AN APPLICATION OF YIELD MODELS TO A CALIFORNIA OCEAN SHRIMP POPULATION

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ABSTRACT

Two types of yield models were utilized to analyze fishery data from California's northernmost bed of ocean shrimp, *Pandalus jordani.* The Schaefer form of stock production model was applied to catch and effort data for the years 1954 through 1969. Age-structured catch data for 1955 through 1968 were analyzed by the Murphy method to obtain mortality rates and biomass estimates. Catchability coefficients and a growth curve were also estimated. Attempts to fit spawner-recruit models to estimates obtained from the age-structured catch data were inconclusive; so, age specific mortality and growth estimates were only used to fit a yield-per-recruit model.

After comparing the results from the two models, the Schaefer model was deemed most suitable for managing this fishery. The model estimated the maximum sustainable yield at 2.46 million pounds. A strategy for managing the fishery under a quota system was proposed.

The fishery for ocean shrimp, *Pandalus jordani,* in California has a unique importance despite the fact that it does not rank high among the State's fisheries in terms of pounds landed or value of the landings. This unique importance exists since the fishery developed after discovery of the shrimp beds by the California Department of Fish and Game's exploratory fishing and because it has been under continuous quota control by the California Fish and Game Commission since the fishery's inception in 1952 (Dahlstrom 1961, 1970). It is also the only California commercial fishery whose catch is fully regulated under a quota system.

This paper is limited to a discussion of the population and fishery which range along the coast from the mouth of the Mad River in California to the Rogue River in Oregon. This fishery consists primarily of regulated California vessels, but there is a small Oregon fleet not covered by California's regulations while fishing north of the California border. Three smaller populations which occur farther south in California are not considered here.

Initially, quotas were set arbitrarily at one-fourth the estimated biomass on the bed. Biomass was originally estimated from an examination of commercial catch data and later from research vessel cruise data. In later years, quota recommendations were at least partially directed toward allowing what was deemed an appropriate spawning stock to remain at the end of the season. Spawning stock values were based on estimated preseason year class abundance and estimated survival over the fishing season.

Estimating procedures which assume commercial or research fishing gear catches all shrimp in the water column above the swept path must inherently be negatively biased since escapement over, around, and through the gear occurs. The methods just discussed are of this type. A more complete account of the basis for quota recommendations prior to 1969 is found in Dahlstrom (1961, 1970), Dahlstrom and Gotshall (1969), and Gotshall (in press).

Over the history of this fishery substantial amounts of data have been collected. Of relevance to this paper are catch and effort data, estimated age and sex composition of landings,

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and research vessel biomass estimates for 1965 through 1968. These data were used in applying a stock production model and a dynamic pool model. The general characteristics of these two models were discussed by Schaefer and Beverton (1963) under the designations of "Schaefer Approach" and "Beverton-Holt Approach," respectively.

STOCK PRODUCTION MODEL

From an operational viewpoint stock production models possess the advantage of requiring only catch and effort data, which are usually available at relatively little expense, for their fitting. Another desirable characteristic is the inclusion of density dependent effects, even though they are treated grossly and population response to density is assumed to be instantaneous. Pella and Tomlinson (1969) discuss the assumptions implicit in the model. The most notable fisheries application of this type model was to yellowfin tuna of the eastern Pacific by Schaefer (1954, 1957), who developed a method for fitting the model to a population in a nonequilibrium state.

Pella (1967) examined a number of methods for estimating parameters of the Schaefer model and concluded that a surface searching technique for minimizing the summed, squared deviations between observed catches and catches predicted by an integrated form of the Schaefer model was generally most satisfactory.

Pella and Tomlinson (1969) generalized the Schaefer model to allow asymmetry in the integrated form and gave the population growth rate as

$$\frac{dP(t)}{dt} = HP^m(t) - KP(t) - qf(t)P(t), \quad (1)$$

where H, K, m, and q are constants. $P(t)$ represents the population size at time t, $f(t)$ is the fishing intensity at t, q is the catchability coefficient, and m determines the amount of asymmetry in the equilibrium yield curve. In the Schaefer model, $m = 2$ and the equilibrium curve is a parabola. The integral of (1) from time 0 to t with f constant is

$$P(t) = \left[\frac{H}{K+qf} - \left(\frac{H}{K+qf} - P(0)^{1-m} \right) \times e^{-(K+qf)(1-m)t} \right]^{\frac{1}{1-m}} \quad (2)$$

and Pella and Tomlinson (1969) used a numerical approximation of

$$C(t) = \int_0^t qf(t)P(t)\,dt \quad (3)$$

for computer calculation of expected catch over the interval. Pella (1967) gives the integrated form of (3) for the Schaefer model.

A computer program, GENPROD, (Pella and Tomlinson, 1969) for fitting the generalized model to catch and effort data uses the criterion of least squares between observed and predicted catches. Fox (1971) discusses least squares for estimating parameters in (2) and suggests alternatives which may be preferable to that used by GENPROD.

CATCH AND EFFORT DATA

Catch and effort data have been collected since the beginning of the fishery in 1952, but data from the first 2 years of the fishery are not used in this study because there was little effort and low catch-per-effort values indicated that fishermen had not fully acquired the skills needed for successfully catching shrimp. California landings were obtained from market receipts, and effort by California vessels was obtained from compulsory logbooks carried by all California trawlers. Oregon landings and effort were supplied by the Oregon Fish Commission (Jack Robinson, Oregon Fish Commission, personal communication).

California vessels were restricted to use of beam trawls until otter trawls became legal in 1963. Oregon vessels have used otter trawls since their entry into the fishery in 1960. A correction factor was used to convert California beam trawl effort to otter trawl effort for 1954 to 1962.

Fishing power of beam trawls relative to otter trawls was estimated from 40 pairs of catch-per-hour statistics. These paired statistics consisted of the average weekly catch-per-hour for each gear within a 10-fm depth interval bounded by a 10-min by 10-min block area. The data were collected during 1960 through 1962 when Oregon vessels were using otter trawls and California vessels were still restricted to beam trawls. California Department of Fish and Game trawler logbooks and information supplied by the Oregon Fish Commission were the sources of the records (Tom Jow, California Department of Fish and Game, personal communication).

With otter trawl taken as the standard gear, the relative log fishing power of beam trawls was computed by Robson's (1966) method except the two gear types were used in a manner analogous to his treatment of individual vessels. If the logarithm of catch-per-hour is normally distributed and the other assumptions of Robson's model hold, then his method produces B_i, an unbiased estimate of relative log fishing power, β_i, for the ith gear. However, exp (B_i) is a biased estimate of exp (β_i). An unbiased estimator for exp (β_i) is given by Laurent (1963) as

$$\widehat{\exp(\beta_i)} = \left[\exp(B_i)\right]\left[1 + \sum_{j=1}^{\infty} \frac{(-1)^j}{j!2^j} \times \frac{(n-k-1)^j \left[v(B_i)\right]^j}{(n-k-1)(n-k+1)\dots(n-k+2j-3)}\right], \tag{4}$$

where $v(B_i)$ is an unbiased estimate of the variance of B_i with $n-k-1$ degrees of freedom. Robson's method provides $v(B_i)$ and our computer program for calculating fishing power carries the series expansion in (4) to 15 terms. This computer program is described by Berude and Abramson (1972) and a FORTRAN listing is contained in Abramson (1971).

The estimated fishing power of beam trawls relative to otter trawls in the shrimp fishery was 0.71; all beam trawl effort used in this study was adjusted by that factor.

FITTING THE PRODUCTION MODEL

Usable catch and effort data covered a period of 16 years, each divided into open and closed seasons. Each season was treated as a separate interval in the fitting procedure and thus population estimates were obtained at 32 points in time. Table 1 shows catch, adjusted effort, and time for the series of seasons used to fit the generalized production model.

When initially fitting GENPROD to the data the parameters representing optimum effort (F_{opt}), catchability coefficient (q), maximum catch-per-effort $(U_{\max})$, and the ratio of initial population to maximum population (r) were unrestricted. Pella and Tomlinson (1969) give these parameters as transformations of those in (2). The equation was fitted with the parameter m taking values from 1.4 to 2.6 by increments of 0.2. Results showed that number or distribution of data points was not sufficient to determine the value of m with any degree of precision and that very small population estimates accompanied by excessively large q values were being obtained.

The first problem was handled by setting $m = 2$, since the symmetric or Schaefer model seemed best in face of the uncertainty. The catchability coefficient was fixed at a value which minimized the sum of the squared deviations between GENPROD'S estimates of $P(t)$ and research vessel cruise estimates of population biomass at seven time points when both were available. The research vessel biomass estimates were obtained from surveys conducted in the spring and fall of 1965, 1966, and 1967 and the fall of 1968 (Gotshall, in press). Gotshall's catch in weight per standard haul was expanded on an areal basis to provide estimates for the entire survey area; as mentioned previously, these are negatively biased. Based on this procedure, $q = 8.5 \times 10^{-5}$ was the best value. The final fit of the Schaefer model was made with GENPROD's computing parameters KK and N set equal to 5 and 10, respectively. KK is related to the fineness of the surface searching procedure, and N involves the accuracy of the numerical integration used to estimate expected catch. These computing parameters are explained fully in Pella and Tomlinson (1969).

GENPROD estimated a maximum equilibrium catch $(C_{\max})$ of 2.46 million pounds, an effort level required to obtain this catch under equilibrium conditions (F_{opt}) of 6,049 otter trawl

TABLE 1.—Estimates of Schaefer model parameters, observed catch and effort, predicted population size and catch, population and catch in millions of pounds, effort in thousands of hours.

Time interval		Population size end of interval	Applied effort	Observed catch	Predicted catch	Catch/effort	
Begin	End					Observed	Predicted
May 54	Aug. 54	8.63	0.206	0.169	0.150	0.819	0.727
Sept. 54	Apr. 55	9.07	0.0	0.0	0.0		
May 55	Oct. 55	8.82	0.733	0.505	0.557	0.689	0.760
Nov. 55	Apr. 56	9.11	0.0	0.0	0.0		
May 56	Sept. 56	8.57	1.11	0.896	0.836	0.803	0.750
Oct. 56	Apr. 57	9.00	0.0	0.0	0.0		
May 57	Oct. 57	8.59	1.05	0.748	0.783	0.713	0.746
Nov. 57	Apr. 58	8.96	0.0	0.0	0.0		
May 58	Sept. 58	8.18	1.61	1.14	1.17	0.706	0.726
Oct. 58	Apr. 59	8.76	0.0	0.0	0.0		
May 59	Sept. 59	7.83	2.01	1.69	1.41	0.841	0.702
Oct. 59	Mar. 60	8.45	0.0	0.0	0.0		
Apr. 60	Oct. 60	7.36	2.90	1.80	1.93	0.623	0.667
Nov. 60	May 61	8.22	0.0	0.0	0.0		
Jun. 61	Nov. 61	7.75	1.70	1.46	1.15	0.859	0.677
Dec. 61	Mar. 62	8.21	0.0	0.0	0.0		
Apr. 62	Oct. 62	6.39	4.70	2.98	2.87	0.635	0.611
Nov. 62	Mar. 63	7.23	0.0	0.0	0.0		
Apr. 63	Oct. 63	5.82	4.85	2.30	2.66	0.475	0.549
Nov. 63	Apr. 64	6.91	0.0	0.0	0.0		
May 64	Oct. 64	6.63	2.28	1.20	1.31	0.525	0.575
Nov. 64	Apr. 65	7.56	0.0	0.0	0.0		
May 65	Oct. 65	6.17	4.14	1.62	2.39	0.392	0.578
Nov. 65	Apr. 66	7.20	0.0	0.0	0.0		
May 66	Oct. 66	6.13	3.76	1.61	2.12	0.427	0.563
Nov. 66	Feb. 67	6.85	0.0	0.0	0.0		
Mar. 67	Oct. 67	6.22	3.71	2.26	2.05	0.608	0.553
Nov. 67	Apr. 68	7.24	0.0	0.0	0.0		
May 68	Oct. 68	6.72	2.54	2.67	1.50	1.052	0.592
Nov. 68	Feb. 69	7.36	0.0	0.0	0.0		
Mar. 69	Oct. 69	6.03	4.82	3.11	2.71	0.644	0.563
Nov. 69	Apr. 70	7.09	0.0	0.0	0.0		

Parameter estimates

C_{max}	F_{opt}	P_{opt}	q	U_{opt}	U_{max}	r	P_{max}	H	K
2.46	6.05	4.79	8.5×10^{-5}	0.407	0.814	0.884	9.58	-1.07×10^{-7}	−1.03

hours, and an optimum population size (P_{opt}) of 4.79 million pounds. Other parameter estimates, as defined by Pella and Tomlinson, and the complete output from the program are shown in Table 1.

Figure 1 shows both the expected catch as predicted by the model and the observed catch plotted against time. The fit appears to be generally quite good, although it has worsened during the most recent 5 years. The statistic R, derived by Pella and Tomlinson to measure the improvement in estimating catch from this model rather than from the mean catch, was 0.91. However, a somewhat spurious R is obtained when intervals with no catch are included in the data. This occurs because the model always predicts a zero catch from zero effort and the arithmetic mean cannot make such a prediction. Recalculating R from only periods when effort was applied yielded 0.75.

Figure 2 shows the fitted line ($m = 2$) for catch per unit effort versus effort in the equilibrium state and the observed catches per hour by year. However, the population should not have been in equilibrium during the period studied since the level of effort fluctuated from year to year.

The actual catch exceeded the estimated maximum equilibrium yield (2.46 million pounds) during the period 1954 through 1969 only three times (Table 1): 1962, 1968, and 1969. Effort has always been substantially below the estimated level which would produce the maximum sustainable yield. A literal interpretation of these results would indicate the population has been underexploited until recently.

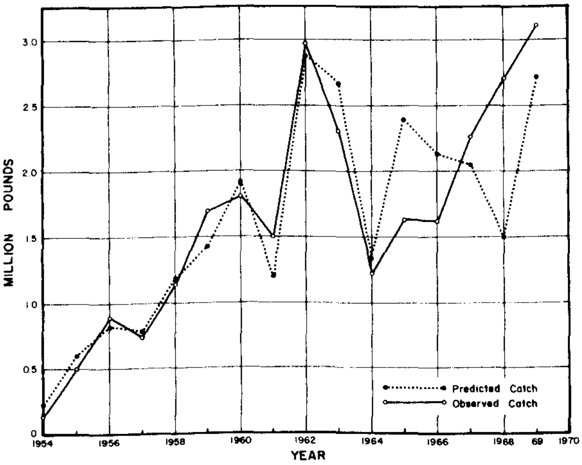

FIGURE 1.—Ocean shrimp catches predicted by GENPROD and observed catches for the years 1954 through 1969.

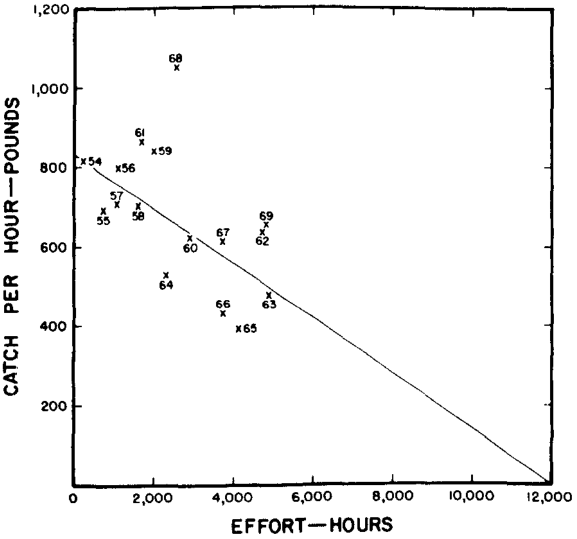

FIGURE 2.—Fitted model ($m = 2$) for catch-per-hour as a function of hours under equilibrium conditions.

It is a problem in actual management situations to deduce how well a model such as this represents a population. In years when the observed catch-per-unit effort deviates substantially from the corresponding expected value, it cannot be determined whether deviations are due to an actual departure from the expected population size or due to a temporary change in the catchability. In the management strategy which we will discuss later, we are assuming the population size is being predicted correctly by the model and we are essentially ignoring deviations between the observed and expected catch insofar as they may represent actual population deviations.

DYNAMIC POOL MODEL

Catch data by age categories, both in weight and numbers, were utilized to estimate mortality, growth, and recruitment parameters necessary in a dynamic pool model.

AGE-STRUCTURED CATCH DATA

Catches from the population were landed at Eureka and Crescent City, Calif., and Brookings, Oreg. Landings data were obtained mainly from the Pacific Marine Fisheries Commission Data Series (1965-1969). Catches south of the California-Oregon border were recorded in that publication in tables for PMFC Area 96, but those from north of the border were included in, but did not comprise all of, the catch reported from PMFC Area 88. Catches within PMFC Area 88 south of the Rogue River were obtained from the Oregon Fish Commission (Jack Robinson, Oregon Fish Commission, personal communication). Catches made in the more recent years were obtained from the California Department of Fish and Game Shellfish Program (Daniel Gotshall and Walter Dahlstrom, California

Department of Fish and Game, personal communication). Virtually all catches were made during single day trips.

Landings were stratified into port-months, with Eureka-Crescent City as "California" and Brookings as "Oregon." Relative age frequency and weight at age were determined from samples of most port-month catches. Values used for California strata not sampled were either the average of preceding and following strata or the nearest sampled strata of the same season. The Oregon Fish Commission provided values for all Oregon strata.

Several methods of drawing samples from within strata were used by California. For all but very recent years, the methods were equivalent to assuming a simple random sample of shrimp from within strata. These sampled shrimp were aged by carapace length measurements, and the fraction falling into a specific age group determined its relative frequency. In recent years a simple random sample of boatloads was assumed drawn, and the length composition of a subsample from each boatload was weighted by the estimated number of shrimp in the load. Estimates by strata, done separately for Oregon and California, were combined to obtain the values in Table 2.

The average weight at age was determined by two methods: (1) the aged shrimp were placed into length frequency groups, a length-weight key was used to convert length to weight, and average weight for each age group was calculated; (2) the aged shrimp were weighed and an average weight computed directly for each age group. The study of aged catch data was performed for the 1955 through 1968 seasons. All aged shrimp fell into age groups 0, I, II, or III, but the 0 group was rare and omitted from the study.

Catch by age category for 2,598 million shrimp (22.88 million pounds) harvested during 1955 through 1968 are listed by month in Table 2. During the first 7 of these years, the fishery was active during 39 months and captured an estimated 954 million shrimp, excluding age 0, yielding a monthly average of 24.5 million. These shrimp weighed about 8.25 million pounds, averaging 212,000 lb. per month of fishing and 0.0086 lb. per shrimp. The fishery was active during 46 months of the second 7 years and caught an estimated 1,644 million shrimp, excluding age 0, for a monthly average of 35.7 million. These weighed about 14.63 million pounds, averaging 318,000 lb. per month and 0.0089 lb. per shrimp. The relative frequencies in numbers during the first 7 years were: 0.559 for age I, 0.422 for age II, and 0.019 for age III. During the second 7 years the frequencies were 0.495 for age I, 0.463 for age II, and 0.042 for age III. The reliability of the age frequency values is uncertain due to the aging method.

GROWTH CURVE

A growth in weight curve was obtained empirically by plotting average weights of shrimp by month and age for all seasons 1955 through 1968 (Tables 2 and 3, Figure 3). Dahlstrom (1970) and Gotshall (California Department of

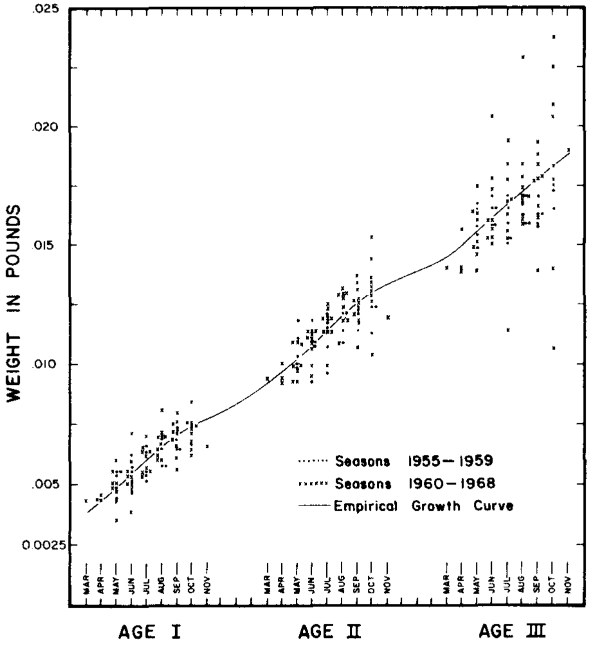

FIGURE 3.—Ocean shrimp growth in weight by month from sampling commercial landings. Seasons included are 1955 through 1968.

TABLE 2.—Aged catch[1] and catch-per effort (C.P.E.) statistics.
[Pounds and numbers in thousands.]

Season	Month	Age group	Relative frequency	Average weight (lb.)	C.P.E. numbers	Pounds	Numbers
1955	May	I	0.226	0.0055	9.9	4.3	782
		II	.747	.0118	32.7	30.4	2586
		III	.027	.0169	1.2	1.6	93
	June	I	.408	.0046	26.2	21.0	4531
		II	.576	.0118	37.0	75.3	6397
		III	.016	.0172	1.0	3.1	178
	July	I	.425	.0055	38.4	30.8	5603
		II	.565	.0122	51.0	90.8	7449
		III	.010	.0172	.9	2.3	132
	Aug.	I	.378	.0057	28.8	28.0	4897
		II	.603	.0122	46.0	95.3	7812
		III	.019	.0172	1.4	4.2	246
	Sept.	I	.588	.0064	49.9	47.7	7441
		II	.397	.0123	33.7	62.0	5024
		III	.015	.0172	1.3	3.3	190
	Oct.	I	.588	.0067	19.3	2.1	309
		II	.397	.0125	13.0	2.6	208
		III	.015	.0175	.5	.1	8
1956	May	I	.342	.0044	45.2	19.7	4479
		II	.608	.0099	80.4	78.8	7963
		III	.050	.0156	6.6	10.2	655
	June	I	.140	.0051	13.1	16.1	3166
		II	.833	.0093	78.2	174.4	18839
		III	.027	.0159	2.5	9.7	611
	July	I	.140	.0051	13.0	13.7	2697
		II	.843	.0097	78.5	157.7	16240
		III	.017	.0154	1.6	5.0	327
	Aug.	I	.161	.0057	13.0	27.8	4857
		II	.818	.0110	66.0	271.2	24680
		III	.021	.0169	1.7	10.7	634
	Sept.	I	.230	.0064	11.4	14.2	2221
		II	.753	.0115	37.3	83.6	7271
		III	.017	.0164	.8	2.7	164
1957	May	I	.366	.0053	29.5	22.0	4159
		II	.629	.0104	50.7	74.4	7147
		III	.005	.0167	.4	.9	57
	June	I	.592	.0057	33.4	21.9	3807
		II	.403	.0110	22.7	28.5	2592
		III	.005	.0167	.3	.5	32
	July	I	.592	.0063	33.0	11.2	1785
		II	.403	.0116	22.5	14.1	1215
		III	.005	.0167	.3	.3	15
	Aug.	I	.652	.0070	60.9	229.0	32515
		II	.343	.0122	32.0	208.6	17105
		III	.005	.0161	.5	4.0	249
	Sept.	I	.597	.0072	40.0	52.4	7235
		II	.386	.0122	25.8	57.0	4678
		III	.017	.0161	1.1	3.3	206
	Oct.	I	.597	.0074	50.9	9.0	1221
		II	.386	.0125	32.9	9.9	790
		III	.017	.0167	1.4	.6	35
1958	May	I	.429	.0048	30.1	41.6	8648
		II	.541	.0101	38.0	110.0	10906
		III	.030	.0150	2.1	9.1	605
	June	I	.429	.0055	37.2	82.3	14987
		II	.541	.0108	46.9	203.2	18899
		III	.030	.0159	2.6	16.6	1048
1958	July	I	.492	0.0053	29.7	30.9	5842
		II	.496	.0108	29.9	63.3	5890
		III	.012	.0152	.7	2.2	142
	Aug.	I	.596	.0064	64.8	121.7	18992
		II	.394	.0115	42.9	144.3	12555
		III	.010	.0161	1.1	5.1	319
	Sept.	I	.786	.0068	70.8	209.9	30649
		II	.209	.0118	18.8	95.9	8150
		III	.005	.0159	.5	3.1	195
1959	May	I	.599	.0055	87.2	104.0	18919
		II	.355	.0112	51.7	126.0	11213
		III	.046	.0159	6.7	23.1	1453
	June	I	.748	.0062	87.8	268.8	43542
		II	.240	.0115	28.2	160.6	13971
		III	.012	.0167	1.4	11.6	699
	July	I	.595	.0066	53.1	211.4	31921
		II	.382	.0118	34.1	241.1	20494
		III	.023	.0161	2.1	19.9	1234
	Aug.	I	.648	.0069	47.9	91.1	13120
		II	.329	.0120	24.3	80.3	6661
		III	.023	.0164	1.7	7.6	466
	Sept.	I	.720	.0073	66.8	207.3	28403
		II	.258	.0123	23.9	125.7	10178
		III	.022	.0164	2.0	14.2	868
1960	May	I	.601	.0042	45.1	48.2	11464
		II	.382	.0100	28.7	73.2	7287
		III	.017	.0153	1.3	5.0	324
	June	I	.689	.0050	53.2	94.0	18882
		II	.289	.0115	22.3	91.5	7920
		III	.022	.0162	1.7	9.8	603
	July	I	.798	.0055	78.5	321.7	58428
		II	.192	.0114	18.9	160.5	14058
		III	.010	.0172	1.0	12.6	732
	Aug.	I	.700	.0060	63.4	281.4	47073
		II	.276	.0110	25.0	204.0	18560
		III	.024	.0162	2.2	26.1	1614
	Sept.	I	.699	.0066	53.5	210.6	31760
		II	.264	.0122	20.2	146.6	11995
		III	.037	.0168	2.8	28.3	1681
	Oct.	I	.698	.0076	33.3	51.3	6790
		II	.280	.0132	13.4	35.9	2724
		III	.022	.0179	1.0	3.8	214
1961	June	I	.454	.0052	48.8	52.3	10053
		II	.531	.0112	57.1	131.5	11759
		III	.015	.0162	1.6	5.4	332
	July	I	.441	.0063	36.1	79.0	12559
		II	.549	.0126	44.9	196.7	15635
		III	.010	.0170	.8	4.8	285
	Aug.	I	.337	.0069	25.8	118.9	17195
		II	.643	.0131	49.2	428.3	32809
		III	.020	.0172	1.5	17.6	1021
	Sept.	I	.400	.0076	31.6	62.0	8152
		II	.574	.0135	45.3	158.1	11698
		III	.026	.0190	2.1	10.1	530
	Oct.	I	.222	.0073	15.7	23.4	3181
		II	.758	.0146	53.5	158.5	10860
		III	.020	.0227	1.4	6.5	287

TABLE 2.—Continued.

Season	Month	Age group	Relative frequency	Average weight (lb.)	C.P.E. numbers	Pounds	Numbers
1961	Nov.	I	.633	0.0066	33.1	4.8	728
		II	.345	.0121	18.0	4.8	397
		III	.022	.0192	1.2	.5	25
1962	Apr.	I	.460	.0043	38.6	44.8	10512
		II	.465	.0093	39.1	99.0	10626
		III	.075	.0141	6.3	24.1	1714
	May	I	.460	.0047	41.8	121.8	25684
		II	.465	.0100	42.3	259.1	25964
		III	.075	.0148	6.8	62.0	4188
	June	I	.460	.0050	30.9	80.2	15998
		II	.465	.0113	31.2	182.9	16172
		III	.075	.0180	5.0	46.8	2608
	July	I	.414	.0053	22.4	86.4	16208
		II	.537	.0120	29.0	253.3	21024
		III	.049	.0179	2.6	34.3	1918
	Aug.	I	.560	.0059	32.6	249.2	42011
		II	.395	.0129	23.0	382.9	29633
		III	.045	.0231	2.6	78.0	3376
	Sept.	I	.560	.0068	41.2	281.6	41305
		II	.410	.0128	30.2	387.7	30241
		III	.030	.0180	2.2	39.7	2213
	Oct.	I	.550	.0064	51.3	95.1	14976
		II	.380	.0136	35.4	140.8	10347
		III	.070	.0177	6.5	33.7	1906
1963	Apr.	I	.160	.0043	11.2	7.5	1751
		II	.770	.0094	54.0	79.3	8427
		III	.070	.0142	4.9	10.9	766
	May	I	.162	.0034	10.6	20.7	6028
		II	.750	.0092	49.0	257.7	27909
		III	.088	.0177	5.7	57.9	3275
	June	I	.171	.0038	8.9	26.1	6811
		II	.730	.0096	37.9	280.4	29077
		III	.099	.0155	5.1	61.1	3943
1963	July	I	.165	.0055	6.8	39.5	7236
		II	.725	.0114	29.7	361.7	31796
		III	.110	.0154	4.5	74.4	4824
	Aug.	I	.274	.0061	11.1	99.6	16226
		II	.674	.0125	27.3	497.1	39915
		III	.052	.0176	2.1	54.1	3079
	Sept.	I	.289	.0061	12.5	46.8	7687
		II	.661	.0119	28.7	209.3	17582
		III	.050	.0163	2.2	21.7	1330
	Oct.	I	.330	.0068	14.4	20.1	2943
		II	.660	.0127	28.9	74.8	5886
		III	.010	.0108	.4	1.0	89
1964	May	I	.460	.0059	26.4	65.6	11102
		II	.523	.0110	30.1	139.3	12622
		III	.017	.0165	1.0	6.8	410
	June	I	.430	.0071	23.5	95.2	13352
		II	.550	.0107	30.0	182.3	17078
		III	.020	.0206	1.1	12.8	621
	July	I	.505	.0069	24.1	68.8	9924
		II	.474	.0124	22.6	115.9	9315
		III	.021	.0196	1.0	8.1	413
	Aug.	I	.548	.0081	30.4	183.7	22554
		II	.431	.0132	23.9	233.4	17739
		III	.021	.0186	1.2	16.0	864
1964	Sept.	I	.548	0.0071	29.5	9.7	1359
		II	.431	.0128	23.2	13.7	1069
		III	.021	.0179	1.1	.9	52
	Oct.	I	.548	.0076	26.7	17.8	2351
		II	.431	.0131	21.0	24.3	1849
		III	.021	.0186	1.0	1.7	90
1965	May	I	.612	.0049	29.4	98.2	20029
		II	.275	.0098	13.2	88.4	9000
		III	.113	.0141	5.4	52.3	3698
	June	I	.704	.0048	47.7	168.8	35439
		II	.266	.0109	18.0	146.3	13390
		III	.030	.0154	2.0	23.3	1510
	July	I	.868	.0056	61.5	456.1	81360
		II	.119	.0115	8.4	128.8	11154
		III	.013	.0164	.9	19.9	1219
	Aug.	I	.858	.0069	35.6	177.7	25734
		II	.126	.0130	5.2	49.1	3779
		III	.016	.0172	.7	8.2	480
	Sept.	I	.865	.0075	30.9	105.2	14112
		II	.124	.0132	4.4	26.7	2023
		III	.011	.0185	.4	3.3	179
	Oct.	I	.807	.0074	30.6	48.7	6545
		II	.176	.0137	6.7	19.5	1427
		III	.017	.0211	.6	2.9	138
1966	May	I	.147	.0050	7.2	11.4	2273
		II	.795	.0093	38.9	114.4	12295
		III	.058	.0151	2.8	13.5	897
	June	I	.230	.0053	11.1	70.7	13397
		II	.735	.0100	35.6	426.8	42812
		III	.035	.0158	1.7	32.2	2039
	July	I	.252	.0062	13.4	94.5	15364
		II	.717	.0100	38.1	437.6	43714
		III	.031	.0116	1.6	22.0	1890
	Aug.	I	.292	.0064	9.9	33.4	5223
		II	.687	.0119	23.3	146.1	12289
		III	.021	.0161	.7	6.1	376
	Sept.	I	.410	.0072	12.6	36.1	5022
		II	.586	.0127	18.0	91.4	7178
		III	.004	.0181	.1	.9	49
	Oct.	I	.424	.0072	15.8	19.4	2682
		II	.567	.0134	21.1	47.9	3587
		III	.009	.0206	.3	1.2	57
1967	Mar.	I	.674	.0042	51.7	10.4	2482
		II	.159	.0094	12.2	5.5	586
		III	.167	.0142	12.8	8.7	615
	Apr.	I	.730	.0044	60.7	45.6	10444
		II	.176	.0101	14.6	25.5	2518
		III	.094	.0158	7.8	21.2	1345
	May	I	.723	.0048	47.3	11.9	2461
		II	.176	.0110	11.5	6.6	599
		III	.101	.0162	6.6	5.6	344
	June	I	.739	.0055	61.8	294.7	53862
		II	.225	.0115	18.8	188.9	16399
		III	.036	.0152	3.0	39.8	2624
	July	I	.804	.0063	79.0	672.5	106859
		II	.172	.0120	16.9	273.8	22860
		III	.024	.0157	2.4	50.2	3190

TABLE 2.—Continued.

Season	Month	Age group	Relative frequency	Average weight (lb.)	C.P.E. numbers	Pounds	Numbers
1967	Aug.	I	.868	0.0071	61.9	359.7	50317
		II	.107	.0131	7.6	81.5	6203
		III	.025	.0171	1.8	24.7	1449
	Sept.	I	.840	.0079	30.0	76.8	9734
		II	.130	.0138	4.6	20.8	1506
		III	.030	.0195	1.1	6.8	348
	Oct.	I	.784	.0084	24.4	15.1	1807
		II	.171	.0154	5.3	6.1	394
		III	.045	.0239	1.4	2.5	104
1968	May	I	.193	.0054	17.1	77.9	14303
		II	.726	.0109	64.4	584.2	53801
		III	.081	.0166	7.2	99.4	6003
	June	I	.268	.0059	46.6	153.0	26091
		II	.715	.0112	124.3	776.9	69609
		III	.017	.0175	3.0	28.9	1655
1968	July	I	.380	0.0065	40.9	154.7	23947
		II	.611	.0120	65.7	460.6	38504
		III	.009	.0186	1.0	10.5	567
	Aug.	I	.277	.0066	15.0	46.1	6982
		II	.685	.0120	37.1	206.8	17267
		III	.038	.0180	2.1	17.2	958
	Sept.	I	.161	.0056	9.4	4.2	761
		II	.814	.0108	47.5	41.6	3848
		III	.025	.0141	1.5	1.7	118
	Oct.	I	.164	.0062	10.1	.5	81
		II	.812	.0105	50.2	4.2	402
		III	.024	.0142	1.5	.2	12

[1] Catches of 0 age groups are not included.

TABLE 3.—Average weight in pounds by age. From aged catch landed in northern California and southern Oregon, 1955-1968.

Month	Age I	Age II	Age III
March	0.0038	0.0092	0.0146
April	.0043	.0098	.0152
May	.0049	.0104	.0158
June	.0055	.0110	.0164
July	.0060	.0116	.0168
August	.0060	.0121	.0174
September	.0065	.0127	.0180
October	.0070	.0132	.0185
November	.0074	.0135	.0190
January	.0086	.0140	

Fish and Game, personal communication) indicated that shrimp grow faster in the open season than during the closed season. Hence, the empirical curve was drawn to show seasonal differences in the growth rate. A more objective fit of the data could be obtained, but it would not alter the results enough to change the conclusions contained herein.

The curve shows relatively constant (linear) growth in weight during the open season, but slower growth during the closed period. The shrimp apparently do not approach an asymptotic weight prior to reaching maximum age in the fishery, and growth in weight could be described as linear during the exploited phase. Obviously, there was considerable variation, increasing with age.

Annual average count per pound for ages I, II, and III combined varied from 94 in 1961 to 142 in 1965 (Figure 4). Monthly values varied from 76 to 155 with an average for all years of 114.

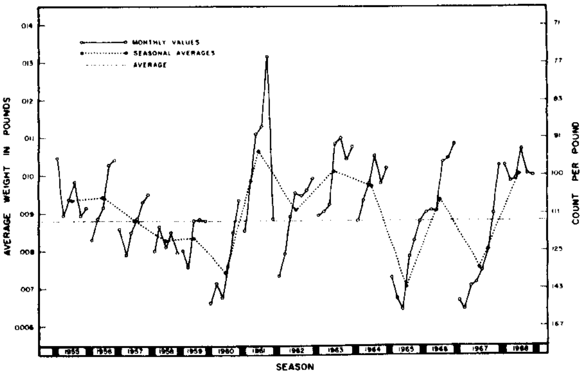

FIGURE 4.—Average size of ocean shrimp in the landings by month, season, and overall.

Because of the variation exhibited by the size at age data, it is possible that significant random or systematic errors are contained in the age composition data and that the subsequent analyses of these data will be correspondingly affected.

PARAMETER ESTIMATION WITH THE MURPHY METHOD

Reference Values

We used the generalized Murphy catch equation (Tomlinson, 1970) to analyze aged catch data. Gotshall (in press) provides estimates of natural mortality and biomass based upon a fishery independent randomized trawling scheme (Abramson, 1968). Since the biomass estimates are inherently negatively biased regardless of the catchability of shrimp and the mortality estimates may deviate from the population parameters in either direction, we decided to choose a natural mortality which would provide Murphy Method biomass estimates of a magnitude similar to those obtained from the randomized trawling scheme.

An annual natural mortality coefficient of $M = 1.44$ applied to all age groups yielded the appropriate biomass estimates. This is within the range of the M values given in Gotshall's (in press) Table 6 and cannot be considered significantly different from those estimates in view of the sizes of the standard errors shown in his Table 9.

Constructing Catch Ratios

Ratios of number caught in month $i+1$ to number caught in month i were calculated for all age III catches, giving values useful for within-season estimation of fishing mortality. To estimate across the closed seasons, the ratio of catch at age III in the first catch-month of season $i+1$ to catch at age II in the last catch-month of season i was calculated. For example, with 2 seasons and 3 months in each season, the ratios computed by this scheme would be: $R(1) = C_{III}(2)/C_{III}(1)$; $R(2) = C_{III}(3)/C_{III}(2)$; $R(3) = C_{III}(4)/C_{II}(3)$; $R(4) = C_{III}(5)/C_{III}(4)$; $R(5) = C_{III}(6)/C_{III}(5)$, where the catches used represent monthly catches by age (Table 4) and a closed season exists between months 3 and 4.

An additional assumption is that the exploitation rate (E) during the last month of each season is equal for ages II and III. Thus in the example, $E_{II}(3) = E_{III}(3)$. This assumption is necessary to allow estimation across the closed season.

Using these ratios for age III within season and age III to age II between seasons and assuming various exploitation rates for the last month of fishing in 1968, it was possible to make numerous estimates of the exploitation rates at age III during each month of fishing from 1955 to 1968. The Murphy method with backward calculation (Tomlinson, 1970) was used. The technique is similar to one used by Murphy (1965, 1966), except that Murphy used years instead of months, combined some age groups within years, had no years without catches, and did not treat year classes separately.

The data were separated into catches from year classes 1952 through 1967. Using the same hypothetical example as before (Table 4), the catch data can be put in the order $C_{I}(1)$, $C_{I}(2)$, $C_{I}(3)$, 0, $C_{II}(4)$, $C_{II}(5)$, $C_{II}(6)$, 0, $C_{III}(7)$, $C_{III}(8)$, $C_{III}(9)$. The catch ratios are computed as $C_{I}(2)/C_{I}(1)$, $C_{I}(3)/C_{I}(2)$, 0, $C_{II}(4)/C_{I}(3)$, $C_{II}(5)/C_{II}(4)$, $C_{II}(6)/C_{II}(5)$, 0, $C_{III}(7)/C_{II}(6)$, $C_{III}(8)/C_{III}(7)$, $C_{III}(9)/C_{III}(8)$. Since these catches all came from the same cohort, the Murphy method can be used to estimate $E_{I}(1)$, $E_{I}(2), \ldots, E_{III}(8)$, given that $E_{III}(9)$ is known. The previous analysis of age III data gave estimates of E at age III during the last month of fishing in each season, and these were used as starting values for backward calculation on each year class from 1952 through 1965. It was necessary in estimating E for the 1966 and 1967 year classes to choose values which gave an average population size in 1968 similar to the results obtained from fitting the Schaefer model.

Additional Modifications and Assumptions

Two additional assumptions fundamental to the results are: (1) ages II and III were exploited at the same rate, on the average, over the entire time period; (2) the catchability coefficient (q), computed as monthly catch-per-effort in weight divided by estimated average population weight for the combined age groups during the month, was reasonably constant over the entire time period. In order to satisfy these two assumptions, it was necessary to alter some

TABLE 4.—Hypothetical structure of age-structured shrimp catches and exploitation rates as arranged for analysis by the Murphy method.

Season	Catch-month	Catches by ages			Exploitation rates by ages		
		Age I	Age II	Age III	Age I	Age II	Age III
1	1	$C_{I}(1)$	$C_{II}(1)$	$C_{III}(1)$	$E_{I}(1)$	$E_{II}(1)$	$E_{III}(1)$
1	2	$C_{I}(2)$	$C_{II}(2)$	$C_{III}(2)$	$E_{I}(2)$	$E_{II}(2)$	$E_{III}(2)$
1	3	$C_{I}(3)$	$C_{II}(3)$	$C_{III}(3)$	$E_{I}(3)$	$E_{II}(3)$	$E_{III}(3)$
	Closed season						
2	4	$C_{I}(4)$	$C_{II}(4)$	$C_{III}(4)$	$E_{I}(4)$	$E_{II}(4)$	$E_{III}(4)$
2	5	$C_{I}(5)$	$C_{II}(5)$	$C_{III}(5)$	$E_{I}(5)$	$E_{II}(5)$	$E_{III}(5)$
2	6	$C_{I}(6)$	$C_{II}(6)$	$C_{III}(6)$	$E_{I}(6)$	$E_{II}(6)$	$E_{III}(6)$
	Closed season						
3	7	$C_{I}(7)$	$C_{II}(7)$	$C_{III}(7)$	$E_{I}(7)$	$E_{II}(7)$	$E_{III}(7)$
3	8	$C_{I}(8)$	$C_{II}(8)$	$C_{III}(8)$	$E_{I}(8)$	$E_{II}(8)$	$E_{III}(8)$
3	9	$C_{I}(9)$	$C_{II}(9)$	$C_{III}(9)$	$E_{I}(9)$	$E_{II}(9)$	$E_{III}(9)$

of the E values from the age III analysis used as starting values for the year class solutions.

An additional problem occurred which resulted in some final changes that were arbitrary and difficult to explain. For some years, especially 1955 through 1959, estimates of population size were quite low and q very high. It was demonstrated that a good transfer from age III to age II across the closed season did not occur for the year classes involved. Therefore, with year classes 1953 through 1958, 1962, 1963, and 1966, the estimation from the last catch-month at age II to the first catch-month at age I disregarded estimates during age III. It is hoped that the final result justifies these arbitrary decisions. It was also noted from the dots on Figure 3 that a growth curve from the sample data (Table 2) for seasons 1955 through 1959 indicates faster growth during the closed season than during the open season. This seems extremely doubtful in light of other contrary evidence and indicates that the problem was caused by inaccurate aging. Since age III shrimp make up such a small fraction of the catch and population biomass, it was not considered to seriously discredit final results.

Fishing Mortality Estimates

Estimation of monthly instantaneous fishing mortality coefficients, (F), was accomplished for each age group in each month by applying the Murphy method, as described above, to catches in numbers (Table 2). Since $M = 0.12$ was used as monthly instantaneous natural mortality for all months and ages, monthly exploitation rates, E, and monthly survival rates, s, may be obtained from

$$E = F\left[1 - e^{-(F+0.12)}\right]/(F + 0.12),$$

and

$$s = e^{-(F+0.12)}.$$

The estimates of F (Table 5) varied considerably, but age I was always exploited at a rate lower than ages II and III. During the 7 years, 1955-1961, average estimated F was 0.015 for age I, 0.056 for age II, and 0.057 for age III. In the 7 years, 1962-1968, $F(\text{I}) = 0.023$, $F(\text{II}) = 0.116$, and $F(\text{III}) = 0.159$. Averages for all 14 years were $F(\text{I}) = 0.019$, $F(\text{II}) = 0.088$, and $F(\text{III}) = 0.089$. Thus, as previously stated for a condition of estimation, ages II and III were exploited at about the same rates.

Converting fishing mortality to exploitation (Table 6), it was estimated that the fishery was removing about 5% of ages II and III and 1% of age I each month of fishing. Fishing intensity increased over the years and during 1962-1968 exploitation was nearly double that of 1955-1961 for each age. During the period 1961-1967, July and August were the most important months, followed by May, June, and September, while April and October were of little importance. Average F (Table 10) during these years, for

TABLE 5.—Monthly instantaneous fishing mortality coefficients.

Year	Month	Age group		
		I	II	III
1955	May	0.001	0.011	0.006
	June	.003	.032	.013
	July	.004	.044	.011
	Aug.	.004	.055	.023
	Sept.	.007	.042	.021
	Oct.	.001	.002	.001
	Mean	.0033	.0310	.0125
1956	May	.003	.021	.042
	June	.002	.058	.046
	July	.002	.060	.029
	Aug.	.005	.112	.066
	Sept.	.003	.040	.020
	Mean	.0030	.0582	.0406
1957	May	.003	.022	.014
	June	.003	.009	.009
	July	.002	.005	.005
	Aug.	.038	.080	.096
	Sept.	.010	.026	.098
	Oct.	.002	.005	.020
	Mean	.0097	.0245	.0433
1958	May	.006	.039	.071
	June	.012	.081	.154
	July	.005	.030	.026
	Aug.	.020	.077	.068
	Sept.	.038	.060	.050
	Mean	.0162	.0574	.0738
1959	May	.016	.037	.090
	June	.042	.055	.053
	July	.036	.098	.113
	Aug.	.017	.038	.052
	Sept.	.043	.070	.120
	Mean	.0308	.0596	.0856
1960	May	.008	.030	.019
	June	.015	.038	.042
	July	.055	.080	.060
	Aug.	.052	.133	.168
	Sept.	.042	.109	.241
	Oct.	.010	.030	.040
	Mean	.0303	.0700	.0783
1961	June	.009	.048	.021
	July	.013	.077	.031
	Aug.	.020	.209	.137
	Sept.	.011	.098	.090
	Oct.	.005	.114	.059
	Nov.	.001	.005	.006
	Mean	.0098	.0918	.0590
1962	Apr.	.006	.034	.040
	May	.018	.099	.120
	June	.013	.016	.094
	July	.015	.122	.085
	Aug.	.045	.231	.193
	Sept.	.053	.354	.171
	Oct.	.022	.179	.200
	Mean	.0246	.1564	.1290
1963	Apr.	0.002	0.026	0.030
	May	.007	.105	.161
	June	.009	.140	.270
	July	.011	.204	.555
	Aug.	.029	.386	.762
	Sept.	.016	.267	.812
	Oct.	.007	.123	.100
	Mean	.0116	.1787	.3843
1964	May	.017	.071	.021
	June	.024	.119	.038
	July	.021	.080	.029
	Aug.	.055	.198	.072
	Sept.	.004	.015	.005
	Oct.	.008	.030	.010
	Mean	.0215	.0855	.0292
1965	May	.017	.069	.298
	June	.035	.128	.174
	July	.098	.137	.190
	Aug.	.037	.058	.098
	Sept.	.024	.036	.044
	Oct.	.013	.030	.040
	Mean	.0373	.0763	.1407
1966	May	.002	.057	.097
	June	.017	.262	.301
	July	.023	.423	.456
	Aug.	.009	.183	.140
	Sept.	.010	.142	.022
	Oct.	.006	.090	.030
	Mean	.0112	.1928	.1743
1967	Mar.	.001	.002	.030
	Apr.	.007	.011	.078
	May	.002	.003	.024
	June	.045	.100	.228
	July	.108	.180	.432
	Aug.	.063	.062	.325
	Sept.	.014	.018	.110
	Oct.	.003	.005	.040
	Mean	.0304	.0476	.1584
1968	May	.025	.098	.207
	June	.054	.162	.074
	July	.059	.116	.030
	Aug.	.020	.064	.060
	Sept.	.002	.017	.009
	Oct.	.001	.002	.001
	Mean	.0268	.0765	.0635

TABLE 6.—Monthly exploitation rates.

Year	Month	Age group I	Age group II	Age group III
1955	May	0.0004	0.0107	0.0056
	June	.0029	.0301	.0121
	July	.0041	.0408	.0103
	Aug.	.0040	.0505	.0218
	Sept.	.0069	.0387	.0194
	Oct.	.0003	.0019	.0009
1956	May	.0030	.0195	.0385
	June	.0024	.0531	.0422
	July	.0023	.0547	.0266
	Aug.	.0046	.0995	.0599
	Sept.	.0024	.0370	.0187
1957	May	.0031	.0203	.0134
	June	.0032	.0085	.0086
	July	.0017	.0045	.0046
	Aug.	.0349	.0720	.0860
	Sept.	.0091	.0241	.0883
	Oct.	.0017	.0047	.0187
1958	May	.0058	.0362	.0642
	June	.0114	.0736	.1346
	July	.0051	.0281	.0240
	Aug.	.0188	.0695	.0623
	Sept.	.0348	.0549	.0460
1959	May	.0146	.0346	.0812
	June	.0385	.0504	.0482
	July	.0332	.0881	.1011
	Aug.	.0160	.0356	.0482
	Sept.	.0396	.0638	.1067
1960	May	.0076	.0277	.0181
	June	.0141	.0350	.0387
	July	.0501	.0727	.0552
	Aug.	.0481	.1172	.1457
	Sept.	.0385	.0976	.2024
	Oct.	.0097	.0279	.0370
1961	June	.0083	.0442	.0290
	July	.0119	.0696	.0290
	Aug.	.0185	.1778	.1207
	Sept.	.0101	.0880	.0810
	Oct.	.0045	.1016	.0541
	Nov.	.0012	.0047	.0056
1962	Apr.	.0061	.0311	.0371
	May	.0171	.0885	.1064
	June	.0121	.0686	.0842
	July	.0140	.1084	.0767
	Aug.	.0416	.1946	.1657
	Sept.	.0483	.2820	.1485
	Oct.	.0208	.1550	.1712
1963	Apr.	0.0017	0.0246	0.0282
	May	.0067	.0943	.1402
	June	.0086	.1230	.2235
	July	.0104	.1744	.4037
	Aug.	.0267	.3028	.5062
	Sept.	.0147	.2211	.5282
	Oct.	.0064	.1090	.0898
1964	May	.0162	.0644	.0199
	June	.0223	.1054	.0347
	July	.0192	.0730	.0270
	Aug.	.0501	.1698	.0656
	Sept.	.0036	.0141	.0048
	Oct.	.0070	.0279	.0094
1965	May	.0162	.0629	.2433
	June	.0328	.1130	.1509
	July	.0880	.1206	.1635
	Aug.	.0346	.0528	.0878
	Sept.	.0222	.0338	.0407
	Oct.	.0119	.0279	.0370
1966	May	.0024	.0524	.0870
	June	.0160	.2179	.2458
	July	.0210	.3262	.3471
	Aug.	.0082	.1577	.1230
	Sept.	.0090	.1247	.0208
	Oct.	.0055	.0810	.0279
1967	Mar.	.0013	.0022	.0277
	Apr.	.0063	.0107	.0703
	May	.0017	.0029	.0219
	June	.0414	.0895	.1929
	July	.0968	.1554	.3323
	Aug.	.0573	.0569	.2622
	Sept.	.0133	.0166	.0983
	Oct.	.0028	.0050	.0370
1968	May	.0233	.0876	.1767
	June	.0492	.1409	.0676
	July	.0538	.1033	.0281
	Aug.	.0187	.0587	.0552
	Sept.	.0023	.0157	.0081
	Oct.	.0003	.0019	.0009

June to September, was 0.16 for age II, 0.21 for age III, and only 0.03 for age I.

Biomass Estimation

The Murphy method produces estimates of population size in numbers at the beginning of each catch interval. The present study also required estimates of biomass. Murphy (1966) stated he computed biomass by dividing the catch in weight by the appropriate estimate of *E* from his analysis of numbers in the catch. This would result in a positively biased biomass estimate, since it is equivalent to multiplying the number alive at the beginning of a catch interval by the average weight during the interval.

Two ways of computing the correct estimates of biomass utilizing Murphy's method to estimate numbers are possible:

(1) multiply the estimated average weight at the beginning of each interval by the number estimated for the population by the Murphy method. That is,

$\widehat{P^*_{ij}} = \hat{C}_{ij}\hat{w}^*_{ij}/\hat{E}_{ij}$ = estimated biomass to begin interval j, age i,

$\hat{w}^*_{ij}$ = estimated average weight for age i at beginning of interval j,

$\hat{C}_{ij}$ = estimated number of age i caught during interval j.

(2) Multiply the average number of age i alive during interval j by the average weight of age i individuals during this interval. That is,

$\hat{\hat{P}}_{ij} = \hat{\hat{N}}_{ij}\hat{\hat{w}}_{ij}$ = estimated average biomass of age i, during j.

$\hat{\hat{w}}_{ij}$ = average weight in the catch of age i, during interval j.

$\hat{\hat{N}}_{ij}$ = average number alive during interval j of age i.

For this study, the second method was used with average population numbers, $\hat{\hat{N}}_{ij}$, being given by

$\hat{\hat{N}}_{ij} = \hat{N}_{ij}(1-e^{-t_j\hat{Z}_{ij}})/(t_j\hat{Z}_{ij})$

$\hat{N}_{ij} = \hat{C}_{ij}/\hat{E}_{ij}$,

t_j = fraction of year elapsed during interval j; $t_j = 1/12$ for all intervals.

$\hat{Z}_{ij}$ = total annual instantaneous mortality coefficient during j.

Total population biomass for ages I through III was computed as

$$\hat{\hat{P}}_j = \sum_{i=1}^{3} \hat{\hat{N}}_{ij}\hat{\hat{w}}_{ij} = \text{average biomass available during interval } j\text{, and the}$$

catchability coefficient from

$$\hat{q}_j = \sum_{i=1}^{3} \hat{C}_{ij}\hat{\hat{w}}_{ij}/(\hat{\hat{P}}_j f_j);\ f_j \text{ is effort expended during interval } j.$$

Estimates of within-season monthly population biomass varied from a high of 12.0 million pounds in May 1955 to a low of 3.4 million pounds in October 1964 (Table 7). Population changes estimated by the Murphy method follow trends estimated by the Schaefer model (Figure 5), except Schaefer model estimates exhibit considerably less within season change. This difference in range of within season change was caused by the different ways in which the two models treat growth and recruitment. The Schaefer model assumes a continuous process for combined growth and recruitment, whereas the Murphy method treats growth as continuous (Figure 3) and recruitment as instantaneous (Table 7).

Estimates of monthly catchability (q) (Table 7) had extreme variation and showed an average within season increase (Figure 6). However, the within season changes were inconsistent and obscured by the variation. Monthly estimates of q varied from 21.3×10^{-5} in June 1968 to 3.8×10^{-5} in May 1955. Yearly averages had less variation and appeared to show no long-term trend (Figure 7). Average q over all months was about 9.0×10^{-5} which agreed closely with the value 8.5×10^{-5} used for the Schaefer model.

Spawning Biomass and Recruitment

Female spawning biomass consisted of all ages II and III shrimp plus some fraction of age I shrimp. Some data from commercial catch samples on the fraction of age I shrimp func-

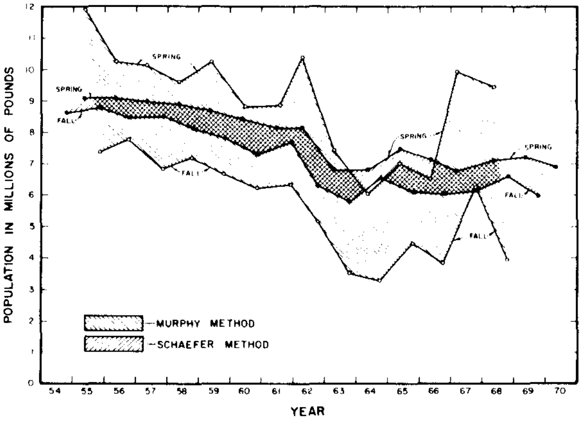

FIGURE 5.—Comparison of annual maximum and minimum population sizes as estimated by the Schaefer model and the Murphy method.

TABLE 7.—Ocean shrimp population biomass in thousands of pounds by age and month.

Year	Month	Ages			Total	Est. $q \times 10^6$
		I	II	III		
1955	May	9,100	2,670	265	12,035	38
	June	6,788	2,318	237	9,343	61
	July	7,119	2,051	208	9,378	91
	Aug.	6,538	1,731	181	8,450	89
	Sept.	6,467	1,481	157	8,105	51
	Oct.	5,942	1,300	140	7,382	41
1956	May	6,265	3,770	246	10,281	107
	June	6,385	3,008	212	9,605	86
	July	5,649	2,637	175	8,461	101
	Aug.	5,619	2,432	164	8,215	101
	Sept.	5,570	2,089	134	7,793	66
1957	May	6,701	3,425	67	10,193	68
	June	6,434	3,155	58	9,647	46
	July	6,229	2,940	51	9,220	51
	Aug.	6,067	2,625	42	8,734	95
	Sept.	5,405	2,207	34	7,646	82
	Oct.	4,872	1,975	29	6,876	118
1958	May	6,710	2,811	129	9,650	58
	June	6,739	2,501	108	9,348	80
	July	5,704	2,096	84	7,884	62
	Aug.	6,053	1,885	75	8,013	116
	Sept.	5,574	1,598	62	7,234	99
1959	May	6,650	3,372	256	10,278	113
	June	6,441	2,922	222	9,585	93
	July	5,895	2,459	176	8,530	92
	Aug.	5,337	2,084	146	7,567	86
	Sept.	4,830	2,795	119	6,744	121
1960	May	5,987	2,453	255	8,695	57
	June	6,217	2,419	276	8,912	62
	July	5,894	2,001	209	8,104	82
	Aug.	5,378	2,537	155	7,070	97
	Sept.	5,048	1,343	117	6,508	100
	Oct.	4,967	1,196	96	6,259	71
1961	June	5,875	2,737	172	8,784	105
	July	6,234	2,566	155	8,955	90
	Aug.	5,983	2,054	128	8,165	104
	Sept.	5,749	1,614	112	7,475	119
	Oct.	4,885	1,391	110	6,386	145
	Nov.	3,893	960	80	4,933	93
1962	Apr.	6,888	2,956	601	10,445	59
	May	6,714	2,631	518	9,863	73
	June	6,197	2,424	500	9,121	66
	July	5,764	2,076	404	8,244	63
	Aug.	5,519	1,660	404	7,583	73
	Sept.	5,358	1,095	232	6,685	106
	Oct.	4,265	785	168	5,218	177
1963	Apr.	4,101	3,000	358	7,459	84
	May	2,892	2,448	360	5,700	104
	June	2,830	2,008	227	5,065	95
	July	3,546	1,772	134	5,452	81
	Aug.	3,466	1,289	71	4,826	92
	Sept.	2,979	785	27	3,791	120
	Oct.	2,939	609	10	3,558	132
1964	May	3,790	1,970	317	6,077	83
	June	3,972	1,538	341	5,851	87
	July	3,349	1,438	278	5,065	92
	Aug.	3,363	1,177	222	4,762	123
	Sept.	2,542	908	183	3,633	145
	Oct.	2,378	810	167	3,355	148
1965	May	5,676	1,281	176	7,133	49
	June	4,764	1,147	134	6,045	75
	July	4,657	942	105	5,704	80
	Aug.	4,751	852	84	5,687	57
	Sept.	4,409	732	75	5,216	57
	Oct.	3,831	651	73	4,555	73
1966	May	4,481	1,999	140	6,620	67
	June	4,131	1,628	107	5,866	75
	July	4,191	1,035	48	5,274	92
	Aug.	3,804	799	43	4,646	76
	Sept.	3,754	645	40	4,439	73
	Oct.	3,332	533	39	3,904	103
1967	Mar.	7,401	2,371	293	10,065	51
	Apr.	6,853	2,246	274	9,373	57
	May	6,708	2,154	236	9,098	51
	June	6,566	1,895	174	8,635	70
	July	6,212	1,522	116	7,850	94
	Aug.	5,741	1,309	76	7,126	80
	Sept.	5,407	1,172	62	6,641	48
	Oct.	5,049	1,148	62	6,259	51
1968	May	3,105	5,991	480	9,476	95
	June	2,853	4,804	389	8,046	213
	July	2,636	3,971	347	6,954	154
	Aug.	2,295	3,219	286	5,800	100
	Sept.	1,694	2,474	192	4,360	134
	Oct.	1,667	2,116	170	3,953	155

tioning as females were made available from unpublished sources, but a good method for predicting the fraction of age I shrimp that would function as females was not found. Thus, a simple mean was computed from the data available for years 1957 through 1967 (Table 8). It is assumed that this mean proportion (0.33) predicts the fraction of the biomass of age I shrimp alive in September that will be females and that the sum of the September biomass of ages II and III, plus the fraction of age I functioning as females in September, is directly proportional to spawning biomass during the spawning season.

Recruitment was defined as the number of age I shrimp alive on May 1 of each year. Thus, the female biomass in September of season i is proportional to the biomass which will spawn sometime prior to May of season $i+1$ and the progeny of this spawning will be recruited to the fishery at the beginning of season $i+2$. Two

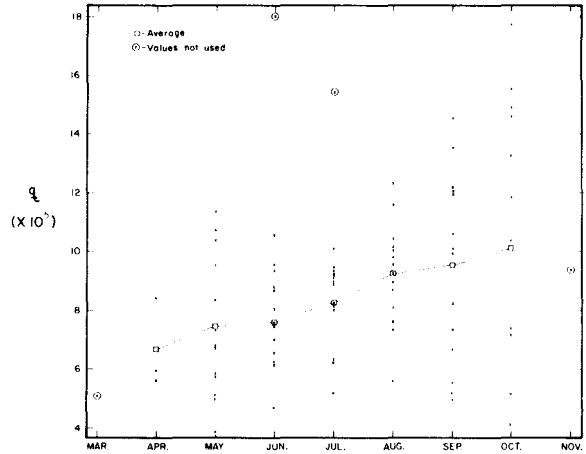

FIGURE 6.—Within-season changes in catchability coefficient (q). Line connects seasonal mean values. Circled points were not used to compute means.

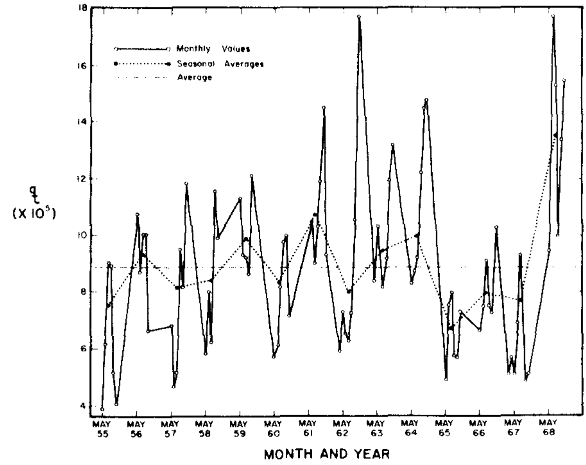

FIGURE 7.—Monthly catchability coefficients (q). Dashed line shows 1955-1968 mean.

TABLE 8.—Estimated percentage of numbers of shrimp functioning as spawners at age I.

Year	Percent	Year	Percent
1957	43	1963	21
1958	17	1964	36
1959	36	1965	51
1960	42	1966	24
1961	11	1967	54
1962	30	Mean	33

models for predicting recruitment from population biomass were tried.

$$\text{Model I}: \quad R_{i+2} = aS_i \, e^{-bS_i};$$

$$\text{Model II}: \quad R_{i+2} = cS_i \, e^{-dP_{i+1}}$$

where

R_{i+2} = number of age I shrimp on May 1, season $i+2$.

S_i = average biomass of functioning females during September of season i.

P_{i+1} = average total biomass (ages I, II, and III) during September of season $i+1$.

a, b, c, and d are constants.

Model I assumes the number of eggs produced is proportional to spawning biomass and that survival from egg to recruitment is influenced by this same spawning biomass. Model II assumes the number of eggs is proportional to spawning biomass and that survival from egg to recruit is a function of average biomass competing for the population space. September of season $i+1$ was selected for Model II because this seemed likely to be proportional to the average biomass encountered by age 0 shrimp, and data were available for all Septembers. May 1 was selected for recruitment since most seasons opened on this date.

Both models of recruitment were fitted by using transformations and a linear model (Paulik and Gales, 1965). The transformed equations are:

$$\text{Model I}: \quad \log_e(R_{i+2}/S_i) = \log_e(a) - bS_i;$$

$$\text{Model II}: \quad \log_e(R_{i+2}/S_i) = \log_e(c) - dP_{i+1}.$$

Estimates of recruitment by the Murphy method varied from a high of 1.5 billion shrimp on May 1, 1962, to a low of 0.6 billion on May 1, 1968. Spawning stocks producing recruitment varied from 4.5 million pounds in September 1959 to 1.8 million pounds in September 1963 (Table 9).

The range in recruitment observed at any given spawning stock size was very large relative to the range in size of spawning stock, and the fitting of Model I did not result in a meaningful

TABLE 9.—Recruits vs. spawners and population biomass.

Year (i)	$^1R_{i+2}$	^2S_i	$^3P_{i+1}$
1955	1,341.6	3,772	7,793
1956	1,491.0	4,061	7,646
1957	1,295.8	4,025	7,234
1958	1,508.4	3,499	6,744
1959	^{4}1,365.6	4,508	6,508
1960	1,502.0	3,126	7,475
1961	899.7	3,623	6,685
1962	685.3	3,095	3,791
1963	1,236.4	1,795	3,633
1964	947.1	1,930	5,216
1965	1,447.6	2,262	4,439
1966	613.9	1,924	6,641

[1] R_{i+2} = Recruits in millions on May 1 of year $i+2$.

[2] S_i = Spawners in thousands of pounds during September of year i.

[3] P_{i+1} = Population in thousands of pounds during September of year $i+1$.

[4] Estimated from June.

relationship (Figure 8). Model II, which considers the effect of the population competing with the prerecruits, did not account for the variation in recruitment either. Consequently, a realistic spawner-recruit relationship could not be determined from the available data.

Yield per Recruit

Because a well-defined spawner-recruit relationship could not be determined the use of a self-generating model of the dynamic pool type, such as Walters (1969), is not feasible. We can, however, utilize the age-structured catch data to examine this type of model under the assumption that recruitment is constant.

We feel that the greatest confidence can be placed in the estimates of instantaneous fishing mortality (F_{ij}) for 1961 through 1967 (Table 5). For this reason, these values were combined to yield average monthly values ($\bar{F}_{ij}$). The averages were computed as simple arithmetic means to give vectors of average fishing mortality by month and age for April through October (Table 10), and allow for computations of yield per recruit. Yield per million recruits was computed by step-wise integration (Ricker, 1958; Paulik and Bayliff, 1967). For a season of l months, a year class would be exposed to fishing for $n = 3l$ months and protected for $3(12-l)$ months. This would give a total lifetime after recruitment of $L=36$ months. The yield can be expressed as

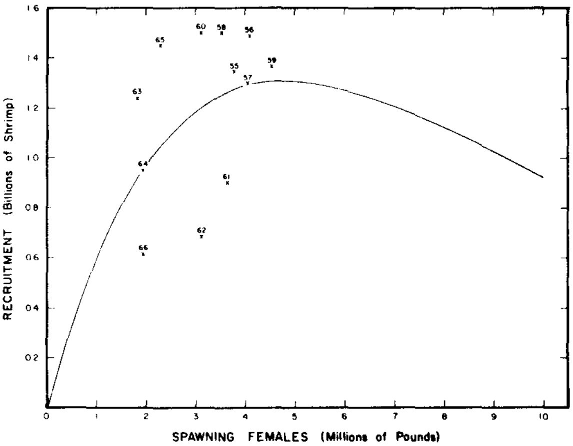

FIGURE 8.—Number of recruits on May 1 of year $i+2$ produced by spawning biomass of September, year i. Smooth curve represents Model I of the text.

$$Y = \sum_{k=1}^{L} y_k = \sum_{i=1}^{3} \sum_{j=1}^{12} C_{ij}\bar{w}_{ij};\; k = 12(i-1)+j$$

where

$\bar{w}_{ij}$ = average weight taken from the empirical growth curve,

C_{ij} = $L_k\, E_{ij}$ = number caught in month j of year i,

E_{ij} = $\bar{F}_{ij}(1-e^{-Z_k})/Z_k$ = monthly exploitation rate in month j of year i

Z_k = $(\bar{F}_{ij} + M)$ = total monthly instantaneous mortality (note that Z was previously used

TABLE 10.—Mean monthly instantaneous fishing mortality coefficients, $\bar{F}_{ij}$, by age group.

Month	Age group		
	I	II	III
April	0.005	0.024	0.049
May	.011	.067	.120
June	.022	.123	.162
July	.041	.175	.254
August	.037	.190	.247
September	.019	.133	.179
October	.009	.082	.068

to represent the annual mortality coefficient),

$L_k = R \exp\left[-\sum_{h=1}^{k-1} Z_h\right]$ = number survivors to begin interval k,

Z_h = M during months closed to fishing.

R = number of recruits = 1,000,000.

M = 0.12.

The yields at various levels of fishing intensity were predicted by multiplying all $\bar{F}_{ij}$ by a constant equal to the intensity change desired and recalculating catches for all months. Estimates of expected yield in numbers and expected average weight per shrimp were also provided by this procedure. By setting appropriate values of $F = 0$, yields for various seasons and entry ages were computed.

If $M = 0.12$ is the monthly instantaneous mortality coefficient and if growth in weight at age is taken from the empirical growth curve (Table 3), a year class of shrimp that is not fished will reach its maximum biomass during the period July to August as age I. The biomass will then decline rapidly and by July to August as age II it will be about one-half the maximum.

The estimated yield per recruit for the period 1961 to 1967 was 0.00165 lb. per shrimp. Since the average annual catch during that 7-year period was 1.918 million pounds, it would have required an average of 1.162 billion recruits on April 1 to support the catch. The Murphy method estimates an average recruitment on May 1 of about 1.155 billion. Thus, the analysis of yield per recruit is in good agreement with the Murphy method results.

Given 1.155 billion recruits on April 1, it would require a yield per recruit of 0.00216 lb. per shrimp to obtain a total harvest approximately equal to the maximum sustainable yield estimated from the Schaefer model. To have obtained a yield-per-recruit of 0.00216 during those 7 years would have required an increase in fishing mortality of about 75% (Figure 9). This additional yield could not have been obtained by shortening the season or changing age at recruitment unless a substantial increase in fishing mortality accompanied the changes (Figures 9 and 10). With the distribution of fishing effort observed during 1961-1967, the average total monthly instantaneous fishing mortality ($\sum \sum F_{ij}$) operating against a year class during 3 seasons was estimated to be 2.0176. While maintaining total fishing mortality at 2.0176,

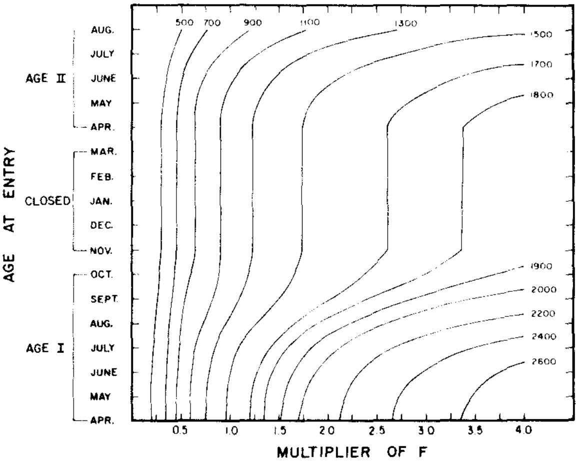

FIGURE 9.—Yield in pounds per million recruits as a function of age at entry into the fishery and fishing mortality. Fixed population parameters used were 1961-1967 means.

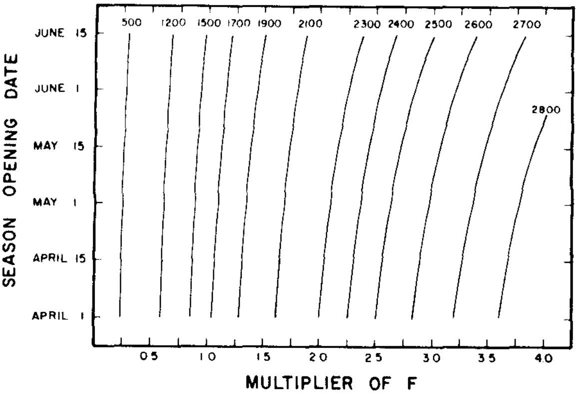

FIGURE 10.—Yield in pounds per million recruits as a function of season opening date and monthly fishing mortality coefficient. October 31 season closing date assumed and fixed population parameters used were 1961-1967 means.

the annual yield could theoretically be increased to about 2.8 million pounds by shifting fishing mortality so that I's and II's suffered equal rates. This would involve a 75% reduction in fishing mortality at ages II and III and assumes that the population with the new age structure would continue to produce 1.155 billion recruits. Such a change would also produce a reduction of 26% in the average size of shrimp in the catch and pose the problem of how the mortality pattern could be so altered.

INTER-MODEL COMPARISONS

Because we were unable to determine a spawner-recruit relationship and produce a self-generating form of the dynamic pool model, a realistic comparison of the results from the two types of models is not possible. In addition, the yield-per-recruit model treats natural mortality and growth parameters as constant while in the Schaefer model they are components of density dependent terms.

It is of interest to note that the biomass estimates obtained from the age-structured catch data by the Murphy method are in general agreement with the corresponding estimates of the Schaefer model. Although this does not compare the yield-per-recruit and Schaefer models, we feel it indicates some support of the Schaefer model from a semi-independent source. Another point of agreement between the yield-per-recruit and Schaefer models was that, given the average recruitment over the 1961-67 period, the former required a 75% increase in fishing mortality to produce the Schaefer model's maximum sustainable yield while the average annual effort expended during that period would require a 68% increase to reach the optimum effort level of the Schaefer model. However, as can be seen from Figure 9, maximum yield-per-recruit is predicted to occur at a much higher effort level under the previously mentioned assumption of constant parameters.

It seems clear from the foregoing discussion of results relative to the two models that management procedures should be based on the Schaefer model at the present time.

PROPOSED MANAGEMENT STRATEGY

Fitting an equation such as the Schaefer model to a set of actual catch and effort data may be viewed as merely an interesting exercise unless one has to make actual management recommendations based upon the results. Then the situation becomes somewhat sticky. It is obvious that a simple deterministic model such as Schaefer's will not precisely describe the dynamics of a fish population. At best, there will be fluctuations in recruitment, growth, and catchability which will cause some consternation to the manager attempting to use such a model.

In the case of the shrimp fishery, the management strategy we propose treats the Schaefer model estimates as exactly correct, responds to indicated deviations from the optimum population size in a relatively arbitrary but conservative manner, and integrates the Oregon and California fishing. This conservative strategy attempts a gradual reduction in the biomass when the model estimates it to be above P_{opt} and a rapid increase in the stock size when it is estimated to be below P_{opt}. To formulate this procedure, let Q be the catch quota (California + Oregon) and $C_e(P) = HP^2 - KP$ be the equilibrium yield obtainable from a population of size P. With $P(t)$ the population when the next fishing season commences,

$$Q = \frac{P(t) - P_{\text{opt}}}{2} + C_e\left(\frac{P(t) + P_{\text{opt}}}{2}\right); \quad P(t) > P_{\text{opt}},$$

$$Q = P(t) - P_{\text{opt}} + C_e\Big(P(t)\Big); \quad P(t) < P_{\text{opt}}.$$

When the model predicts the stock is in the surplus condition we are, then, proposing to harvest one-half of this surplus plus the predicted sustainable yield at the point midway between $P(t)$ and P_{opt}. A predicted stock deficit evokes a procedure which harvests the sustainable yield at $P(t)$ minus the amount by which the stock falls short of P_{opt}. For example, the 1970 California shrimp quota of 3.4 million pounds was set by the above method with $P(t) - P_{\text{opt}} =$

$7.1 - 4.8 = 2.3$ and C_e (5.9) $= 2.3$ for a recommended yield of 3.4 million pounds (Table 1). It was assumed the Oregon fleet's catch from Oregon waters would be negligible.

A more radical strategy such as harvesting all of the surplus stock could be employed, but the attendant risks would be higher. These risks would include a possible disturbance of whatever stability exists in the population, particularly with reference to age structure. It might also be argued that the observed catch-per-effort should be used to adjust $P(t)$ before making the quota calculation described above. Here again, a substantial risk would be involved if the observed catch-per-effort were much higher than the expected since with our methods it could not be determined whether such an anomaly was due to abnormal catchability or to a real increase in the stock size. The quota-setting procedure we recommended above does respond in a limited way to a higher than expected catch. Since the fishery operates under a quota, a catch-per-effort which is higher than expected will result in the quota being filled with a lower than expected amount of effort and usually in a shorter time period. An examination of (2) shows that this will increase $P(t)$ and thus result in a larger quota for the season beginning at time t.

This fishery must be carefully followed in the future to observe how well the model based upon current parameters describes the observed catch and effort pattern. An equation such as this which is fitted to data from only 16 years cannot be considered definitive from a statistical estimation viewpoint and, in addition, there is a chance the population parameters will actually be changing. For example, one cannot avoid speculating about the effect of the large Pacific coast hake fishery on the shrimp natural mortality rate. Since hake may be a substantial predator upon shrimp (Gotshall, 1969), a reduction of the hake population due to a large fishery might increase the abundance of ocean shrimp.

Beyond the technical management problems which we have discussed at length, there is the institutional problem of a single state attempting to manage an interstate fishery. While the catch from Oregon waters by Oregon-based vessels has usually been so small that it affects the population negligibly, at times it has been substantial. A sustained change in conditions could nullify the effect of California's quota mechanism.

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