SOME STATISTICAL CONSIDERATIONS OF THE DESIGN OF TRAWL SURVEYS FOR ROCKFISH (SCORPAENIDAE)

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ABSTRACT

This study is in two parts. The first part reviews statistical theory for choosing among random, stratified random, and systematic sample survey schemes when strata are of equal size and receive equal sampling effort. The theory is applied to data collected during a pilot trawl survey for rockfish in Queen Charlotte Sound, British Columbia, and a full scale survey along the coasts of Washington, Oregon, and California. The results indicate that on a scale of about 80 km, a systematic survey scheme provides more precise estimates than the other schemes. However, the differences in precision are slight and probably should not outweigh other factors such as logistical constraints in the design of trawl surveys. The second part of the study reviews statistical theory for sampling from negative binomial distributions. Results of the Queen Charlotte Sound pilot survey indicate that except for fish with very low densities, numerous tows of short distances are relatively more precise than fewer tows of longer distances for trawl surveys for rockfish.

The Fisheries Conservation and Management Act of 1976 requires the development of fishery management plans for each marine fishery under jurisdiction of the United States. The requirement emphasizes the need to assess the status of U.S. fisheries. Estimates of stock abundance are essential for fishery assessment, and trawl surveys often are used to estimate absolute or relative stock abundance where suitable data from a fishery itself are lacking.

Very little data are available from the complex fisheries for rockfish (genus *Sebastes*) of the Pacific coast of North America. The fisheries are complex because many species and types of gear are involved. Often landing statistics do not specify species, and catches are seldom sufficiently sampled for age, length, and sex composition. Catch per effort data also are not reported by species and are difficult to interpret because of temporal changes in fishing power and target species.

Because of this lack of needed data, the Northwest and Alaska Fisheries Center of the National Marine Fisheries Service initiated a large scale trawl survey of rockfish stocks from southern California to the Aleutian Islands. The first stage of the 4-yr survey was to conduct pilot surveys in the Monterey Bay area, Calif., and Queen Char-

lotte Sound, British Columbia, during 1976. The overall goal of the pilot surveys was to provide information for design of the full scale survey. Gunderson and Nelson[2] describe the pilot survey and present preliminary results of the effort. A full scale survey was conducted in 1977 off the coasts of Washington, Oregon, and California. Design of the 1977 survey was partly based on results of the pilot surveys. Gunderson and Sample (1980) discussed the 1977 survey.

While trawl surveys have proven to be a useful tool for assessing fish stocks, problems still remain in their design and analysis. This paper presents analyses of results of the Queen Charlotte Sound pilot survey and 1977 survey. The analyses were aimed at answering three questions: 1) Should the full scale survey design be based on a random, stratified random, or systematic scheme? 2) Do results of the 1977 survey indicate that aspects of the design based on the pilot survey were correct? 3) What are the trade offs in precision between distance trawled and number of tows? Ancillary to question one are the questions: 1) Are there significant benefits to be gained by choosing one or a combination of the three sampling schemes? 2) Are there significant biases in estimates of

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either means or variances using one or a combination of the three sampling schemes?

While our analyses are limited to data from the Queen Charlotte Sound pilot survey and the 1977 survey, the questions repeatedly arise in discussions of trawl surveys and thus are of general interest.

COMPARISONS OF RANDOM, STRATIFIED RANDOM, AND SYSTEMATIC SAMPLING

Methodology

Chapter 8 of Cochran (1964) discusses systematic sampling and presents methodology for choosing among random, stratified random, and systematic (every kth) sampling. Similar discussions are found in other sampling texts such as Hansen et al. (1953) and Sukhatme and Sukhatme (1970). The methodology used in comparing the three sampling techniques assumes equal sampling effort in each strata. If prior information indicated that variance differs considerably among strata, the optimal stratified random sampling scheme would not be equal allocation of sampling effort among strata. Unfortunately as shown by Abramson (1968), it can be difficult to obtain meaningful information on within strata variance for trawl surveys even if previous surveys have been made in the area. The methodology also only considers regularly spaced strata of uniform size. While prior knowledge (catch records) made it possible to design strata of unequal size on a large scale basis, knowledge is insufficient to do so on the scale considered in the analysis. The multispecies aspects of the survey made it particularly difficult to devise an optimal stratified random scheme.

In this section we use Cochran's notation. However, instead of examining components of variance for choosing among the three types of sampling as Cochran did, we calculated the variances for each type of sampling. Using the notation of Cochran, let a population of k possible systematic samples be represented by

Member	Systematic sample number				
	1	...	i	...	k
1	y_{11}	...	y_{i1}	...	y_{k1}
.	.		.		.
.	.		.		.
.	.		.		.

j	y_{1j}	...	y_{ij}	...	y_{kj}
.	.		.		.
.	.		.		.
.	.		.		.
n	y_{1n}	...	y_{in}	...	y_{kn}

where y_{ij} is the jth member of the ith systematic sample.

If the y_{ij}'s are arranged as they actually occur for a population of two systematic samples ($k = 2$) with four ($n = 4$) members they appear as follows:

Unit	1	2	3	4	5	6	7	8
Variable	y_{11}	y_{21}	y_{12}	y_{22}	y_{13}	y_{23}	y_{14}	y_{24}.

In a systematic survey a number (i) is chosen between 1 and k and then n members that are k units apart are sampled. Under a scheme of drawing one systematic sample, either units 1, 3, 5, and 7 or 2, 4, 6, and 8 would be observed. Under a stratified random scheme one unit out of each of four strata (units 1 and 2, 3 and 4, 5 and 6, and 7 and 8) would be chosen at random for observation. Under the corresponding random scheme any four of the eight units would be chosen at random to be observed. The example population contains 2 possible systematic samples, 16 possible stratified random samples, and 70 possible random samples. While more than 20% of the possible random samples match with a stratified random sample, <3% match with a systematic sample. A systematic sample is much more structured or constrained than the other schemes. Population variances of the means of random, stratified random, and systematic sampling are calculated as follows:

$$V(\bar{y}_{ran}) = \frac{N-n}{N} \ S^2_{ran}/n \qquad (1)$$

where $V(\bar{y}_{ran})$ = variance of the mean calculated from random sampling ($\bar{y}_{ran}$),

$$N = k \times n,$$

$$S^2_{ran} = \sum_{i=1}^{k} \sum_{j=1}^{n} (y_{ij} - \bar{y})^2 /N{-}1,$$

$$\bar{y} = \sum_{i=1}^{k} \sum_{j=1}^{n} y_{ij} /N,$$

$$V\,(\bar{y}_{st}) = \frac{(N-n)}{N}\;S^2_{st}/n \qquad (2)$$

where $V\,(\bar{y}_{st})$ = variance of the mean calculated from stratified random sampling $(\bar{y}_{st})$,

$$S^2_{st} = \frac{1}{n(k-1)}\;\sum_{j=1}^{n}\;\sum_{i=1}^{k}\;(y_{ij} - \bar{y}_{.j})^2,$$

$$\bar{y}_{.j} = \sum_{i=1}^{k}\;y_{ij}/\,k,\text{ and}$$

$$V\,(\bar{y}_{sys}) = \frac{1}{k}\;\sum_{i=1}^{k}\;(\bar{y}_{i.} - \bar{y})^2 \qquad (3)$$

where $V\,(\bar{y}_{sys})$ = variance of the mean calculated from systematic sampling $(\bar{y}_{sys})$,

$$\bar{y}_{i.} = \sum_{j=1}^{n}\;y_{ij}/n.$$

If k systematic samples are taken from a population that is sufficiently large to ignore the finite population correction factor then the variance estimates become:

$$S^2\,(\bar{y}_{ran}) = S^2_{ran}/n \qquad (4)$$

where $S^2\,(\bar{y}_{ran})$ = estimate of $V\,(\bar{y}_{ran})$,

$$S^2_{ran} = \sum_{i=1}^{k}\;\sum_{j=1}^{n}\;(y_{ij} - \bar{y})^2\;/(nk-1),$$

$$\bar{y} = \sum_{i=1}^{k}\;\sum_{j=1}^{n}\;y_{ij}/kn,$$

$$S^2\,(\bar{y}_{st}) = S^2_{st}/n \qquad (5)$$

where $S^2\,(\bar{y}_{st})$ = estimate of $V\,(\bar{y}_{st})$,

$$S^2_{st} = \frac{1}{n(k-1)}\;\sum_{j=1}^{n}\;\sum_{i=1}^{k}\;(y_{ij} - \bar{y}_{.j})^2,$$

$$y_{.j} = \sum_{i=1}^{k}\;y_{ij}/k,\text{ and}$$

$$S^2\,(\bar{y}_{sys}) = \frac{1}{k-1}\;\sum_{i=1}^{k}\;(\bar{y}_{i.} - y)^2 \qquad (6)$$

where $S^2\,(\bar{y}_{sys})$ = estimate of $V\,(\bar{y}_{sys})$,

$$\bar{y}_{i.} = \sum_{j=1}^{n}\;y_{ij}/n.$$

Results of Pilot Survey

In the case of the Queen Charlotte survey, two random starting points were chosen and then tows were made along four transects for each of the two systematic samples. The transects within a systematic sample were approximately 16.1 km (10 mi) apart and bottom topography dictated some deviations from the desired transects. Because preferred depths differ among species of rockfish (e.g., *Sebastes alutus* is relatively scarce in shallow waters, while *S. proriger* is relatively scarce in deep waters), attempts were made to distribute sampling effort among 18.3 m (10-fathom) depth intervals within the depth range of concern, 91.4 m (50 fathoms) and 292 m (160 fathoms). Examination of the data indicated that, to obtain reasonable sample sizes, observations should be divided into only three depth intervals: 91-145 m (79 fathoms), 146-181 m (80-99 fathoms), and >181 m.

The Queen Charlotte data were organized in two ways to examine the relative precision of the three sample schemes. We first arranged tows at depths >181 m into a hypothetical population of four systematic samples for each species. While the original sample design called for two systematic samples, the two random starting points resulted in all transects being about 8.1 km (5 mi) apart. Each systematic sample contains two members. Furthermore, each hypothetical population is composed of x_{is} = the average catch (kilograms) per 1.8 km (nautical mile) of species s of all tows taken >181 m in transect i of the Queen Charlotte survey. Under the preceding definition the hypothetical population of systematic samples of species s is

Member   *Systematic sample*

	1	2	3	4
1	$y_{11} = X_{1s}$	$y_{21} = X_{2s}$	$y_{31} = X_{3s}$	$y_{41} = X_{4s}$
2	$y_{12} = X_{5s}$	$y_{22} = X_{6s}$	$y_{32} = X_{7s}$	$y_{42} = X_{8s}$.

In this case $k = 4$ and $n = 2$. The y_{ij}'s are averages of 1-5 tows (Table 1). The resulting estimates of variance apply only to these hypothetical populations and particular mixture of tows per average (y_{ij}). It was not possible to construct similar hypothetical populations for the other depth intervals because of missing cells.

Values of V ($\bar{y}_{ran}$), V ($\bar{y}_{st}$), and V ($\bar{y}_{sys}$) for the first hypothetical populations are shown in Table 2. Comparison of the precision of the three sampling methods indicates that systematic sampling would be the most precise (has lowest variance) scheme for 8 of the 15 species. Ties occurred for the other seven species. Assuming that each species represents an independent observation, the sign test indicated that systematic sampling gave

TABLE 1.—Number of tows taken during the Queen Charlotte survey by stratum, systematic sample, member, and group of hypothetical populations.

First group of hypothetical populations Systematic sample				
Member	1	2	3	4
1	1	1	1	2
2	2	2	4	5

Second group of hypothetical populations Systematic sample			
Depth (m)	Member	1	2
91-145	1	2	8
	2	6	2
146-181	1	6	5
	2	2	1
>181	1	2	3
	2	4	9

TABLE 2.—Variances of means of catch (per 1.8 km towed) from the first hypothetical populations of Queen Charlotte rockfish. Calculations are made under random, stratified random, and systematic sampling schemes.

Population	Variance		
	Random	Stratified random	Systematic
Sebastes alutus	6,595.069	5,791.227	3,312.254
S. flavidus	0.188	0.188	0.188
S. pinniger	0.003	0.003	0.003
S. paucispinis	0.766	0.734	0.620
S. brevispinis	6.601	6.431	6.273
S. elongatus	0.002	0.002	0.002
S. proriger	0.002	0.002	0.002
S. babcocki	225.734	238.282	191.356
S. crameri	0.107	0.083	0.060
S. zacentrus	73.723	74.613	71.624
S. diploproa	0.344	0.329	0.329
S. entomelas	0.117	0.117	0.117
S. reedi	74.625	74.625	74.625
S. aleutianus	0.246	0.266	0.163
S. helvomaculatus	0.056	0.057	0.054

better results than stratified random sampling and random sampling at the 1% level of significance, and that stratified random sampling did not give significantly better results than random sampling.

Because of the uneven distribution of tows per transect, we organized the data in another fashion to determine if the relative precision of the three sampling schemes is affected by organization of the data.

We next grouped the data into three depth intervals: 91-145 m, 146-181 m, and >181 m. In order to avoid missing cells it was necessary to create hypothetical populations of only two systematic samples with two members. We did so as follows: X_{isd} = the average of catch (kilograms) per 1.8 km of species s in depth interval d of all tows in transects i and $i+1$. The hypothetical population of systematic samples of species s from depth interval d is

Member
(stratum)   *Systematic sample*

	1	2
1	$y_{11} = X_{1 \text{ and } 2, s, d}$	$y_{21} = X_{3 \text{ and } 4, s, d}$
2	$y_{12} = X_{5 \text{ and } 6, s, d}$	$y_{22} = X_{7 \text{ and } 8, s, d}$.

In this case $k = 2$ and $n = 2$. The y_{ij}'s are averages of 1 to 9 tows (Table 1). The values of V ($\bar{y}_{ran}$), V ($\bar{y}_{st}$), and V ($\bar{y}_{sys}$) are shown in Table 3.

The results indicate that systematic sampling produces more precise estimates of rockfish densities than either random or stratified random sampling. However, the sign test revealed that systematic sampling is not significantly better than stratified random sampling and only better than random sampling at the 5% level of significance. Stratified random sampling was not significantly better than random sampling. While systematic sampling appears to be the most precise of the three survey design schemes, there were many cases in which two or more of the schemes would be equally precise. In many other cases little precision would be lost if either stratified random or random designs were chosen.

Results of 1977 Survey

The 1977 survey design included both stratified random and systematic sampling strategies. The coast was stratified into three types of areas: high density sampling, low density sampling, and no

TABLE 3.—Variances of means of catch (per 1.8 km towed) from second hypothetical populations of Queen Charlotte rockfish. Calculations are made under random, stratified random, and systematic sampling schemes.

Population	Depth interval (m)	Variance		
		Random	Stratified random	Systematic
Sebastes alutus	91-145	82.313	59.446	55.285
S. alutus	146-181	2,745.670	3,803.277	1,125.770
S. alutus	>181	1,892.520	1,270.058	2,502.014
S. flavidus	91-145	8.442	12.750	0.102
S. flavidus	146-181	0.259	0.193	0.160
S. flavidus	>181	0.063	0.063	0.063
S. pinniger	91-145	257.220	234.198	228.577
S. pinniger	146-181	0.507	0.759	0.400
S. pinniger	>181	0.001	0.001	0.001
S. paucispinis	91-145	2.657	3.987	1.503
S. paucispinis	146-181	1.560	0.287	0.402
S. paucispinis	>181	0.260	0.197	0.121
S. brevispinis	91-145	60.440	19.718	13.783
S. brevispinis	146-181	8.703	13.020	11.262
S. brevispinis	>181	2.328	2.194	3.464
S. elongatus	91-145	0.129	0.100	0.102
S. elongatus	146-181	0.327	0.317	0.345
S. elongatus	>181	0.001	0.001	0.001
S. proriger	91-145	1,019.197	686.814	657.281
S. proriger	146-181	5.152	5.560	6.494
S. proriger	>181	0.002	0.002	0.002
S. babcocki	91-145	2.090	1.584	1.169
S. babcocki	146-181	3.241	0.339	0.339
S. babcocki	>181	49.730	50.671	24.671
S. crameri	91-145	0.001	0.001	0.001
S. crameri	146-181	0.057	0.051	0.029
S. crameri	>181	0.061	0.037	0.074
S. zacentrus	91-145	0.006	0.006	0.006
S. zacentrus	146-181	0.220	0.217	0.217
S. zacentrus	>181	29.261	30.641	30.641
S. diploproa	91-145	0.000	0.000	0.000
S. diploproa	146-181	0.000	0.000	0.000
S. diploproa	>181	0.165	0.162	0.162
S. entomelas	91-145	0.006	0.006	0.006
S. entomelas	146-181	0.609	0.724	0.378
S. entomelas	>181	0.048	0.048	0.048
S. reedi	91-145	0.000	0.000	0.000
S. reedi	146-181	0.302	0.302	0.302
S. reedi	>181	30.710	30.710	30.710
S. aleutianus	91-145	0.000	0.000	0.000
S. aleutianus	146-181	0.002	0.002	0.002
S. aleutianus	>181	0.087	0.057	0.018
S. helvomaculatus	91-145	0.000	0.000	0.000
S. helvomaculatus	146-181	0.001	0.001	0.001
S. helvomaculatus	>181	0.022	0.024	0.020

sampling. Areas in which historical fisheries data indicated high abundances of important rockfish species were assigned high density sampling. In these areas, transects were set at 8.1 km (5-mi) intervals. The typical high density area used in the study was about 81 km (50 mi) long. Transects in other areas were set at 16.1 km (10-mi) intervals unless bottom topography precluded sampling. Each transect was divided into four 91 m (50-fathom) depth strata between 91 and 457 m (250 fathoms). Sampled depths were then chosen at random within each depth stratum of a transect. The number of samples within a depth stratum was proportional to the bottom area of that stratum.

The survey design was based on several factors. The large scale stratification along the coast was an attempt to make sampling proportional to expected densities of important rockfish. This was done with the knowledge that there often are positive correlations among means and variances of fish densities. Depths were randomly chosen because it was known that often within an area densities of many species of rockfish sometimes only occur over a narrow depth interval. Thus, unless depths were chosen at random, bias could occur. Sampling was proportional to bottom area, because bottom area is used to convert fish densities to abundance estimates. Systematic transects were taken to ensure adequate aerial coverage for one intended use of the data, because of logistics and the results of the pilot survey.

Four high density areas had sufficient sampling effort to be included in the study. Eight or more adjacent transects were sampled in one or more depth strata in each chosen area. Area 1 was between lat. 34°33' and 35°19' N, area 2: lat. 35°19' and 35°59' N, area 3: lat. 39°7' and 39°53' N, and area 4: lat. 44°59' and 45°50' N. If more than one sample was taken from a depth stratum of a transect, one sample was chosen at random for the study. As in the case of the first Queen Charlotte hypothetical populations, populations of four or five systematic samples of two members each were created from the data. The results again indicated that systematic samples were slightly more precise than either random or stratified random (Table 4). The sign test indicated that systematic sampling was more precise than random at the 1% level of significance and stratified random at the 10% level. Stratified random sampling was not significantly less precise than random.

The data were also arranged into two systematic samples with four or five members each. Systematic sampling was more precise than random at the 1% level of significance, but was not significantly more precise than stratified random (Table 5). Stratified random sampling was not significantly less precise than random sampling.

Discussion

The results of this study indicate that on a scale of about 80 km along the coast systematic sampling for rockfish is slightly more precise than random sampling or a stratified random scheme with regularly spaced strata of equal size and

TABLE 4.—Variances of means of catch (per 1.8 km towed) from hypothetical populations of rockfish that were constructed from 1977 survey data. Calculations are made under random, stratified random, and systematic sampling schemes. Hypothetical populations are composed of either four or five systematic samples with two members.

Population	Systematic samples	Area	Depth interval (m)	Variance		
				Random	Stratified random	Systematic
Sebastes alutus	4	4	183-273	6,276.924	6,232.073	5,426.781
S. alutus	4	4	366-457	46.552	49.621	51.403
S. flavidus	4	2	91-182	0.090	0.102	0.074
S. pinniger	4	1	91-182	8.420	6.604	6.604
S. pinniger	4	2	91-182	0.185	0.215	0.422
S. paucispinis	4	1	91-182	9,630.529	9,888.812	8,989.353
S. paucispinis	4	2	91-182	34.452	39.381	27.818
S. paucispinis	5	2	183-273	24.181	23.665	30.904
S. paucispinis	5	3	91-182	37.490	42.176	81.772
S. brevispinis	4	4	183-273	2.185	2.290	1.421
S. elongatus	4	1	91-182	3.651	3.930	3.039
S. elongatus	4	2	183-273	0.060	0.054	0.054
S. elongatus	5	3	91-182	3.062	2.994	2.994
S. elongatus	4	4	183-273	63.100	66.665	93.148
S. babcocki	4	2	183-273	3.974	3.434	3.434
S. babcocki	4	2	274-365	4.998	5.820	8.106
S. babcocki	4	3	366-457	0.789	0.469	0.469
S. babcocki	4	4	183-273	7.761	6.086	6.125
S. babcocki	4	4	366-457	90.164	85.971	85.971
S. crameri	4	2	274-365	32.481	27.882	26.134
S. crameri	5	2	366-457	3.516	3.200	3.200
S. crameri	4	4	183-273	1,903.722	1,948.233	1,741.187
S. crameri	4	4	366-457	0.230	0.251	0.110
S. zacentrus	4	4	183-273	15.078	12.492	12.492
S. diploproa	4	2	183-273	1,255.342	1,389.344	1,317.551
S. diploproa	4	2	274-365	2,861.498	3,304.897	5,275.483
S. diploproa	5	2	366-457	2,727.360	1,979.702	948.597
S. diploproa	4	3	366-457	159.021	68.670	91.114
S. diploproa	4	4	183-273	683.478	700.234	779.467
S. diploproa	4	4	366-457	1.827	1.966	1.216
S. entomelas	4	1	91-182	467.709	468.780	476.280
S. entomelas	4	2	91-182	1.012	1.001	0.981
S. entomelas	4	2	183-273	3.050	3.103	2.633
S. entomelas	5	3	91-182	0.376	0.423	0.846
S. entomelas	4	4	183-273	3.002	3.057	2.129
S. aleutianus	4	4	366-457	589.997	688.030	399.129
S. goodei	4	1	91-182	119.235	119.105	121.329
S. goodei	4	2	91-182	4,439.160	4,391.472	4,226.895
S. goodei	4	2	183-273	416.868	468.696	301.373
S. goodei	5	3	91-182	258.380	221.057	220.347
S. jordani	4	1	91-182	467.621	406.738	399.790
S. jordani	4	2	91-182	26,765.442	27,254.716	25,958.703
S. jordani	4	2	183-273	7.256	8.158	5.457
S. jordani	5	3	91-182	0.575	0.570	0.570
S. saxicola	4	1	91-182	11,033.375	8,981.548	8,450.645
S. saxicola	4	2	91-182	145.957	162.133	120.545
S. saxicola	4	2	183-273	11,177.436	11,261.334	11,301.092
S. saxicola	4	2	274-365	0.244	0.235	0.235
S. saxicola	5	2	366-457	0.104	0.066	0.057
S. saxicola	5	3	91-182	3,402.868	3,070.065	3,068.507
S. saxicola	4	3	366-457	3.106	3.304	2.843
S. saxicola	4	4	183-273	4.630	4.187	3.308
S. rufus	4	2	183-273	10.247	11.501	10.092
S. rufus	4	2	274-365	6.588	6.561	7.374
S. rufus	5	2	366-457	0.513	0.566	0.870
S. aurora	4	2	274-365	6.835	7.591	6.215
S. aurora	5	2	366-457	22.452	24.477	20.423
S. aurora	4	3	366-457	54.214	45.845	45.465
S. aurora	4	4	366-457	0.507	0.511	0.782
S. melanostomus	4	2	274-365	0.566	0.148	0.148
S. melanostomus	5	2	366-457	22.167	20.848	24.547
Average				1,398.584	1,369.394	1,314.689

observations. It was also noted that our present state of knowledge precludes more optimally designed stratified random schemes on the scale considered. It appears that the decision to space transects of the 1977 survey in a systematic fashion was correct. While the data do indicate that systematic sampling is more precise than stratified random, the differences are slight and

TABLE 5.—Variances of means of catch (per 1.8 km towed) from hypothetical populations of rockfish that were constructed from 1977 survey data. Calculations are made under random, stratified random, and systematic sampling schemes. Hypothetical populations are composed of two systematic samples with four or five members.

Population	Members	Area	Depth interval (m)	Variance		
				Random	Stratified random	Systematic
Sebastes alutus	4	4	183-273	1,569.231	1,576.140	3,193.380
S. alutus	4	4	366-457	15.517	5.162	3.195
S. flavidus	4	2	91-182	0.030	0.034	0.063
S. pinniger	4	1	91-182	2.807	3.261	6.439
S. pinniger	4	2	91-182	0.062	0.072	0.141
S. paucispinis	4	1	91-182	3,210.176	3,203.033	2,889.063
S. paucispinis	4	2	91-182	11.484	13.142	24.503
S. paucispinis	5	2	183-273	6.045	5.266	3.534
S. paucispinis	5	3	91-182	9.373	10.639	1.145
S. brevispinis	4	4	183-273	0.546	0.226	0.336
S. elongatus	4	1	91-182	1.217	1.436	2.441
S. elongatus	4	2	183-273	0.015	0.016	0.026
S. elongatus	5	3	91-182	0.766	0.656	0.656
S. elongatus	4	4	183-273	15.775	17.090	11.696
S. babcocki	4	2	183-273	0.994	1.102	0.130
S. babcocki	4	2	274-365	1.666	0.909	0.063
S. babcocki	4	3	366-457	0.263	0.197	0.002
S. babcocki	4	4	183-273	30.055	31.103	38.440
S. babcocki	4	4	366-457	1.940	0.551	1.000
S. crameri	4	2	274-365	10.827	12.414	2.403
S. crameri	5	2	366-457	0.879	0.372	0.372
S. crameri	4	4	183-273	475.931	378.362	426.423
S. crameri	4	4	366-457	0.077	0.107	0.035
S. zacentrus	4	4	183-273	3.770	0.485	0.281
S. diploproa	4	2	183-273	313.836	243.015	367.489
S. diploproa	4	2	274-365	953.833	869.422	1,065.206
S. diploproa	5	2	366-457	681.840	375.130	53.290
S. diploproa	4	3	366-457	53.007	25.279	0.191
S. diploproa	4	4	183-273	170.870	178.211	122.324
S. diploproa	4	4	366-457	0.609	0.803	0.090
S. entomelas	4	1	91-182	155.903	156.260	158.760
S. entomelas	4	2	91-182	0.337	0.402	0.856
S. entomelas	4	2	183-273	0.763	0.275	0.723
S. entomelas	5	3	91-182	0.094	0.106	0.000
S. entomelas	4	4	183-273	0.751	0.849	0.250
S. aleutianus	4	4	366-457	196.666	199.264	398.003
S. goodei	4	1	91-182	39.745	38.155	35.106
S. goodei	4	2	91-182	1,479.720	1,367.750	1,531.744
S. goodei	4	2	183-273	104.217	127.159	335.989
S. goodei	5	3	91-182	64.595	72.201	140.660
S. jordani	4	1	91-182	155.874	163.576	85.794
S. jordani	4	2	91-182	8,921.814	9,116.721	7,983.423
S. jordani	4	2	183-273	1.814	2.179	0.004
S. jordani	5	3	91-182	0.144	0.137	0.137
S. saxicola	4	1	91-182	3,677.792	335.847	431.081
S. saxicola	4	2	91-182	48.652	55.079	106.864
S. saxicola	4	2	183-273	2,794.359	3,362.293	3,068.052
S. saxicola	4	2	274-365	0.081	0.069	0.069
S. saxicola	5	2	366-457	0.026	0.007	0.000
S. saxicola	5	3	91-182	850.717	916.415	388.878
S. saxicola	4	3	366-457	1.035	1.101	0.640
S. saxicola	4	4	183-273	1.158	1.285	2.789
S. rufus	4	2	183-273	2.562	1.899	2.161
S. rufus	4	2	274-365	2.196	2.447	3.331
S. rufus	5	2	366-457	0.128	0.144	0.000
S. aurora	4	2	274-365	2.278	2.502	0.902
S. aurora	5	2	366-457	5.613	2.157	1.988
S. aurora	4	3	366-457	18.071	12.091	14.440
S. aurora	4	4	366-457	0.169	0.045	0.051
S. melanostomus	4	2	274-365	0.189	0.008	0.004
S. melanostomus	5	2	366-457	5.542	4.153	3.686
Average				427.483	375.349	375.586

probably should not outweigh other factors such as logistical constraints in the design of trawl surveys.

The sign test used to test the significance of differences among sample designs assumed that values for each species were independent. To examine this assumption we calculated correlation coefficients for each species pair in each combination of depth and area. Only samples containing at least one occurrence of each species of a pair were included. The average of the absolute value of the correlation coefficient is

0.324. This indicates that the assumption of independence is reasonable.

Even though our results indicate that systematic sampling is slightly more precise for the type of survey studied, the consequence of using a systematic design when another design may be more appropriate should be considered.

We first examine the effects of using a systematic design when in actuality the data are randomly distributed. Under these conditions the expected value of $S^2 y_{st}$ of Equation (5) is equal to the expected value of S^2_{sys} of Equation (6) and is related to the expected value of $S^2 \bar{y}_{ran}$ of Equation (4) as follows:

$$E\ (S^2_{st}) = \frac{nk-1}{nk-n}\ E\ (S^2_{ran}).$$

Thus, random sampling will produce the lowest variance and if total sampling effort (nk) is constant, the variance of systematic and stratified random sampling will decrease relative to random sampling as n decreases. All three design strategies will result in unbiased estimates of the mean.

If there is a linear trend in the data such as shown below

Transect	1	2	3	4	5	6	7	8
Value	1	1.5	2	2.5	3	3.5	4	4.5,

then as Cochran (1964:217) showed, stratified random sampling is the same or more precise than systematic sampling, which is the same or more precise than random. The discrepancies increase as n increases.

If there are cycles in the data with a periodicity equal to or a multiple of spacing of transects such as

Transect	1	2	3	4	5	6	7	8
Value	1	2	1	2	1	2	1	2,

then systematic sampling is less precise than stratified random sampling, which is less precise than random sampling. The discrepancies increase as n increases. In addition, a single systematic sample would result in a biased estimate of the mean.

Systematic sampling is equal to or more precise than stratified random sampling which is equal to or more precise than random sampling if a popula-

tion in which a plot of correlations between pairs of transects against distance between transects is concave upward and greater than or equal to 0 (Cochran 1964). Since systematic sampling was the most precise in this study, bias due to periodicity in the data should not be a problem.

Often in practice, investigators use a systematic sampling scheme with only one sample and calculate the variance as if the scheme is random. If V $(\bar{y}_{sys})$ is $<$V $(\bar{y}_{ran})$, as it appears to be for rockfishes, the resulting confidence limits would be conservative. The choice between precision and estimation of V $(\bar{y}_{sys})$ would depend on the objectives of the survey and the difference between V $(\bar{y}_{ran})$ and V $(\bar{y}_{sys})$. If V $(\bar{y}_{sys})$ is $<$V $(\bar{y}_{ran})$ and total number of transects is constant, increasing the number of systematic samples (k) in order to estimate V $(\bar{y}_{sys})$ causes the sampling scheme to become more like a random scheme and results in a corresponding increase in V $(\bar{y}_{sys})$ relative to V $(\bar{y}_{ran})$ (compare average variances shown in Tables 4 and 5).

A review by Cochran (1964) of a small number of surveys of terrestrial populations also indicated that systematic sampling is more precise than stratified random. Although Cochran did not state so, it also appeared that systematic sampling would be more precise than random sampling.

Two studies were found in the literature on marine populations. Venrick (1978) found that on the average systematic samples of chlorophyll in the water column produced estimates of total chlorophyll that were closer to the true value than stratified random samples, but was not able to compare precision of estimates for a given water column because only one systematic sample was taken from each column. She expressed a preference for stratified random sampling, because there tended to be more temporal correlations of deviations of the estimated values from the true values for the systematic samples than for the stratified random samples. The deviations were usually $<5\%$ and the temporal correlations probably could have been eliminated if the starting points of the systematic samples were observed at random instead of being fixed at the surface as was done in her study. Fiedler (1978) examined the relative precision of random, systematic, and stratified systematic transect surveys of northern anchovy, *Engraulis mordax*, school groups. He found that random sampling was the least precise of the three schemes. He also found that stratified

systematic sampling was more precise than systematic sampling when school groups were distributed in a highly nonrandom way. Systematic sampling was more precise than stratified systematic sampling when sampling density was high. In other cases there were no significant differences between systematic and stratified systematic sampling.

Fiedler based allocations of sampling effort among strata on results of previous sampling. His results, in conjunction with those of Abramson (1968) and Venrick (1978), indicate the difficulty of determining optimum allocation of sampling effort among strata in the marine environment. The frustration of scientists who have attempted to do so is aptly stated by Venrick: "Study A demonstrated the dependence of the success of a sampling design upon the interaction of that design with the structure of the population being sampled; thus, it would seem that intelligent application of knowledge about the sampled population should improve the design. It was, therefore, disconcerting to find that RSS every 20 m, RSS-1, consistently performed as successfully as did RSS-3 which was designed by a presumably experienced worker (the author) with total knowledge about the population to be sampled."

We hope that improved knowledge on the distributions of populations will eventually result in more efficient allocation of effort among strata of systematic or random sampling schemes in the marine environment. However, we point out that fishermen still have their failures in attempting to restrict their sampling effort to times and areas of high fish catches in spite of years of experience, sophisticated fish finding equipment—presumably flexible sampling plans—and at times recent information from their colleagues.

EXAMINATION OF TRADE OFFS BETWEEN TOW LENGTH AND NUMBER OF TOWS

The distance trawled is an important factor to consider in the design of trawl surveys. Considerations include distance needed to obtain sufficient specimens for biological samples; time required to set and retrieve a trawl, to cover the distance, and to move between trawl locations; and the relationship among precision, tow length, and number of tows.

In this section we first use a negative binomial model with varying element size to estimate the relationship among precision and tow length and number of tows. We next use this relationship and time factors to illustrate the relationship between logistically feasible tow lengths and precision.

Animals in nature are rarely randomly distributed. They usually show some degree of aggregation or contagion. When these populations are sampled, they lead to distributions which are markedly skewed and have a large proportion of zero elements. The negative binomial distribution is often assumed for such populations because of practical performance (Laubscher 1961; Pielou 1969) and theoretical basis (Taylor 1953; Patil and Stiteler 1974). The distribution can be used to provide an estimate of the relationship between sample element size and precision.

The negative binomial distribution often is used to describe observed distributions in both general ecological research (Pielou 1969; Poole 1974) and fisheries research (Taylor 1953; Moyle and Lound 1960; Lambou 1963; Roessler 1965; Clark 1974). The distribution is characterized by two parameters, m the mean number of units per sampling element, and k, an "index of aggregation" (Waters 1959). The value of k varies inversely with the degree of aggregation of the population. The variance of a mean drawn from a population which follows a negative binomial distribution (V_{nb}) is a function of the mean (m) and k,

$$V_{nb} = m + m^2/k.$$

As the degree of aggregation increases, k approaches 0 and when the empty elements are ignored, this distribution approaches Fisher's logarithmic series. As the degree of aggregation decreases, k will approach infinity and the distribution converges to the Poisson.

The negative binomial can be derived from five or more different models, which may be mutually contradictory (Anscombe 1950; Bliss and Fisher 1953). A commonly used procedure to derive the distribution is to assume that it arises from a cluster of objects in space where the clusters follow a Poisson distribution and the number of animals in a cluster are distributed according to Fisher's logarithmic series. Taylor (1953) derived a form of the negative binomial as a probability model specifically to describe the relative abundance of fish species in trawl catches.

The data used for this analysis come from the pilot survey made in Queen Charlotte Sound. Since the negative binomial distribution is a

discrete function, catches are measured in numbers of fish instead of weight as in the preceding section.

Fitting data from a systematically designed survey to the negative binomial distribution is a common practice (Moyle and Lound 1960; Taft 1960; Roessler 1965; Clark 1974). Hairston et al. (1971) made one of the few studies of sampling design for measuring spatial pattern. They found that estimates based on a grid (systematic) pattern were superior to those made from sampling at random. The grid pattern correctly reflected the spatial patterns of 17 of 22 species, while random sampling with the same number of samples correctly reflected only 12 of 22 species.

The standard negative binomial model requires a constant element size. This leads to two problems. The first is that comparisons can only be made at one sampling element size. The second problem is the negative binomial model cannot be fit to data with variation in sample element size.

To specifically deal with these problems, Bissell (1970, 1972) derived a negative binomial model that can be used when sample element size is variable and/or to predict the distribution of events for element sizes which differ from those on which the observations were based. The probability of observing x_i events on the i element which has a size of w_i is

$$P(x_i/w_i) = \left\{ k/(mw_i + k)\right\}^k \quad \left\{ mw_i/(mw_i + k)\right\}^{x_i} \quad \overset{x_i}{\underset{j=i}{\pi}} \left\{ (k + j - 1)/j\right\} \qquad (7)$$

where m = mean value of x_i for element size of unit size

  k = parameter representing the degree of aggregation (Note that k in this section has a different meaning than in the section on sampling schemes)

  w_i = element size (distance towed).

Iterative maximum likelihood solutions (Bissell 1972) gave estimates of values of m, k, and their standard errors relative to the average distance towed. The values were converted to densities (d) with units of numbers per kilometer. Estimates of d, k, and chi-square goodness of fit tests are given in Table 6. These tests were made by calculating the probability of a given number of fish occurring in a trawl of a given length from the probability density function given by Bissell (1972). This probability was cumulated over all trawls and

TABLE 6.—Estimates of mean densities ($\bar{d}$) (in numbers per kilometer), k, standard errors, with chi-square goodness of fit tests for trawl catches made in all depths in the Queen Charlotte survey.

Species	Mean density	SE($\bar{d}$)	k	SE(k)	Chi-square probability
Sebastes alutus	75.368	18.466	0.334	0.055	N.S.
S. flavidus	0.648	0.028	2.608	0.803	$P \leqslant 0.01$
S. pinniger	1.564	0.287	0.398	0.071	N.S.
S. paucispinis	0.508	0.072	2.546	0.814	N.S.
S. brevispinis	2.446	0.426	0.528	0.098	N.S.
S. elongatus	0.519	0.070	3.393	1.253	$P \leqslant 0.01$
S. proriger	30.402	9.612	0.198	0.031	$P \leqslant 0.05$
S. babcocki	1.723	0.273	0.665	0.131	$P \leqslant 0.01$
S. crameri	0.509	0.070	3.233	1.008	$P \leqslant 0.01$
S. zacentrus	4.973	1.428	0.274	0.044	$P \leqslant 0.01$
S. diploproa	0.623	0.102	1.157	0.292	$P \leqslant 0.01$
S. entomelas	0.119	0.028	6.289	1.004	N.S.
S. reedi	1.867	0.412	0.390	0.069	$P \leqslant 0.01$
S. aleutianus	0.205	0.037	6.550	1.215	N.S.
S. helvomaculatus	0.301	0.048	4.424	1.078	$P \leqslant 0.01$

then compared with the observed value using a chi-square test. Values of k from trawls in all depths ranged from 0.2 to 0.6 for the most abundant species. The low values of k indicate that the more abundant species are highly aggregated. The estimates for k are close to the value of k (0.27) that we estimated for S. marinus, an abundant species of rockfish, from Georges Bank from data in the paper by Taylor (1953).

We next divided the trawls into three depth intervals: 91-145 m, 146-181 m, and >181 m. Estimates of mean densities, k, and goodness of fit tests by depth strata are presented in Table 7.

The chi-square tests show that the data combined over all depths are not well represented by the negative binomial model. However, when the data are divided up by depth strata, the agreement is quite good. When the data from low density and high density depth strata are combined, the resulting frequency distribution has too many zero elements and too many high abundance elements. This results in the high chi-square values from trawls at all depths. In comparing the results in Tables 6 and 7, it is obvious that depth stratification is important. The differences between densities of species among depth strata were tested at the 10% level of significance. Of 43 possible comparisons, 27 (or 63%) were significantly different. This can be tested against what would have occurred randomly as a binomial proportion (Hollander and Wolfe 1973). The proportion is significantly different than random (z = 6.77,

TABLE 7.—Estimates of mean densities ($\bar{d}$) (in numbers per kilometer), k, standard errors, with chi-square goodness of fit tests for trawl catches by depth strata in the Queen Charlotte survey.

Species	Depth strata (m)	Mean density	SE($\bar{d}$)	k	SE(k)	Chi-square probability
Sebastes alutus	91-145	7.119	3.098	0.290	0.080	N.S.
S. alutus	146-181	90.816	37.533	0.419	0.132	N.S.
S. alutus	>181	131.914	36.838	0.693	0.197	N.S.
S. flavidus	91-145	1.160	0.253	1.660	0.817	N.S.
S. flavidus	146-181	0.724	0.200	2.037	1.030	N.S.
S. flavidus	>181	0.051	0.003	4.900	1.231	([1])
S. pinniger	91-145	5.200	2.084	0.322	0.090	N.S.
S. pinniger	146-181	0.465	0.136	2.155	1.063	([1])
S. pinniger	>181	0.018	0.017	([2])	—	—
S. paucispinis	91-145	1.051	0.308	0.870	0.323	N.S.
S. paucispinis	146-181	0.262	0.088	2.494	0.944	N.S.
S. paucispinis	>181	0.196	0.064	3.580	1.077	N.S.
S. brevispinis	91-145	4.401	1.435	0.531	0.160	N.S.
S. brevispinis	146-181	3.027	0.993	0.652	0.232	N.S.
S. brevispinis	>181	1.145	0.354	0.679	0.231	$P \leqslant 0.05$
S. elongatus	91-145	0.426	0.106	2.623	1.130	N.S.
S. elongatus	146-181	1.023	0.326	0.857	0.353	N.S.
S. elongatus	>181	0.193	0.062	4.535	1.334	N.S.
S. proriger	91-145	81.620	64.163	0.090	0.022	N.S.
S. proriger	146-181	4.701	1.901	0.440	0.146	N.S.
S. proriger	>181	0.053	0.031	5.968	1.537	([1])
S. babcocki	91-145	0.220	0.068	3.108	0.956	N.S.
S. babcocki	146-181	1.426	0.446	1.274	0.627	N.S.
S. babcocki	>181	4.526	1.395	0.600	0.170	N.S.
S. crameri	91-145	0.017	0.016	([2])	—	—
S. crameri	146-181	0.916	0.280	1.042	0.476	$P \leqslant 0.01$
S. crameri	>181	0.673	0.146	2.549	1.168	N.S.
S. zacentrus	91-145	0.118	0.047	4.040	1.098	N.S.
S. zacentrus	146-181	1.065	0.363	0.622	0.239	N.S.
S. zacentrus	>181	12.991	6.382	0.229	0.061	N.S.
S. diploproa	91-145	0.000				
S. diploproa	146-181	0.000				
S. diploproa	>181	1.563	0.504	0.496	0.154	$P \leqslant 0.01$
S. entomelas	91-145	0.034	0.024	4.442	1.087	([1])
S. entomelas	146-181	0.202	0.073	3.306	1.105	$P \leqslant 0.05$
S. entomelas	>181	0.142	0.053	4.451	1.241	N.S.
S. reedi	91-145	0.000				
S. reedi	146-181	0.066	0.039	4.124	1.177	([1])
S. reedi	>181	5.445	2.332	0.302	0.084	$P \leqslant 0.01$
S. aleutianus	91-145	0.000				
S. aleutianus	146-181	0.087	0.040	([2])	—	—
S. aleutianus	>181	0.517	0.125	2.379	1.035	N.S.
S. helvomaculatus	91-145	0.000				
S. helvomaculatus	146-181	0.022	0.022	3.277	0.908	N.S.
S. helvomaculatus	>181	0.844	0.227	1.098	0.457	$P \leqslant 0.01$

[1] Insufficient nonzero elements to perform chi-square test.
[2] Randomly distributed, $k \to \infty$.

$P < 0.01$). Although the rockfish species tended to be aggregated, the group covers a wide range of spatial patterns.

In sampling from a negative binomial distribution, the precision of a density estimate for any given population depends both on the properties of the population, its density (d) and degree of aggregation (k), and on the characteristics of sampling, sample size (n) and the sample element size (S) (tow length). By modifying the sample characteristics, one can modify the precision of estimates.

Taylor (1953) showed in his Appendix E that reducing sample element size (length of the trawl) was the optimal sampling strategy under the condition that the total sampling area remained constant. That is, if A = area of strata (which is constant over all strata, i.e., $A_1 = A_2 = A_3 \ldots$),

a = area of the sampling element, and n = the number of samples taken in each stratum, then the value $(a/A)n$ is fixed. Therefore, by reducing the length of tow, there must be a corresponding increase in the number of tows. However, in the body of his paper, Taylor implies that it would be advantageous to reduce sample element size even with a constant number of samples. His argument is based on the relationship between the mean and variance for a negative binomial population (V_{nb_1})

$$V_{nb_1} = m + m^2/k.$$

The argument is that as m is reduced by some factor $1/b$, then V_{nb_2} would only be

$$V_{nb_2} = m/b + (m/b)^2/k.$$

While this argument is correct, it does not mean that the estimate of total numbers of fish over the entire strata is more precise with decreasing sample element size. The effect of reducing sample element size on the variance of the estimate of the total number of fish in a stratum under the condition of a fixed sample size is considered below. Using the definitions of A, a, n from above, then N = total possible number of samples in a stratum where the sample element size equals a (i.e., $N = A/a$). The variance of the total number of fish in a stratum from n samples of the standard element size (V_{t_1}) is

$$V_{t_1} = \frac{N_1^2}{n} \, V_{nb_1} \quad \text{or} \quad V_{t_1} = \frac{A^2}{a^2 n} \left[m + \frac{m^2}{k} \right]$$

(Cochran 1964). The variance (V_{t_2}) of the total number of fish in a stratum for the sample element size reduced by $1/b$ is

$$V_{t_2} = \frac{N_2^2}{n} \, V_{nb_2} = \frac{\left[\frac{A}{\left(\frac{a}{b}\right)}\right]^2}{n} \left[\frac{m}{b} + \frac{m^2}{b^2 k} \right]$$

$$= \frac{A^2 b^2}{a^2 n} \left[\frac{m}{b} + \frac{m^2}{b^2 k} \right] = \frac{A^2}{a^2 n} \left[bm + \frac{m^2}{k} \right].$$

The difference in variance between the different sample element sizes is

$$V_d = V_{t_2} - V_{t_1}$$

$$= \frac{A^2}{a^2 n} \left[bm + \frac{m^2}{k} \right] - \frac{A^2}{a^2 n} \left[m + \frac{m^2}{k} \right]$$

$$= \frac{A^2}{a^2 n} \left[m(b - 1) \right]. \tag{8}$$

Although there is actually an increase in overall variance by reducing sample element size with a constant sample size, it will be relatively small compared with the overall variance when m is large in value and/or k small:

$$\text{relative increase} = \frac{\frac{A^2}{a^2 n} \left[m(b - 1) \right]}{\frac{A^2}{a^2 n} \left[m\left(1 + \frac{m}{k}\right) \right]}$$

$$= \frac{b - 1}{1 + \frac{m}{k}}.$$

The purpose of many surveys is to produce total biomass estimates. These total biomass estimates are made by expanding a density estimate (usually in the form of a catch per unit effort measure) (Gunderson and Sample 1980) by the total area (Cochran 1964). Since measurement of the area involved can be made with relatively little error compared with the density estimate, we ignore error in area measurements in the following discussion. The precision of an estimate will vary inversely to its standard error. An index of precision ($P_{\bar{d}}$) is:

$$P_{\bar{d}} = \bar{d}/\text{SE}_{\bar{d}}. \tag{9}$$

This index is the inverse of the coefficient of variation and is used here because it varies directly rather than inversely with precision. The density of a population is equal to the mean of the negative binomial distribution divided by the sample element size (S):

$$\bar{d} = m/S \tag{10}$$

where m = the mean of a number of tows of sample element size (S),

S = a constant sample element size with no variance.

The variance of m is

$$V_m = (m + m^2/k)/n. \tag{11}$$

Therefore the variance of the density estimate is

$$V_{\bar{d}} = (m + m^2/k)/n S^2 \tag{12}$$

and from Equation (10)

$$V_{\bar{d}} = (\bar{d}S + (\bar{d}S)^2/k)/nS^2 \qquad (13)$$

or $\qquad V_{\bar{d}} = \dfrac{\bar{d}}{nS} + \dfrac{\bar{d}^2}{nK} . \qquad (14)$

The standard error of the density is

$$SE_{\bar{d}} = (V_{\bar{d}})^{1/2} \qquad (15)$$

$$= \left(\dfrac{\bar{d}}{nS} + \dfrac{\bar{d}^2}{nK} \right)^{1/2} \qquad (16)$$

and the index of precision is

$$P_{\bar{d}} = (\bar{d})/(\bar{d}/nS + \bar{d}^2/nk)^{1/2}$$

$$= (1/\bar{d}nS + 1/nk)^{-1/2}. \qquad (17)$$

From Equation (17), the precision of a density estimate will decrease as the degree of aggregation increases (i.e., $k \rightarrow 0$). For the case when $k >> \bar{d}$, then the index approaches $(\bar{d}nS)^{1/2}$ and a unit increase in sample size and sample element size are of equal importance. In the case of $\bar{d}$ approximately equal to k then sample element size has almost no effect on precision. When $\bar{d} >> k$, which is often the case for species that support a commercial fishery, the index simplifies to

$$P_{\bar{d}} \rightarrow (nk)^{1/2}. \qquad (18)$$

In these cases, only sample size will affect precision.

More specific evaluation of sampling negative binomial populations can be made by considering the estimates of k for three rockfish species and Equation (17). Since the limiting factor in these surveys is usually ship time and not cost in a direct sense, the evaluation is in terms of the most efficient use of a ship day. The first two species were the two target species in the Queen Charlotte Sound survey: *S. alutus*, a high density, highly aggregated species, and *S. flavidus*, a low density, highly aggregated species. The third species was *S. aleutianus*, a low density more randomly distributed species in Queen Charlotte Sound.

The sampling plan in Queen Charlotte Sound was to perform trawls of 0.5 h on the bottom which covered an average of 2.80 km. The average number of trawls per working day was 4.3. The average working day was 13 h long. Assuming 0.5 h on the bottom per trawl and 4.3 trawls/d, then the average nontrawling time per haul is 2.05 h. The minimum nontrawling time per haul was 1.07 h. The current sampling plan calls for an average of about 5 trawls/d. Using the above times, four possible alternative strategies are: 1) 3 trawls/d with gear at depth for 2.1 h, 2) 4 trawls/d with gear at depth for 1.2 h, 3) 5 trawls/d with gear at depth for 0.5 h, or 4) 6 trawls/d with gear at depth for 0.3 h.

Using the four strategies, values for precision of estimate of density were calculated for *S. alutus*, *S. flavidus*, and *S. aleutianus* and are shown in Figures 1, 2, and 3. The results of this analysis follow directly from the result of the general analysis. When the density to k ratio increases above a critical level, precision is for all practical purposes unaffected by changes in density. For the more randomly distributed species (*S. aleutianus*) the critical ratio occurs at higher density. For more aggregated species (*S. alutus*, *S. flavidus*) sample size (n) is not as effective in increasing $P_{\bar{d}}$ in an absolute sense as in the less aggregated species. Also, since sample size and sample element size are inversely related and precision increases with increased sample size except at very low density, sample element size has little effect on precision for these species except at very low density. Even for the less aggregated species, sample element size has little effect on precision except at low densities.

For a fixed amount of sampling effort, the precision of an estimate from a negative binomial population is a result of the interaction of population factors, density ($\bar{d}$) and degree of aggregation (k); and sampling factors, sample size (n) and sample element size (S). Analysis of the results of the Queen Charlotte Sound survey shows that rockfish species have a wide range of possible combinations of population factors. The analysis of sampling strategies showed that the same sampling plan could have been used for all three species with no significant loss in precision. This is due to the highly aggregated nature of rockfish species. However, for other less aggregated species, such as flatfishes, there would have been a greater difference among sampling schemes. This emphasizes the importance of picking target species on
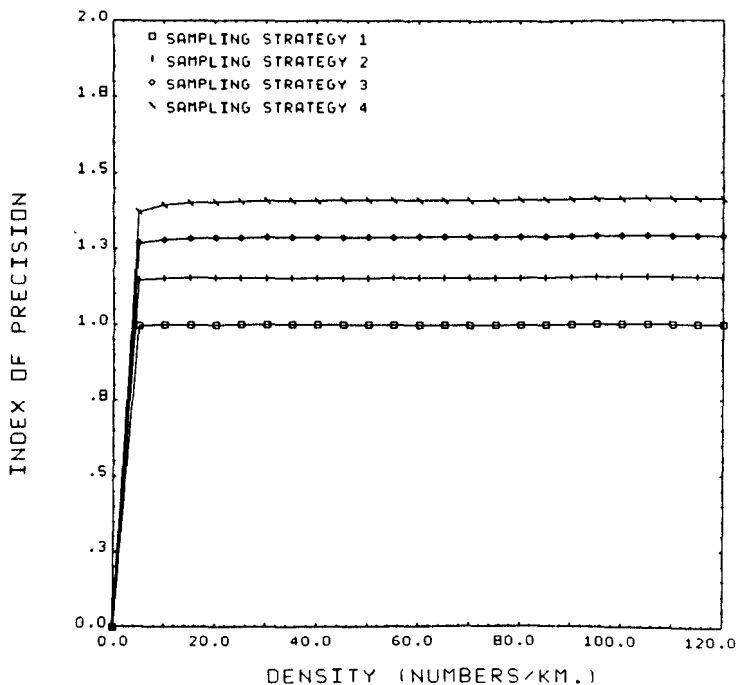
FIGURE 1.—Comparison of precision-density curves of four different sampling strategies for *Sebastes alutus*: 1) Three trawls/d with gear at depth for 2.1 h, 2) four trawls/d with gear at depth for 1.2 h, 3) five trawls/d with gear at depth for 0.5 h, and 4) six trawls/d with gear at depth for 0.3 h.
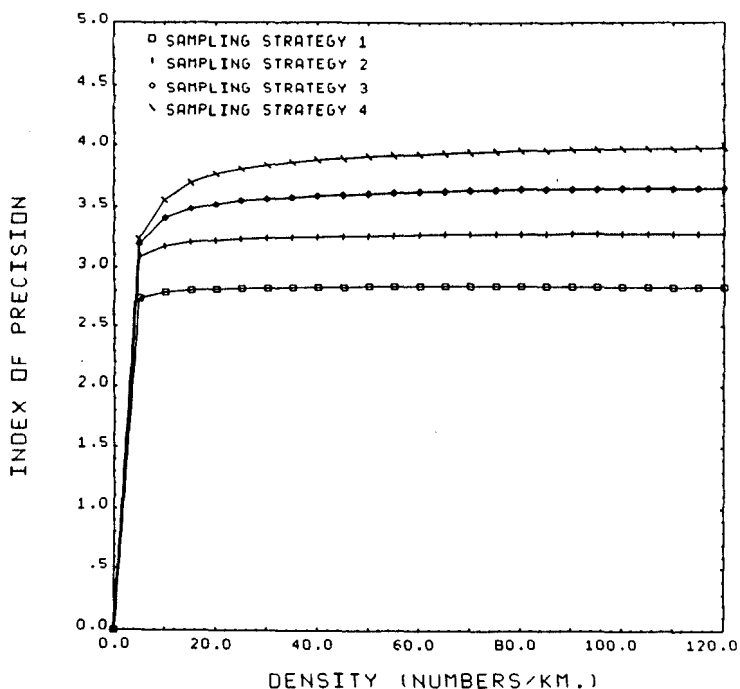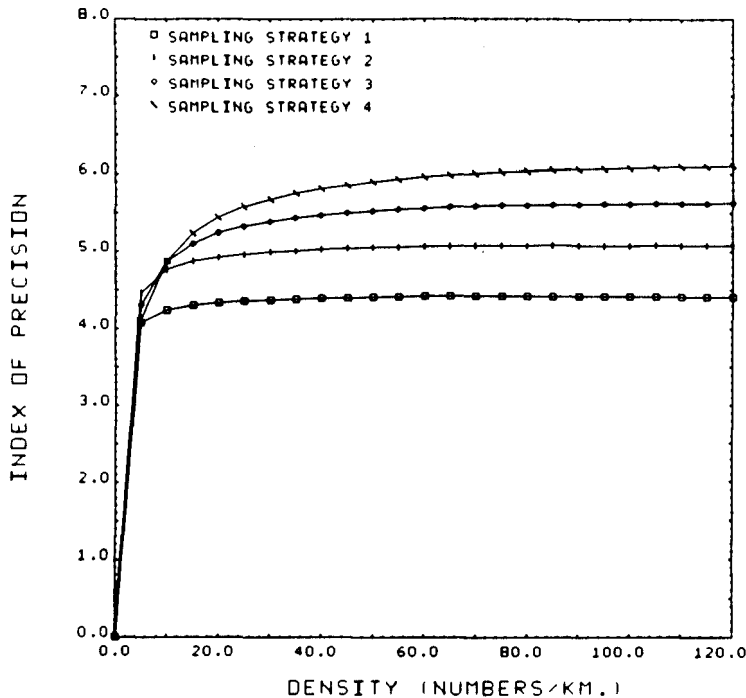
FIGURE 2.—Comparison of precision-density curves of four different sampling strategies for *Sebastes flavidus*: 1) Three trawls/d with gear at depth for 2.1 h, 2) four trawls/d with gear at depth for 1.2 h, 3) five trawls/d with gear at depth for 0.5 h, and 4) six trawls/d with gear at depth for 0.3 h.

FIGURE 3.—Comparison of precision-density curves of four different sampling strategies for *Sebastes aleutianus*: 1) Three trawls/d with gear at depth for 2.1 h, 2) four trawls/d with gear at depth for 1.2 h, 3) five trawls/d with gear at depth for 0.5 h, and 4) six trawls/d with gear at depth for 0.3 h.

which to focus sampling strategies. For rockfish, these are likely to be high density, highly aggregated species. Generally, in sampling strategies for these species, the effects of sample element size would be unimportant. Increases in sample size would be much more important in terms of increased precision; however, increases in sample size would have to be fairly large to make a significant difference.

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