for the present model until more accurate information can be obtained on the behavior of the fish in the run.

On some occasions, the catch in Bristol Bay is limited by the cannery capacity. This capacity can be, of course, adjusted by the industry, but it appears that, according to Mathews (1966), 1 million fish per day is a maximum capacity and we used this value in constraint equation (2). It would appear that the most important assumption implicit in the nature of this constraint is independence among the days; that is to say, we assume that processing a certain number of fish on one day does not affect the processing capacity on the next day. A second assumption is that the maximum capacity is not dependent upon the average size of the fish processed. There is some question, when the cannery operates near peak capacity, as to the effect of overtime payments to cannery employees and to the effect of processing large numbers of fish on the quality of the pack.

For our example, of the 1960 run, the total season constraint was not reached and so this constraint had no effect on any of the examples which we present. It is relevant to note, however, that if this constraint is needed, then the maximum number of fish ever processed (up until 1969) in the Naknek-Kvichak system was 19.1 million.

The next constraint is the escapement constraint. Despite the fact that the level of escapement has, for salmon stocks, been the primary management criterion, there is very little documentation on the proper escapement level for many systems. The latest synthesis of the extensive work on the Naknek-Kvichak system is addressed to the problem of optimum escapement in that particular system as well as others (Burgner, DiCostanzo, Ellis, Harry, Hartman, Kerns, Mathisen, and Royce, 1969), but unfortunately no advice on optimum escapement is given for the Naknek-Kvichak. Another study implies, on the basis of limited data, that escapements beyond about 10 million fish will not result in any increased productivity of smolts (Mathisen, 1969: Fig. 6), and thus if we make a simplifying assumption and assume that marine survival rates are independent of density, we can further assume that an increased return will not accrue from an escapement larger than 10 million fish (very roughly 20 billion eggs).

Despite the fact that we do not "know" the optimal minimal escapement for the Naknek-Kvichak system, it is clear that there must be some level which is, in some sense, optimal. This statement must, of course, be tempered with our knowledge of the cyclical nature of the run.

Because of our uncertainty, we examined the model under a variety of escapement conditions but keeping under each set, what would appear to be, according to studies by Mathisen (1962), a conservatively high male-to-female sex ratio of at least one male for every three females. We found in general, as one might expect, that as we increased escapement we decreased the value of the catch. The objective function was quite sensitive to this manipulation, focusing again upon the need for a well-defined escapement policy. What is not so obvious, however, as we will see subsequently, is that as we change the escapement level, we can obtain considerable changes in the imputed values of the various entities. The 1960 run had an escapement of 31 billion eggs. We estimate that this escapement was partitioned among the entities as follows:

Entity 1, 57.5 % or 8,136,000 fish escaping
Entity 2, 43.4 % or 210,000 fish escaping
Entity 3, 74.8 % or 7,964,000 fish escaping
Entity 4, 30.8% or 389,000 fish escaping

It is interesting to note that in addition to 8,350,000 females which were estimated to have escaped in 1960, there were, in addition, 8,300,000 males that escaped, signifying a nearly 1:1 sex ratio.

Our general approach in presenting our particular 1960 run example was to concentrate upon simulating the actual events in 1960 by using the above run data in equations (13) and (7), the fecundities listed in Table 2, and the logistic value function. We also present some results where we effectively drop equation (13) and replace it with equation (10) using a low escapement of 5 billion eggs in order to demonstrate the sensitivity of the model to various escapement goals.

In Figures 2, 3, 4, and 5, we depict the optimum allocation of the catch which we obtained using the actual 1960 escapement data. The figures also include the smoothed catches derived from Royce's work. From these figures,

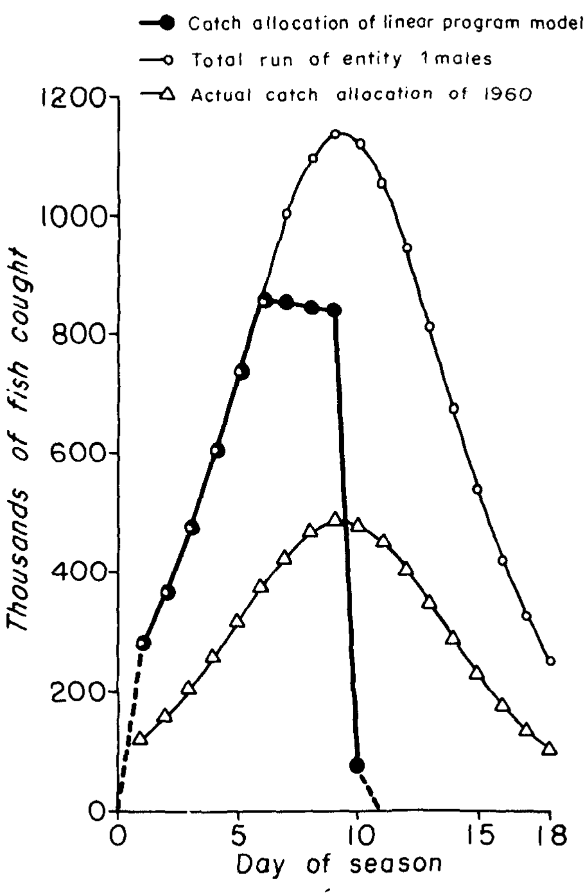

FIGURE 2.—Year 1960 small male catch allocations (entity 1). The actual catch allocations are the smoothed average values obtained from Royce (1965).

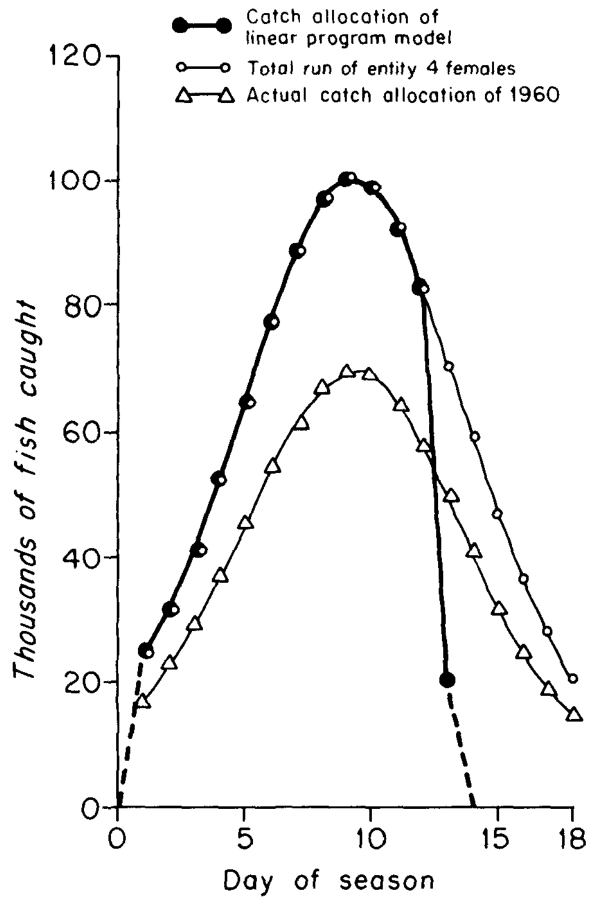

FIGURE 3.—Year 1960 large female catch allocations (entity 4). The actual allocations are the smoothed average values obtained from Royce (1965).

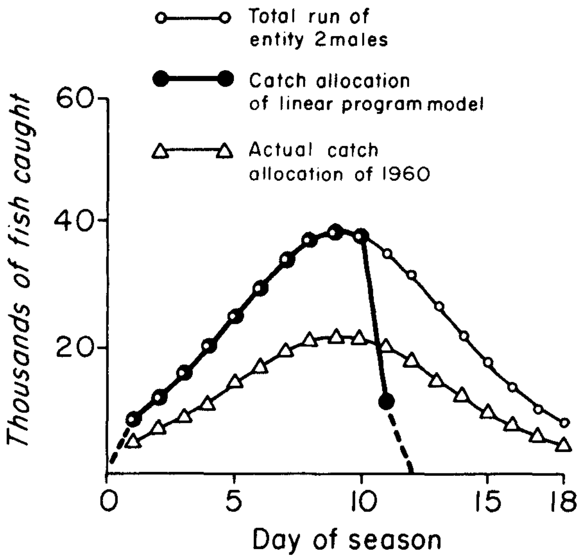

FIGURE 4.—Year 1960 large male catch allocations (entity 3). The actual catch allocations are the smoothed average values obtained from Royce (1965).

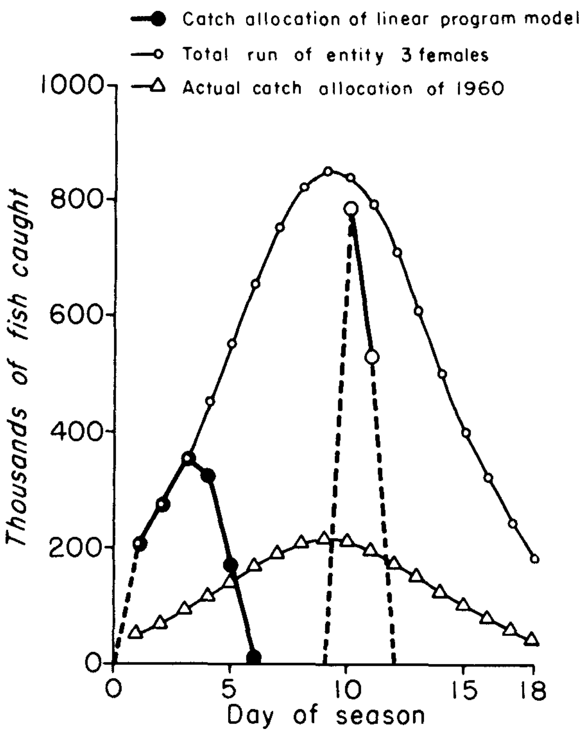

FIGURE 5.—Year 1960 small female catch allocations (entity 3). The actual catch allocations are the smoothed average values obtained from Royce (1965).

we can observe various properties of the optimally allocated catches. For example, in Figures 3 and 4, the catch allocation of the early season is identical to the number of fish in the run. These large male and large female fish are more valuable than the smaller fish and most valuable in the early part of the season because of the value function. Hence, all the large fish available are caught until the season's limit for each of the large-fish entities has been reached. We must be cautious here, however, because as we point out in our discussion, there is a possibility of modifying the characteristic of the run by selective fishing.

In order to interpret Figures 5 and 6, we must realize that the next most valuable fish are the small females. The difference in price between small females and small males increases through the season. This owes to the fact that small females weigh less than small males and thus decrease in value less towards the end of the season. (Recall that the small females are more valuable than small males only because of the eggs which they contain.) We have assumed that the value of the eggs is constant throughout the season.

The optimal solution shows that the cannery is at capacity from days 4 to 10 with as many small males being caught through day 6 as are available, and as many additional small females being caught as the cannery can process. On days 7 to 9 there are enough fish exclusive of the small females to bring the cannery to capacity. However, by day 10, the season's limit of small males has been caught and small females must be caught to keep the cannery at capacity. It must be remembered that the price difference between females and males is greater later in the season. For example, if a small female is caught instead of a small male on the 6th day of the season, the increased profit is \$1.430 — \$1.337 = \$0.093, where the figures are the value of small females and small males, respectively, on day 6 for the logistic value function which we used. But if a female is caught instead of a male on day 10, the increased profit is \$1.371 — \$1.270 = \$0.101, for the same value function. Hence, while the cannery is being operated to capacity and since the total number of fish of any entity is fixed by the seasonal limit, it is more profitable to catch the small, less valuable, males earlier in the season, which may seem contrary to the intuition. The implications of the optimal allocation are considered in the discussion and conclusions section.

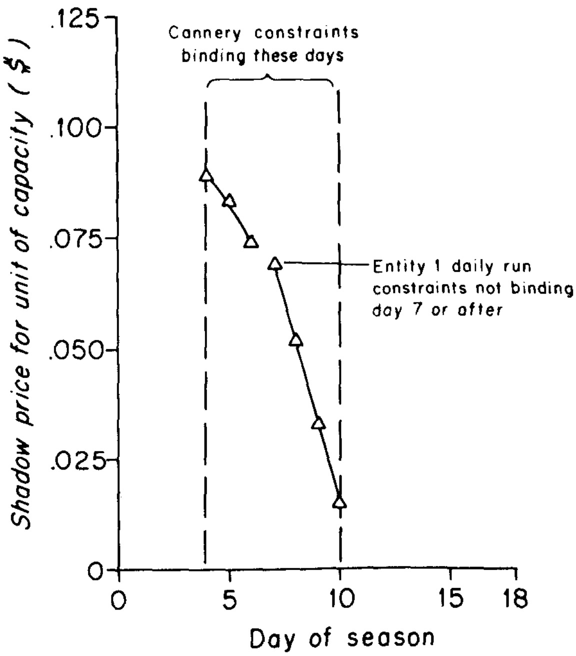

FIGURE 6.—Shadow prices for daily cannery constraints —an example in which seasonal entity limits are imposed.

As indicated previously, the shadow prices are useful in considering various management implications. We consider, as examples, the egg shadow prices, the cannery shadow prices, and the run shadow prices. It might be mentioned somewhat parenthetically that although the shadow prices can be explained and interpreted as in the following paragraphs, in the LP calculation, they are not found in this manner. The shadow prices are calculated as simultaneous results of an iterative solution procedure and include the results of previous iterations. In fact, the shadow prices associated with each constraint at the end of each iteration are used to determine how to manipulate the matrix to improve the objective function in the subsequent iteration. With very involved problems, it might not be possible to examine the shadow prices as below, and in any case, only a good deal of insight into the problem permits their delineation in this manner. One other caution is that while applying the following equations to determine a total increase in profit, care must be taken to see that the same constraints remain binding or nonbinding. Once a constraint changes from binding to nonbinding, the solution basis changes, also changing the relationships between variables and constraints.

The increase in value of the objective function corresponding to a relaxed egg constraint (allowing one more egg in the catch) is the shadow price associated with the egg constraint. The shadow price is dependent upon which, if any, of the constraints in the LP model are binding. If the egg catch constraint[4] is not binding, indicating that the value of this catch scheme is not being limited by this constraint, then the shadow price associated with the egg constraint is zero. If the egg catch constraint is binding, shadow prices associated with the egg constraint, depending upon which of the other constraints are effective, can be calculated.

The imputed value of an egg, its shadow price, thus depends on whether the cannery constraint is binding. Now, if the cannery constraint is binding on day j (indicating that the maximum number of fish are being processed and the addition of a single or marginal fish to the processed catch requires that a fish already existing in the catch must, to maintain the constraint, be replaced by the marginal fish), and the entity 4 run constraints are binding throughout the season (indicating that all large females are caught), then the shadow price associated with the egg catch (ESP) constraint is

$$ESP = (c_{3,j} - c_{1,j})/3{,}700$$

where day j is a day on which the entity 3 run

[4] Although the egg constraint arose from a minimum egg escapement requirement, it was necessary to convert it to a maximum egg catch requirement constraint for use with the LP model, as was described earlier. It is convenient to think of the constraints in terms of "egg catch" for purposes of discussing the shadow prices.

constraint is not binding, i.e., there are entity 3 females available to be caught. The formula indicates that allowing the catch of one more egg essentially allows the catch of 1/3,700 of an entity 3 female which requires the escapement of 1/3,700 of some other fish since the cannery constraint is binding. The fish to be included in the escapement is, of course, the low-valued entity 1 male.

If the cannery constraint on day j is binding, but there is a day when the entity 4 run constraint is not binding, and since the value of a large female per egg is greater than the value of a small female per egg, the value of allowing the catch of an additional egg is

$$ESP = (c_{4,j} - c_{1,j})/4{,}384$$

where day j is a day on which the entity 4 run constraint is not binding.

If the cannery constraint is not binding on day j then catching additional females does not require the escapement of an equal number of males, or, in other words, the addition of the marginal fish does not require the release of a fish extant in the catch. If, however, all the entity 4 run constraints are binding through the season, then

$$ESP = c_{3,j}/3{,}700$$

where day j is a day on which the entity 3 run constraint is not binding. Or, if an entity 4 run constraint is not binding on day j and again since the value of a large female is greater than that of a small female,

$$ESP = c_{4,j}/4{,}384$$

The next shadow price that we will evaluate is the increase in value of the ability to process an additional or marginal fish. The imputed value of processing a marginal fish is called the shadow price of the cannery constraint (CSP), and we must remember that this marginal fish only has an imputed positive value if the cannery constraint is binding. In other words, we can impute a value to an additional unit of cannery capacity. Following the format above, the shadow prices for the cannery constraint can be outlined. For emphasis, we repeat again that if the cannery constraint is not binding, indicating that the value of this catch scheme is not being limited by this constraint, then the shadow price associated with the cannery constraint is zero. If the cannery constraint is binding, shadow prices associated with the cannery constraint, given which other constraints are effective, can be determined.

The objective function will be increased by an amount equal to the value of the additional fish which is included in the new catch scheme (the scheme arising from relaxing the cannery constraint), and hence the most valuable fish will be caught. If run constraints are not binding, the shadow price is

$$CSP_j = c_{2,j}$$

since the large males are the most valuable. As the run constraints become binding for the more valuable entities, the shadow price of the cannery constraints is equal to the value of the most valuable entity available.

If, for example, the egg catch constraint is binding as well as the entity 2 run constraint, the shadow price of the cannery constraint is

$$CSP_j = c_{4,j} - \left(\frac{4{,}384}{3{,}700}\, c_{3,j}\right).$$

The modification of the equation assures that enough eggs will escape (via entity 3 female) to enable the catch of the more valuable entity 4 female.

To emphasize the effect of reduced escapements and concomitant binding egg catch constraints, we employed the 1960 run model but dropped equation (13) and used an effective escapement of 5 billion eggs in equation (10). If the egg catch constraint is binding as well as entity 4 run constraints, the only fish available for catching are the entity 1 (small males), since no more females can be caught without violating the egg catch constraint. Hence

$$CSP_j = c_{1,j}.$$

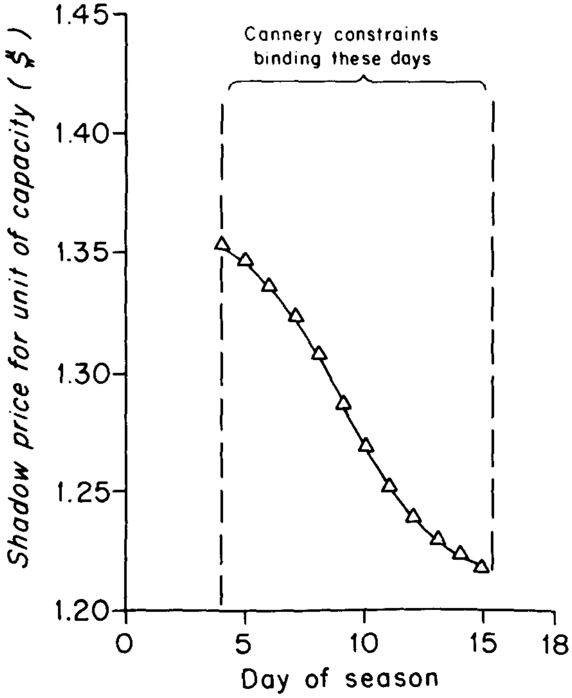

FIGURE 7.—Shadow prices for daily cannery constraints —an example in which seasonal entity limits are not imposed.

As an example of this situation where we have an escapement of 5 billion eggs, consider Figure 7, where the curve representing the cannery shadow prices is exactly the value function curve for entity 1 males. If, however, we contrast the 5 billion egg constraint situation with the actual 1960 run where the seasonal limit constraints are binding for all entities, then the improvement in the objective function attributed to the one unit of increased cannery capacity will only be equal to

$$CSP_j = c_{ij} - c_{ij^*}$$

since to catch a fish of entity i on day j another fish of entity i must be released on day j^* $(j^* \neq j)$ to avoid breaking the seasonal limit constraint. Figure 6 gives an example of this situation. The particular example is taken from the LP model using the logistic value function and actual escapements of 1960 by entity (thus the seasonal limit constraints). In this case, the total number of fish processed remains the same (since season limits are binding for all entities already), but processing a fish earlier in the season can result in an increased profit because of the shape of the value-function curve.

The implications of Figure 6 are quite subtle. In Figure 6, until day 4, the cannery is not at capacity, and hence there is obviously no value associated with an increased unit of capacity. All fish are included in the catch allocation in the early-season, high-value situation. On day 4 the cannery is at capacity and some fish must be included in the escapement (excluded from the catch). Intuitively, we would expect the highest value fish to be caught. This is partly reflected by the continued catch of all entities 2 and 4 fish (the large males and females), but, keeping in mind the fact that catches in all entities are being limited by the seasonal limit constraints and the idea of the decreasing price differential between entity 3 and entity 1 fish, entity 3 fish become part of the escapement. This says that on days 4 to 6, an increase in the capacity of the cannery will result in an entity 3 fish being caught on that day and a less valuable entity 3 fish released later in the season (to avoid breaking the season's limit constraint). We can see in Figure 5 that the last day on which an entity 3 fish can be released is day 11. The value of an entity 3 fish on day 4 is $1.445, the value of an entity 3 fish on day 11 is $1.356. Hence we would expect to gain exactly $0.089 by increasing the cannery capacity by one unit on day 4. Checking Figure 6, we see this is exactly what the graph shows for day 4. Days 5 and 6 can be calculated similarly.

On days 7 to 10, entity 1 daily run constraints are no longer binding, and hence an additional unit of cannery capacity could result in an entity 1 fish being caught on day 7. From Figure 5 the least valuable entity 1 fish in the catch scheme is on day 10, and one of these would go into the escapement. The increased value is

$$c_{1,7} - c_{1,10}$$

or

$$\$1.324 - \$1.270$$

or a profit increase of $0.054. In Figure 6, the shadow price of the cannery constraint on day 7 is shown as \$0.069. Note, however, that when an entity 1 fish is released on day 10 that the cannery is no longer at capacity on day 10, hence an entity 3 fish could be caught on day 10 if an entity 3 fish was released on day 11. This is an additional contribution to the shadow price for day 7 of

$$c_{3,10} - c_{3,11}$$

or

$$\$1.371 - \$1.356$$

or a profit increase of \$0.015. Adding this to the value of catching an earlier entity 1 fish gives \$0.054 + \$0.015 = \$0.069, and this is exactly what is shown for day 7 in Figure 6. Values for days 8 to 10 can be calculated similarly.

Next let us consider the run shadow prices (*RSP*). The increase in value of the objective function corresponding to relaxed run constraints allowing one more fish of any entity in the catch is the shadow price of that run constraint. If the cannery constraint is not binding, if seasonal run constraints are not used or are not binding, and if egg catch or male escapement constraints are not binding, there is no restriction other than the run constraint limiting the catch. Since there is a run constraint for each entity and day

$$RSP_{ij} = c_{ij}.$$

However, if the cannery constraint is effective on day j

$$RSP_{ij} = c_{ij} - c_{i^*j}$$

where entity i^* ($i^* \neq i$) is the fish that must escape to allow the catch of entity i since the cannery is already processing as many fish as possible. And if the egg catch constraint and/or the male escapement constraint is binding, appropriate numbers of fish must be released when additional fish are caught. As an example of the behavior of the system when the 5 billion egg constraint is used, consider Figure 8. As shown in Figure 8, daily cannery constraints are binding from days 4 to 15, and, in addition,

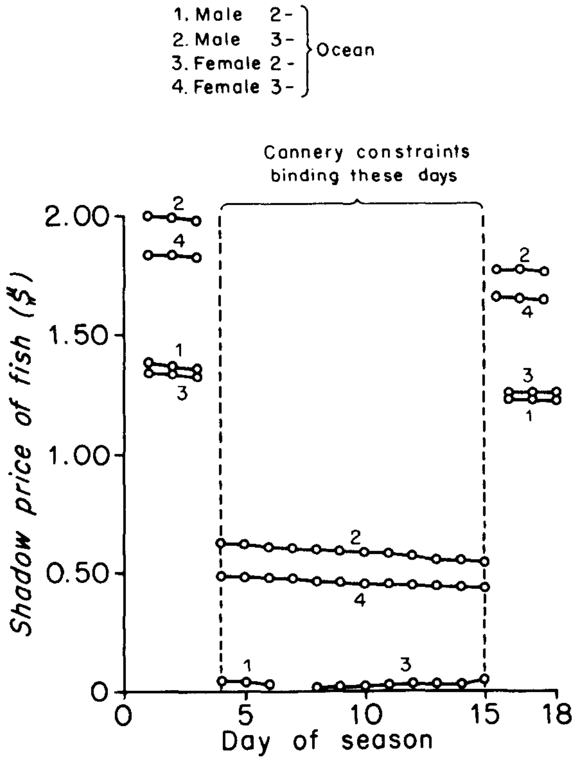

FIGURE 8.—Shadow prices for daily entity run constraints—an example in which seasonal entity limits are not imposed.

the egg catch constraint is binding. Although it is not illustrated in the figure, it is essential, in understanding the problem, to realize that the entire escapement satisfying the egg escapement constraint is made up of small (entity 3) females, and that the entire escapement satisfying the male escapement constraint is made up of small (entity 1) males. Also, all fish of all entities are in the catch scheme on days 1 to 3 and 16 to 18 (all days on which the cannery constraints are not binding).

In Figure 8, on days 1 to 3, the cannery constraints are not binding; and since entity 2 escapement is not being used to satisfy any constraints of any form, the run shadow prices on days 1 to 3 will be equal to the value function (this can be checked with the values listed in Appendix Table 2). In addition, since the male catch constraint is not binding, the run shadow price for entity 1 is also equal to its value function for the first 3 days of the season. This is not true for entity 3 or 4 since the egg-catch constraint is binding, indicating that no more eggs can be "caught" without breaking the constraint. Hence, when an entity 4 female is caught on a day 1 to 3 (catching 4,384 eggs), 4,384 eggs must be released on some other day of the season. This can be done with the smallest loss by releasing 4,384/3,700 entity 3 females (entity 3 females have 3,700 eggs). To further decrease the loss, the above fractional parts of an entity 3 should be released on a day on which there are entity 1 or 2 available to be caught, since this will keep the cannery constraint intact and not violate the male catch constraint since it is not binding. Remembering that the price differential between entity 1 and entity 3 increases over the season, it is desirable to catch the additional entity 1 fish as early as possible, which is on day 7. Then (note that all of the larger males are already in the catch scheme):

$$\begin{aligned} RSP_{4,1} &= c_{4,1} - \frac{a_4}{a_3} c_{3,7} + \frac{a_4}{a_3} c_{1,7} \\ &= \$1.941 - \frac{4{,}384}{3{,}700} \; (\$1.419) \\ &\quad + \frac{4{,}384}{3{,}700} \; (\$1.324) \\ &= \$1.828, \end{aligned}$$

which can be seen on the graph as the shadow price for entity 4 day 1. To calculate the value of the run shadow price for entity 3 in days 1 to 3, we must realize that catching an entity 3 female requires the release of a female some other time during the season in order to avoid breaking the egg catch constraint. Since the entity 3 females are of lesser value than the fractional part of an entity 4 female which must be released to account for the 3,700 extra eggs caught, the release of this fish on a day on which there are entity 1 fish available will lessen the loss. As before, the price differential between entities 1 and 3 is smaller early in the season, and the earliest date on which an entity 1 male is available is day 7. For example,

$$\begin{aligned} RSP_{3,1} &= c_{3,1} - c_{3,7} + c_{1,7} \\ &= \$1.453 - \$1.419 + \$1.324 \\ &= \$1.358, \end{aligned}$$

which is the value shown in Figure 8.

The run shadow price for entity 1 day 4 results from the catch of entity 1 on day 4 requiring the release of entity 3 (to avoid breaking the cannery constraint) which, in turn, allows the catch of an entity 3 and the release of an entity 1 on day 11 (the last day on which there are entity 3's *not* in the catch scheme). Essentially, what we have done is to exchange the catch of entity 1 and entity 3 for a time when the price differential is more favorable.

$$\begin{aligned} RSP_{1,4} &= c_{1,4} - c_{3,4} + c_{3,7} - c_{1,7} \\ &= \$1.353 - \$1.445 + \$1.419 - \$1.324 \\ &= \$0.003, \end{aligned}$$

which is the value shown in Figure 8. Run shadow prices for entity 1 days 5 and 6 can be figured similarly. For entity 1 days 7 to 15, the daily run constraints are not binding, and hence, the run shadow prices are zero.

Run shadow prices for entity 2 days 4 to 15 (still referring to Figure 8) can be calculated as the difference between the value of an entity 2 and entity 1 on those days. The entity 1 must be released in order to satisfy the cannery constraints for those days. In addition, for days 4 to 6, calculating the value of this new availability of an entity 1 released in order to catch an entity 2 is essentially the same situation as calculating the shadow price of entity 1 on those days, and hence, the value of the scheme and the run shadow price is increased by that additional amount.

Run shadow prices for entity 3 days 4 to 7 in Figure 8 are zero since the run constraints of entity 3 for those days are not binding. For entity 3 days 8 to 15, they can be calculated as follows: If another entity 3 is caught on day 8, an entity 1 must be released on day 8 to maintain the cannery constraints. The catch requires the escape of an entity 3 fish on another day to maintain the egg catch constraint. In turn, if the release of the entity 3 is on a day which a male entity is available, that entity can be caught without breaking the cannery constraint, e.g.,

$$RSP_{3,8} = c_{3,8} - c_{1,8} - c_{3,7} + c_{1,7}.$$

(Note that day 7 is the *first* day on which there is a male entity not already in the catch scheme, and hence, the price differential is smallest.)

$$RSP_{3,8} = \$1.404 - \$1.307 - \$1.419 + \$1.324$$
$$= \$0.002,$$

which is that value shown in Figure 8.

Run shadow price for entity 4 days 4 to 15 can be calculated by releasing and catching the fractional parts 4,384/3,700 of entities 3 and 1 as required to maintain cannery and egg catch constraints, in a manner similar to that for days 1 to 3. The run shadow prices for the male entities for days 16 to 18 (those days after the cannery constraints are no longer binding) are simply equal to the value of those entity-days, since they are not involved in satisfying any constraints of any form. The run shadow prices for the female entities on these days are a little more difficult to arrive at, since as they are caught, an equal number of eggs must be released, resulting in a slack cannery constraint which can be filled by catching additional male entities when available.

Consider, in contrast, Figure 9, where the run shadow prices are shown for a case in which we used the actual escapement for the 1960 run in constraint equation (13). The seasonal limit constraints are binding for all entities in this example. It follows that in every case when calculating a run shadow price for an entity day, the inclusion of an additional unit of any entity on that day implies that a unit of that entity must escape on some other day of the season to maintain the equality of the seasonal limit constraint for that entity.

In Figure 9, shadow prices are plotted for the daily run constraints for the particular example in which seasonal limits were imposed to achieve the actual escapements of 1960. Neither the egg catch constraint nor the male catch constraint was binding; the seasonal limit constraint was binding for each entity. Cannery constraints were binding from days 4 to 10, as indicated on the figure. In calculating the run shadow price for any entity on any day, note that whenever an additional fish is included in the catch scheme, a fish of that same entity must be eliminated from the catch scheme on some other day in order to maintain a valid seasonal limit constraint for that entity. Thus, for entity 1 days 1 to 3, the run shadow price can be calculated as the value of a fish included on the day being considered minus the value of the lowest valued fish of entity 1 in the catch scheme which must be released to keep the constraints

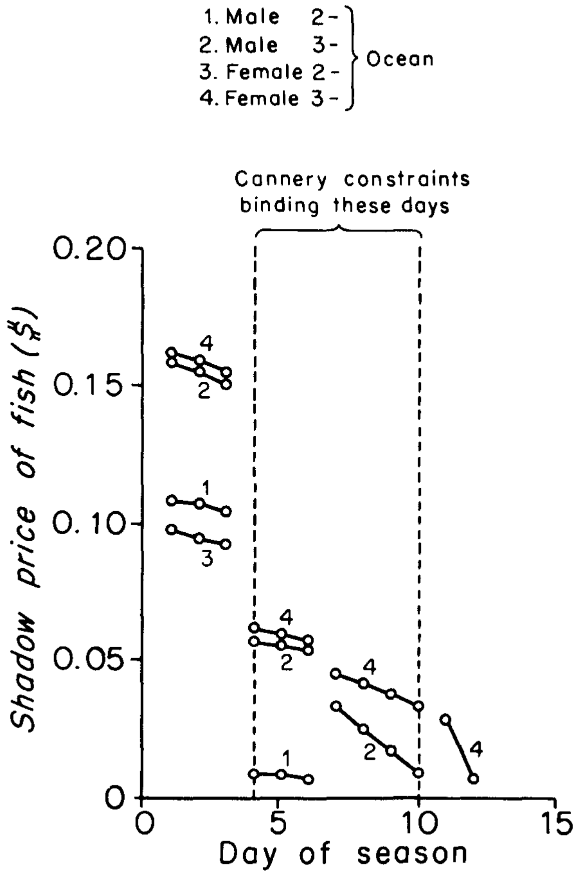

FIGURE 9.—Shadow prices for daily entity run constraints—an example in which seasonal entity limits are imposed.

effective. If this low-valued fish is released on a day on which the cannery constraint was binding, then the cannery constraint is no longer binding and a fish of another entity can be caught on that day if a fish of that other entity is available. For example,

$$\begin{aligned} RSP_{1,1} &= c_{1,1} - c_{1,10} + c_{3,10} - c_{3,11} \\ &= \$1.362 - \$1{,}270 + \$1.371 - \$1.356 \\ &= \$0.107, \end{aligned}$$

which is the value shown in Figure 9. The run shadow prices for entities 2 to 4 on days 1 to 3 are simply calculated as the value of the entity on that day minus the value of that entity on the lowest priced, and hence, last day on which it is included in the catch scheme. For example,

$$\begin{aligned} RSP_{3,1} &= c_{3,1} - c_{3,11} \\ &= \$1.453 - \$1.356 \\ &= \$0.097. \end{aligned}$$

Checking Figure 5, we see that day 11 was the last day on which entity 3 fish were caught, and Figure 9 shows that the value ($0.097) is correct.

Run constraints for entity 3 are not binding after day 3, so run shadow prices associative are zero.

To obtain the run shadow price for entity 1 for days 4 to 6, one may, for example, subtract from the value of entity 1 day 4, the value of entity 3 day 4 (which must be released to maintain the daily cannery constraints), subtract the value of entity 1 day 10 (which must be released to maintain the season entity 1 limit), and add the value of entity 3 day 10 (which can be caught since an entity 1 has been released on day 10); or

$$\begin{aligned} RSP_{1,4\text{-}6} &= c_{1,4} - c_{3,4} - c_{1,10} + c_{3,10} \\ &= \$1.353 - \$1.445 - \$1.270 + \$1.371 \\ &= \$0.009, \end{aligned}$$

which can be seen in Figure 9.

Run shadow prices for entities 2 and 4 for days 4 to 6 are similar. For illustration, entity 4 day 4 will be calculated:

$$RSP_{4,4} = c_{4,4} - c_{3,4} - c_{4,13} + c_{3,11}.$$

That is, the run shadow price of entity 4 day 4 equals the value of entity 4 day 4 less the value of entity 3 day 4 (to preserve the daily cannery constraint) less the value of entity 4 day 13 (to preserve the seasonal entity 4 limit) plus the value of entity 3 day 11 (since an entity 3 day 4 was excluded, this will be within the entity 3 seasonal limit constraint). Thus

$$\begin{aligned} RSP_{4,4} &= \$1.930 - \$1.445 - \$1.780 + \$1.356 \\ &= \$0.061. \end{aligned}$$

This is the value shown in Figure 9.

The run shadow prices for entity 1 after day 7 are zero since the daily run constraints are no longer binding. For entities 2 and 4 days 7 to 10, the run shadow prices can be calculated similarly. For example,

$$RSP_{2,10} = c_{2,10} - c_{3,10} - c_{2,11} + c_{3,11}.$$

That is, the run shadow price of entity 2 day 10 equals the value of entity 2 day 10 less the value of entity 3 day 10 (to preserve daily cannery constraints) less the value of entity 2 day 11 (the last day on which entity 2's are caught and, hence, the cheapest entity 2 which can be released to preserve the seasonal limit) plus the value of entity 3 day 11 (since an entity 3 was released on day 10, this will not fracture the seasonal entity 3 limit constraint). Thus

$$\begin{aligned} RSP_{2,10} &= \$1.836 - \$1.371 - \$1.812 + \$1.356 \\ &= \$0.009, \end{aligned}$$

which is shown as the correct value in Figure 9.

The run shadow prices for entity 4 on days 11 and 12 are easily calculated since the cannery constraints are no longer binding and this is the only entity being caught after day 12. Hence,

$$\begin{aligned} RSP_{4,11} &= c_{4,11} - c_{4,13} \\ &= \$1.808 - \$1.780 \\ &= \$0.028. \end{aligned}$$

It is interesting to note that the run shadow prices of entity 4 throughout the season are higher than those of entity 2 (see Figure 9), even though the value for an entity 4 is less, for any given day, than the value of entity 2. This is due to the optimal catch scheme which has catches of entity 4 until day 13, while entity 2 catches are only made until day 11. The effect is that when an additional fish of an entity is caught early in the season (requiring the release of a fish of the same entity later in the season), the entity 2 fish released is from day 11 of value $1.812, the entity 4 fish released is from day 13 of value $1.780. The value of the entity scheme then, decreases least (proportionately) for releasing the entity 4 fish.

DISCUSSION AND CONCLUSIONS

It can be seen that the linear-programming (LP) approach to salmon management, as with all other modelling approaches, involves a variety of assumptions which are either intrinsic to the procedure or to the way the procedure is applied to real-world problems. We have gone into some detail to show the richness of interpretations that the salmon model affords, and we believe that the application of this procedure to salmon management will provide increased guidance to and widen the spectrum of possible management decisions. The procedure we have used for the Naknek-Kvichak run is widely applicable to a variety of situations both within the Naknek-Kvichak sockeye salmon setting and to other salmon runs as well. The setting, the necessary data, and the formulation of the problem really depend on the problem situation. Our purpose was to demonstrate a conceptual method and we have chosen our data and examples accordingly.

There will naturally be differences of opinion in the formulation of the model (that is, different ways of expressing the constraint equations, some of which are indicated) or the appropriate data which should be used for actual management situations. These differences can at times be easily resolved by examining the sensitivity of the model to various data or formulation modifications.

Nevertheless, we should not ignore the assumptions which are implicit in the LP procedure. These are outlined by, for example, Hillier and Lieberman (1967), and constitute three concepts that should be recognized. These involve the linearity property of the model, the problem of divisibility, and the deterministic nature of the LP approach. In addition, it is important to consider that the LP approach models only static situations. First, the linearity property asserts, for example, that the value of any term in the constraint or objective function must be directly proportional to the level of activity involved. Expressed in another way, the relation between the level of an entity and its contribution toward filling the constraint or modifying the objective function must be a straight line passing through the origin, a condition not often met in practice but frequently approximated. Furthermore, there should be no synergism among the terms of the objective or constraint functions. For example, the unit catch value on day j for the ith entity, c_{ij}, cannot be affected, a posteriori, by the unit catch value on day j-1 for the ith entity $c_{i,\ j\text{-}1}$. The problem of divisibility refers to the fact that the LP approach that we have used gives solution values which are not necessarily integers. A usual practice is to round solutions to the nearest integer value, thus avoiding the embarrassment of having, e.g., 7,012,342.631 salmon. In other applications, such as allocating 10 fishing boats, say, to perhaps three fishing grounds, the possibility of having non-integer answers may lead to erroneous conclusions and one of the integer programming techniques would then be most appropriate. Next, the deterministic nature of the LP approach is, of course, a deficiency in the probabilistic real world. The manager must realize that a full stochastic treatment of the salmon allocation as an optimization problem would most likely be a very difficult task. As alternates, an error structure could be applied to various elements in the problem, thus enabling one to explore a variety of probabilistic phenomena, or chance-constrained programming might be employed. Monte Carlo and simulation approaches might also be utilized but these are not per se optimization procedures. Finally, the static nature of the LP approach provides a challenge to application in the sense that the unit values in the objective function, the constraints, and the right-hand sides of the constraint equations must not only be known in advance, but also must not change as a result of any of the allocations in the model.

The above assumptions can be handled in a variety of ways such as those indicated to handle problems of the deterministic nature. For example, we might, in some instances, use quadratic programming to handle the problem of non-linearity or dynamic programming or apply the outlined procedure in real time to handle the static nature of the programming problem, but unfortunately these approaches will present what can be quite complicated computational difficulties which may, in some instances, be insurmountable. It is thus clear that we have made certain approximations, trading off realism for an easily computable solution which certainly provides management guidance.

As we implied previously, we do not consider our departures from realism to seriously affect the utility of the model to provide guidance for decision making. Thus we believe that, for example, fixing the cannery capacity independent of the entities involved (or we could consider the cannery capacity to be fixed at a level which would accept a reasonable mixture of the entities) or using a simple average fecundity of the female entities to represent the average fecundity of the spawning females materially affects our conclusions. These, however, can be evaluated in direct applications by a sensitivity analysis.

Having outlined some cautions with respect to assumptions, we can now examine some of the indications provided by the various trials of the procedure. These involve the value of the fish on the dock, a reduction in processing-season length, changing value of entities during the run, and finally future data needs.

First with respect to the value of the total catch on the dock, we experimented with three value functions which set the daily value of each entity. Using the value functions to determine the value for each entity and day, and the actual distribution of the catch over the 1960 season, a total value of the catch was calculated which corresponds to the use of each of the three value functions. These values of the actual allocation of the catch were compared with the value of the optimal allocation as determined by the linear program as an indication of the value of optimally allocating the catch over the season. The increased value of the optimally allocated catch ranged from approximately $350,000 to $420,000 dependent on which value-function curve was considered. Table 3 shows these results. In the table, a fourth value function is indicated, which is simply a straight-line function such that the value of each entity remained constant through the season. Each of the other value functions was determined such that the average value of each curve was equal to the constant value for that entity for the season.

TABLE 3.—Comparison of the value of the optional allocation with the value of the actual allocation of the catch for the 1960 season.

	Value function 1[1]	Value function 2[2]	Value function 3[3]	Value function 4[4]
Optimal allocation	$13,787,050	$13,927,860	$13,792,555	$13,517,870
Actual allocation	13,378,650	13,506,250	13,439,825	13,517,890
Increased value	$ 408,400	$ 421,610	$ 352,730	$ [5] −20

[1] After day 6, the price dropped 3¢ per pound.
[2] The price was reduced by subtracting a logistic curve that reduced the price of each entity by 3¢ per pound over the season.
[3] The price was reduced by subtracting a quadratic curve that reduced the price of each entity by 3¢ per pound over the season.
[4] The price for each entity remained constant through the season (actual situation.)
[5] Difference due to rounding in the linear programming algorithm.

All three value functions had the effect of placing emphasis, in the optimal solution, on catching fish on the early days of the season. For two of the functions the value for any entity of fish on a given day is less than the value for that entity on the previous day. This is not true in the step function and thus we do not have a unique allocation, but rather a set of allocations under the high values and a set of allocations under the low values. But results are exactly the same; optimal allocations of fish are identical under the three value functions, although the total value of the catch changes somewhat, according to the exact shape of the value-function curve. Again, we emphasize that these gains from allocation can only be obtained by knowing in advance of the run the information that we actually used in the allocation and having the ability to select the entities in the run as they are selected in the allocation.

Next, an examination of the 1960 optimal allocation reflects that this optimal allocation not only increases the value of the fish on the dock, it also shortens the length of time which a cannery needs to operate. Thus, the same amount of fish could be processed in a shorter period of time, by the same labor force, etc. In the optimal allocation for the 1960 run, all of the fish could have been processed in the first 13 days of the season, 5 days less than the actual operation. Naturally, we need to assume that a policy of catching salmon only from the early part of the run would not affect the genetic constituency of the stock. Furthermore, we must be careful here because, as we have emphasized in several places, by our LP assumptions, we cannot, a priori, let the cannery operations on day j-1, for example, affect the cannery operations on day j and we cannot at least in our formulation allow operating at peak capacity to affect quality of the fish or overtime payments since the variables are external to our model.

Another indication is that the values of fish change during the course of the season and that these values change in rather subtle ways depending upon the "rules" that we set forth (e.g., contrast Figures 8 and 9) and that in the fishery the marginal value of less valuable entities in Table 2 can be greater than the more valuable entities in Table 2. These changes in values need to be acknowledged in any management scheme.

Thus, it appears that we have the opportunity to increase the economic efficiency of some salmon runs. This is, of course, not a new concept, having been treated in some detail by, for example, Crutchfield and Pontecorvo (1969). Our approach is slightly different, however, in that we have concentrated on optimality only from the point of view of increasing the value, as we have defined it, of the fish on the dock. Any full treatment of the management problem must, of course, consider the distribution of fishing effort and its ancillary fishing and economic implications.

Now if we accept the premise that conservation is "optimum" allocation of resources in the times-space stream (*cf.* Crutchfield and Pontecorvo, 1969), and if we observe that mathematical programming provides guidance for optimal allocation, and note that LP is a special case of mathematical programming, and suggest that the kinds of information required to allocate salmon among the days of the run in an LP model are not going to be much different from the kinds of information required for more sophisticated programming procedure, then we are led to the conclusion that perhaps we have not addressed ourselves to asking, in our research, the "right questions" concerning salmon management. Following our argument, it would then be implicit that the right questions are contained in our formulation of the LP model. These answers must be feasible to obtain and they would contain either needed data or documented policies which would be reflected in the right-hand side of the constraint equations and, more importantly, provide an opportunity for enlightened dialogue. There is unfortunately a cost associated with asking right questions. This cost involves the cost of doing new work, or that which inevitably results when existing research activities are reallocated. Are these costs worth the expenditure? These, however, are the kind of questions, the answers to which can be guided by the LP problem. For the salmon management model, we impute values to units of cannery capacity, etc., but, and perhaps of equivalent importance, we impute a value, in meaningful terms, to information. Thus, for our salmon problem, we have cleverly avoided indicating how we could catch X_{ij} fish for some i,j. But it is well known that catching can be approximated because it is possible to catch salmon in traps (although this has never been done to any large extent in Bristol Bay) and, upon visual inspection, to distinguish between large and small, male and female fish, and doing this by virtue of *ceteris paribus,* the allocative process, we could add about 0.5 million dollars to the value of the salmon on the dock. This is, of course, not the full picture, because we would have to trade off the added value of salmon (it is a common opinion that salmon caught in traps are of better condition and higher value than the salmon which are taken by gill netting, for example), the reduction in cannery days used to process the fish, the cost of building traps, and the political problems which are described in some detail in Crutchfield and Pontecorvo (1969). It would not, however, be difficult to determine the discounted present value of the various alternate procedures and thus evaluate the wisdom of engaging in any. In this evaluation, we need not be bound by what are perhaps extreme solutions such as traps, but we could examine the value of other selectivity procedures such as modifying gill net selectivity, etc. In general, then, we can evaluate the value of information by approximating that information, employing it in the model, and contrasting the change in the objective function with the objective function when the information is not in the model.

Additional information is needed on the pattern of the run. For the earlier years, this is available in Royce (1965), a publication which needs to be updated and implemented to obtain even rough estimates of the temporal movement of the fish of various entities through the fishery. This might be quite difficult to accomplish with present concepts, and the feasibility of a system which would acoustically monitor the passage of salmon through the entire river system and developing a central computer-oriented unit which would process the signals from all acoustic units and provide, in real time, through appropriate algorithms, rules for catching fish and making observations on escapement is presently being explored.

In our model, because of a lack of information, we used the total run and allocated this proportionately among the days of the fishery to determine the daily run. This emphasizes the need to have, for the purpose of management, a fairly accurate preseason guess of the total magnitude of the run and the X_{ij}'s. These guesses are already being made and the predictions need to be judged on the basis of whether the predictions do better than simply averaging the run for cycle years and simply averaging the run for noncycle years and applying these averages as predictions. The trick then may not be to estimate the average catch but rather to determine which years are cycle years.

We have included cannery capacity in a rather simple way in our model and this is a subject that also needs additional data since the cannery capacity constraint can be formulated in a variety of ways. It would be interesting to explore in a simulation setting the behavior of the slack variables in the cannery constraint. This is because it seems quite likely that there is a positive correlation between the cost of operating a cannery and the magnitude of the slack variable in the cannery constraint. If the run was constant from year to year, then it would be relatively easy to determine an optimal value for the magnitude of the slack variable in the cannery constraint. But the run varies considerably from year to year, and so in those years when the cannery constraint might be too low, we have an opportunity cost which appears as a slack variable in the dual formulation of the cannery constraint. It would seem then that the best value of the cannery constraint would be somewhere in between the capacity for a maximum run and a minimum run and that this might be investigated by employing the LP model in a simulation setting.

We have also employed egg and sex ratio constraints in our model. The egg constraints require information on fecundity and escapement. There is not much information on fecundity but this should be either easily obtainable or easily approximated. Again, the static nature of the LP problem makes it difficult to attribute a value to an egg for years in the future. This is, of course, important, emphasizing the need of thinking not, as is conventionally done, in terms of the forthcoming year, but rather in terms of, for example, a series of years maximizing (*cf.* Riffenburgh, 1969) economic benefits. In other words, the utilizers of resources may not be interested (even though they may think they are) in management on a year-to-year basis; rather, they are interested in some long-run satisfactory behavior of the time stream of economic benefits. Alternatively, though, we must be cautious of on-the-average management schemes which are typically presented in fishery applications. This is because a particular management scheme might be on-the-average quite profitable in the long run but might frequently completely bankrupt the system for the first 20 years of operation.

The problem of sex ratio is quite important because it appears that the objective function would be quite sensitive to selectively decreasing the number of males in the escapement and thus increasing the catch perhaps substantially. As indicated previously, Mathisen's study (1962) gives us some guidance on this subject and it would appear that, in some instances, the 3:1 ratio might be conservative. Furthermore, it should be mentioned that a year-to-year modification of sex ratio might be a useful cushion for approaching stability for some economic aspect of the fishery. Finally, the problem of escapement eludes us because in the wealth of literature on the subject there appears to be very little that is useful in setting the egg-minimum constraint. It is generally agreed that the stock-recruitment relation for salmon is the familiar Ricker-type curve. It is well known that the variability in these relations is quite large (in the case of the Naknek-Kvichak run, attempting to draw similarities between stock and recruitment places tremendous stresses on the imagination anyhow) and as a consequence, if the dome-shaped model holds, a minimum escapement set sufficiently, but not unreasonably high, could, on the average be reducing the return rather than increasing the return.

It might be difficult even after several years of setting the minimum escapement value at too high a level, to detect, owing to the variability in the system, the effect of this policy. If this is true, then again we are asking the wrong questions by studying the stock and recruitment model per se. We are faced with a system that is so variable, either intrinsically or in terms of measurement techniques, or both, that a large number of data points is required before we can evaluate the relation between the empirical data and the theory and then use the theory to predict. There is but one point a year and so we are asking nature to "stand still" for a large number of years. Given these observations and our past experience, we wonder whether it might not be more appropriate for management purposes to avoid looking at stock and recruitment per se, to intensify study of the physiology and behavior of very young stages of fish, and thus examine fundamental problems of cause and effect, vis-à-vis the variables that influence the magnitude of egg production and survival of these eggs and larvae or other young stages through the first several months of their life. And finally, in the meantime, would it be more appropriate to consider measuring stock and recruitment in terms of transition probabilities which might be estimated by computing the median stock and the median recruitment? Stock sizes which are below the median would be poor, those which are above, good, and similarly with recruitment. The empirical data could then be used to estimate probabilities of good-good, good-poor, poor-good, and poor-poor transitions. We need not in this procedure be restricted to medians, but could in fact use any fractile, and in fact we need not be restrained by fractiles because we might want to place the dividing line at some "optimal value" and explore the consequences.

In conclusion, then, we have formulated a LP model for salmon runs and have shown how it might be related to the Naknek-Kvichak run. We see in this relationship that, given information on the structure of the run, we can both increase the value of the fish on the dock and at the same time reduce processing time. Whether it is worth obtaining the information in terms of the indicated data and the ability to select fish from the run to approach this allocation and whether decreased processing time is, in fact, a saving, are questions that must be answered by the processing industry in light of the increased value of salmon on the dock. If our estimate of increased value is approximately correct, we can see that allocation can add an interesting value to the catch, but far greater additions could come from reducing the escapement, if this is possible, and alleviating the open-access related problems. Perhaps the most interesting feature of the model is the richness of interpretations that LP affords in the salmon situation and the nature of questions and data needs raised by the model. Finally, we emphasize that, as Hillier and Lieberman (1967) note, "A practical problem which completely satisfies all of the assumptions of LP is very rare indeed. However, the LP model is often the most accurate representation of the problem, which will yield a reasonable recommendation for action before implementation is required."

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APPENDIX TABLE 1.—Constants used in value-function equations for each entity and day of the 1960 run.

		Function 1[1]	Function 2[2]	Function 3[3]
Entity 1	IP_1	1.382	1.365	1.331
	b_1	.153	.153	.000417
Entity 2	IP_2	1.998	1.974	1.925
	b_2	.222	.222	.000583
Entity 3	IP_3	1.470	1.455	1.425
	b_3	.135	.135	.000472
Entity 4	IP_4	1.964	1.944	1.903
	b_4	.186	.186	.000694

[1] For entity i on day j,
Value $_{i,j} = IP_i$, for $j < 6$; Value $_{i,j} = IP_i - .03X$ (weight of entity i in pounds), for $j > 6$.

[2] For entity i on day j,

$$\text{Value}_{i,j} = IP_i - \frac{b_i}{1+e^{-(4.5+.5j)}}$$

[3] For entity i on day j,
Value $_{i,j} = IP_i - b_i X j^2$

APPENDIX TABLE 2.—Total run, total catch, and value functions for each entity and day of the 1960 season.

Entity	Day	Total run	Total catch	Value function 1[1]	Value function 2[2]	Value function 3[3]
1	1	280,351	119,149	1.382	1.362	1.331
	2	368,062	156,426	1.382	1.361	1.329
	3	475,118	201,925	1.382	1.358	1.327
	4	600,110	255,046	1.382	1.353	1.323
	5	737,489	313,432	1.382	1.347	1.319
	6	876,389	372,465	1.382	1.337	1.314
	7	1,000,833	425,354	1.229	1.324	1.308
	8	1,092,325	464,238	1.229	1.307	1.301
	9	1,134,780	482,281	1.229	1.288	1.293
	10	1,120,032	476,014	1.229	1.270	1.284
	11	1,050,967	446,661	1.229	1.253	1.274
	12	940,395	399,668	1.229	1.240	1.264
	13	806,344	342,696	1.229	1.230	1.251
	14	666,514	283,268	1.229	1.224	1.238
	15	534,438	227,136	1.229	1.219	1.225
	16	418,185	177,729	1.229	1.216	1.210
	17	321,003	136,426	1.229	1.215	1.195
	18	242,797	103,188	1.229	1.214	1.178
2	1	9,590	5,427	1.998	1.970	1.924
	2	12,590	7,126	1.998	1.967	1.922
	3	16,252	9,199	1.998	1.963	1.919
	4	20,528	11,618	1.998	1.957	1.914
	5	25,227	14,278	1.998	1.948	1.908
	6	29,978	16,967	1.998	1.934	1.900
	7	34,235	19,377	1.776	1.914	1.891
	8	37,365	21,149	1.776	1.890	1.881
	9	38,817	21,970	1.776	1.863	1.869
	10	38,313	21,968	1.776	1.836	1.856
	11	35,950	20,347	1.776	1.812	1.841
	12	32,168	18,207	1.776	1.792	1.825
	13	27,582	14,479	1.776	1.778	1.808
	14	22,800	12,905	1.776	1.769	1.789
	15	18,281	10,347	1.776	1.763	1.769
	16	14,305	8,096	1.776	1.759	1.747
	17	10,980	6,215	1.776	1.756	1.724
	18	8,305	4,700	1.776	1.754	1.700
3	1	211,080	53,192	1.470	1.453	1.425
	2	277,120	69,834	1.470	1.451	1.423
	3	357,724	90,146	1.470	1.449	1.421
	4	451,832	113,861	1.470	1.445	1.418
	5	555,267	139,927	1.470	1.439	1.415
	6	659,846	166,281	1.470	1.430	1.410
	7	753,542	189,892	1.335	1.419	1.405
	8	822,427	207,252	1.335	1.404	1.398
	9	854,393	215,307	1.335	1.387	1.391
	10	843,289	212,488	1.335	1.371	1.383
	11	791,289	199,404	1.335	1.356	1.375
	12	708,038	178,426	1.335	1.345	1.365
	13	607,110	152,992	1.335	1.336	1.355
	14	501,829	126,460	1.335	1.330	1.343
	15	402,387	101,401	1.335	1.326	1.331
	16	314,858	79,344	1.335	1.324	1.318
	17	241,688	60,905	1.335	1.322	1.304
	18	182,805	46,066	1.335	1.321	1.290
4	1	24,985	17,289	1.964	1.941	1.902
	2	32,803	22,700	1.964	1.939	1.901
	3	42,344	29,302	1.964	1.935	1.898
	4	53,484	37,010	1.964	1.930	1.894
	5	65,727	45,483	1.964	1.922	1.888
	6	78,106	54,050	1.964	1.910	1.882
	7	89,197	61,724	1.778	1.894	1.874
	8	97,351	67,367	1.778	1.874	1.866
	9	101,135	69,985	1.778	1.851	1.856
	10	99,820	69,075	1.778	1.828	1.845
	11	93,665	64,816	1.778	1.808	1.832
	12	83,811	57,997	1.778	1.792	1.819
	13	71,864	49,730	1.778	1.780	1.804
	14	59,402	41,106	1.778	1.772	1.789
	15	47,631	32,960	1.778	1.767	1.772
	16	37,270	25,790	1.778	1.763	1.754
	17	28,609	19,797	1.778	1.761	1.735
	18	21,639	14,974	1.778	1.760	1.714

[1] After day 6, the price dropped 3¢ per pound.
[2] The price was reduced by subtracting a logistic curve that reduced the price of each entity by 3¢ per pound over the season.
[3] The price was reduced by subtracting a quadratic curve that reduced the price of each entity by 3¢ per pound over the season.